Stat 13, Intro. to Statistical Methods for the Life and Health Sciences.

- 1. Some recommended book problems.
- 2. Blinding.
- 3. Portacaval shunt example.
- 4. Bradley Effect example.
- 5. Coverage, adherer bias and clofibrate example.
- 6. More about confounding factors.
- 7. Confounding and lefties example.
- 8. Comparing two proportions using numerical and visual summaries, good or bad year example.
- 9. Comparing 2 proportions with Cls + testing using simulation, dolphin example.
- 10. Comparing 2 props. with theory-based testing, smoking and gender example.
- 11. Five number summary, IQR, and geysers.

Read ch5 and 6. The midterm will be on ch 1-6.

A practice midterm is on the course website,

http://www.stat.ucla.edu/~frederic/13/sum23.

We will discuss it Tue and do review Tue.

Hw3 is due Thu Aug31, 10am.

Midterm is Thu Aug31, 10am-11:50am.

1. Some good hw problems from the book.

1.2.18, 1.2.19, 1.2.20, 1.3.17, 1.5.18, 2.1.38, 2.2.6, 2.2.24, 2.3.3, 2.3.25, 3.2.11, 3.2.12, 3.3.8, 3.3.19, 3.3.22, 3.5.23, 4.1.14, 4.1.18, 5.2.2, 5.2.10, 5.2.24, 5.3.11, 5.3.21, 5.3.24, 6.2.23, 6.3.1, 6.3.12, 6.3.22, 6.3.23.

### Blocking and Random Assignment

- The goal in random assignment is to make the two groups as similar as possible in all ways other than the treatment.
- Sometime there are known confounders and you can block on (control for) these variables.
- For example, if our subjects consist of 60% females and 40% males, we can force each group to be 60% female and 40% male, using a matched pair design.
- Blocking makes sense when there are known confounders you want to control for. But randomly assigning subjects to groups makes them as similar as possible in terms of unknown confounders.

### Blinding.

Even in experiments, the treatment and control groups can be different in ways other than the explanatory variable. This is especially true when the response variable is somewhat subjective. Pain is an example. One study found that 1/4 of patients suffering from post-operative pain, when

given a placebo (just a pill of sugar and water) claimed they experienced "significant prompt pain relief".

#### Blinding.

People might not be able to judge their own levels of pain very well, and may be influenced by the belief that they have taken an effective treatment.

Thus in an experiment with such a response variable, researchers should ensure the subject does not know whether he or she received the treatment or the control. This is called blinding.

In a *double-blind* experiment, neither the subject nor the researcher recording the response variable knows the level of the explanatory variable for each subject, i.e. treatment or control.

#### Portacaval shunt example.

The following example shows the importance of doing a randomized controlled experiment.

The portacaval shunt is a medical procedure aimed at curbing bleeding to death in patients with cirrhosis of the liver.

The following table summarizes 51 studies on the portacaval shunt. The poorly designed studies were very enthusiastic about the surgery, while the carefully designed studies prove that the surgery is largely ineffective.

	Degree of enthusiasm		
Design	High Moderate None		
No controls	24	7	1
Controls, but not randomized	10	3	2
Randomized controlled	0	1	3

#### Portacaval shunt example.

Why did the poorly designed studies come to the wrong conclusion?

A likely explanation is that in the studies where patients were not randomly assigned to the treatment or control group, by and large the healthier patients were given the surgery.

This alone could explain why the treatment group outlived the control group in these studies.

	Degre	ee of enthu	ısiasm
Design	High I	Moderate	None
No controls	24	7	1
Controls, but not randomized	10	3	2
Randomized controlled	0	1	3

# Cautions When Conducting Inference, and the controversial "Bradley Effect"

Example 3.5A

## The "Bradley Effect"

- Tom Bradley, long-time mayor of Los Angeles, ran as the Democratic Party's candidate for Governor of California in 1982.
  - Political polls of likely voters showed Bradley with a significant lead in the days before the election.
  - Exit polls favored Bradley significantly.
  - Many media outlets projected Bradley as the winner.
- Bradley narrowly lost the overall race.

# The "Bradley Effect"

- After the election, research suggested a smaller percentage of white voters had voted for Bradley than polls predicted.
- A very large proportion of undecided voters voted for Deukmejian.

## The "Bradley Effect"

- What are explanations for this discrepancy?
  - Likely voters answered the questions with a "social desirability bias".
  - They answered polling questions the way they thought the interviewer wanted them to.
- Discrepancies in polling and elections has since been called the "Bradley effect".
- It has been cited in numerous races and has included gender and other stances on political issues.

- In the 2008 New Hampshire democratic primary
  - Obama received 36.45% of the primary votes.
  - Clinton received 39.09%.
- This result shocked many since Obama seemed to hold a lead over Clinton.
- USA Today/Gallup poll days before the primary, n = 778.
  - 41% of likely voters said they would vote for Obama.
  - 28% of likely voters said they would vote for Clinton.
- How unlikely are the Clinton and Obama poll numbers given that 39.09% and 36.45% of actual primary voters voted for Clinton and Obama?

- We're assuming that the 778 people in the survey are a good representation of those who will vote.
  - The 778 people aren't a simple random sample.
- Pollsters used random digit dialing and asked if respondents planned to vote in the Democratic primary.
  - 9% (a total of 778) agreed to participate.
  - 319 said that they planned to vote for Obama and 218 for Clinton.

Suppose we make the following assumptions:

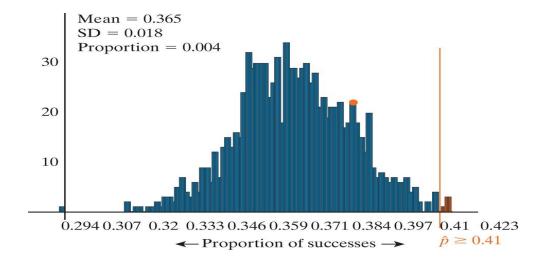
- 1. Random digit dialing is a reasonable way to get a sample of likely voters.
- 2. The 9% who participated are like the 91% who didn't.
- 3. Voters who said they planned to vote actually voted in the primary.
- 4. Answers to who they say they will vote for match who they actually vote for.

Then we expect the sample proportion roughly to agree with the final vote proportion.

- One question is whether the proportion of likely voters who say they will vote for Obama is the same as the proportion of likely voters who actually vote for Obama (observed on primary day to be 0.3645).
- What would the Bradley Effect do in this case?
  - The proportion who say they will vote for Obama would be larger than 0.3645.

- State the Null and Alternative hypotheses.
  - Null: The proportion of likely voters who would claim to vote for Obama is 0.3645.
  - Alternative: The proportion of likely voters who would claim to vote for Obama is higher than 0.3645.

- Simulation of 778 individuals randomly chosen from a population where 36.45% vote for Obama
- The chance of getting a sample proportion of 0.41 successes or higher is very small. 0.004.



- Convincing evidence that the discrepancy between what people said and how they voted is not explained by random chance alone.
- At least one of the 4 model assumptions is not true.

- 1. Random digit dialing is a reasonable way to get a sample of likely voters
  - Roughly equivalent to a SRS of New Hampshire residents who have a landline or cell phone
  - Slight over-representation of people with more than one phone

- 2. The 9% of individuals reached by phone who agree to participate are like the 91% who didn't
  - 91% includes people who didn't answer their phone and who didn't participate
  - Assumes that respondents are like nonrespondents.
  - The *response rate* was very low, but typical for phone polls
  - No guarantee that the 9% are representative.

- 3. Voters who said they plan to vote in the Democratic primary will vote in the primary.
  - There is no guarantee.
- 4. Respondent answers match who they actually vote for.

There is no guarantee.

Because of the wide disparity between polls and the primary, an independent investigation was done with the following conclusions:

- 1. People changed their opinion at the last minute
- 2. People in favor of Clinton were more likely not to respond
- 3. The Bradley Effect
- 4. Clinton was listed before Obama on every ballot These are examples of **nonrandom errors**.

# 5. Coverage, adherer bias and Clofibrate example.

Surveys are observational.

- Coverage is a common issue. Coverage is the extent to which
  the people you sampled from represent the overall population.
  A survey at a fancy research hospital in a wealthy neighborhood
  may yield patients with higher incomes, higher education, etc.
- Non-response bias is another common problem. Poor coverage means the people getting the survey do not represent the general population. Non-response bias means that out of the people you gave the survey to, the people actually filling it out and submitting it are different from the people who did not.
- Same exact issues in web surveys.

# Coverage, adherer bias, and Clofibrate example.

Non-response bias is similar to adherer bias, in experiments.

A drug called clofibrate was tested on 3,892 middle-aged men with heart trouble. It was supposed to prevent heart attacks.

1,103 assigned at random to take clofibrate,

2,789 to placebo (lactose) group.

Subjects were followed for 5 years.

Is this an experiment or an observational study?

Clofibrate patients who died during followup

adherers 15%

non-adherers 25%

total 20%

# Coverage, adherer bias, and Clofibrate example.

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1,103 assigned at random to take clofibrate,

2,789 to placebo (lactose) group.

Subjects were followed for 5 years.

Is this an experiment or an observational study?

It is an experiment. Does Clofibrate work?

Clofibrate patients who died during followup

adherers 15%

non-adherers **25**%

total 20%

adherers 15%

non-adherers 25%

total 20%

\_\_\_\_\_

#### Placebo

adherers 15%

nonadherers 28%

total 21%

Those who took clofibrate did much better than those who didn't keep taking clofibrate. Does this mean clofibrate works?

adherers 15%

non-adherers 25%

total 20%

\_\_\_\_\_

#### Placebo

adherers 15%

nonadherers 28%

total 21%

Those who adhered to placebo also did much better than those who stopped adhering.

adherers 15%

non-adherers 25%

total **20**%

\_\_\_\_\_

#### Placebo

adherers 15%

nonadherers 28%

total **21**%

All in all there was little difference between the two groups.

adherers 15%

non-adherers 25%

total **20**%

\_\_\_\_\_

#### Placebo

adherers 15%

nonadherers 28%

total **21**%

Adherers did better than non-adherers, not because of clofibrate, but because they were healthier in general. Why?

adherers 15%

non-adherers 25%

total **20**%

\_\_\_\_\_

#### Placebo

adherers 15%

nonadherers 28%

total **21**%

Adherers did better than non-adherers, not because of clofibrate, but because they were healthier in general. Why?

- adherers are the type to engage in healthier behavior.
- sick patients are less likely to adhere.

#### 6. More about confounding factors.

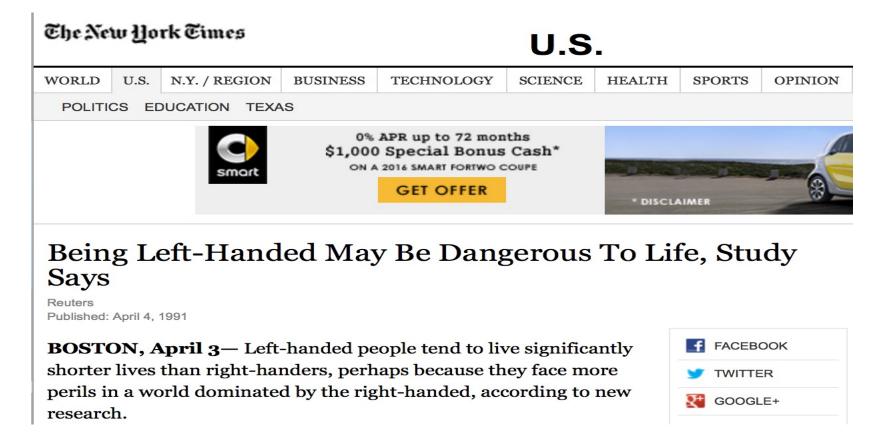
- By a confounding factor, we mean an alternative explanation that could explain the apparent relationship between the two variables, even if they are not causally related. Typically this is done by finding another difference between the treatment and control group. For instance, different studies have examined smokers and non-smokers and have found that smokers have higher rates of liver cancer. One explanation would be that smoking causes liver cancer. But is there any other, alternative explanation?
- One alternative would be that the smokers tend to drink more alcohol, and it is the alcohol, not the smoking, that causes liver cancer.

#### 6. More about confounding factors.

- Another plausible explanation is that the smokers are probably older on average than the non-smokers, and older people are more at risk for all sorts of cancer than younger people.
- Another might be that smokers engage in other unhealthy activities more than non-smokers.
- Note that if one said that "smoking makes you want to drink alcohol which causes liver cancer," that would not be a valid confounding factor, since in that explanation, smoking effective is causally related to liver cancer risk.

- A confounding factor must be plausibly linked to both the explanatory and response variables. So for instance saying "perhaps a higher proportion of the smokers are men" would not be a very convincing confounding factor, unless you have some reason to think gender is strongly linked to liver cancer.
- Another example: left-handedness and age at death.
   Psychologists Diane Halpern and Stanley Coren looked at 1,000 death records of those who died in Southern California in the late 1980s and early 1990s and contacted relatives to see if the deceased were righthanded or lefthanded. They found that the average ages at death of the lefthanded was 66, and for the righthanded it was 75. Their results were published in prestigious scientific journals, Nature and the New England Journal of Medicine.

All sorts of causal conclusions were made about how this shows that the stress of being lefthanded in our righthanded world leads to premature death.



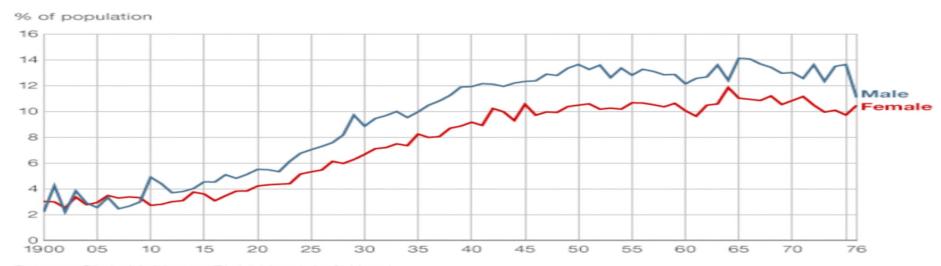
Is this an observational study or an experiment?

- Is this an observational study or an experiment?
   It is an observational study.
- Are there plausible confounding factors you can think of?

#### 7. Lefties example.

• A confounding factor is the age of the two populations in general. Lefties in the 1980s were on average younger than righties. Many old lefties were converted to righties at infancy, in the early 20th century, but this practice has subsided. Thus in the 1980s and 1990s, there were relatively few old lefties but many young lefties in the overall population. This alone explains the discrepancy.

#### Left handedness 1900-1976



Source: Chris McManus Right Hand, Left Hand

#### Unit 2. Comparing Two Groups

- In Unit 1, we learned the basic process of statistical inference using tests and confidence intervals. We did all this by focusing on a single proportion.
- In Unit 2, we will take these ideas and extend them to comparing two groups. We will compare two proportions, two independent means, and paired data.

8. Comparing two proportions using numerical and visual summaries, and the good or bad year example.

Section 5.1

# Example 5.1: Positive and Negative Perceptions

- Consider these two questions:
  - Are you having a good year?
  - Are you having a bad year?

• Do people answer each question in such a way that would indicate the same answer? (e.g. Yes for the first one and No for the second.)

#### Positive and Negative Perceptions

- Researchers questioned 30 students (randomly giving them one of the two questions).
- They then recorded if a positive or negative response was given.
- They wanted to see if the wording of the question influenced the answers.

#### Positive and negative perceptions

- Observational units
  - The 30 students
- Variables
  - Question wording (good year or bad year)
  - Perception of their year (positive or negative)
- Which is the explanatory variable and which is the response variable?
- Is this an observational study or experiment?

# Raw Data in a Spreadsheet

Individual	Type of Question	Response
1	Good Year	Positive
2	Good Year	Negative
3	Bad Year	Positive
4	Good Year	Positive
5	Good Year	Negative
6	Bad Year	Positive
7	Good Year	Positive
8	Good Year	Positive
9	Good Year	Positive
10	Bad Year	Negative
11	Good Year	Negative
12	Bad Year	Negative
13	Good Year	Positive
14	Bad Year	Negative
15	Good Year	Positive

Individual	Type of	Response
	Question	
16	Good Year	Positive
17	Bad Year	Positive
18	Good Year	Positive
19	Good Year	Positive
20	Good Year	Positive
21	Bad Year	Negative
22	Good Year	Positive
23	Bad Year	Negative
24	Good Year	Positive
25	Bad Year	Negative
26	Good Year	Positive
27	Bad Year	Negative
28	Good Year	Positive
29	Bad Year	Positive
30	Bad Year	Negative

#### Two-Way Tables

- A two-way table organizes data
  - Summarizes two categorical variables
  - Also called contingency table
- Are students more likely to give a positive response if they were given the good year question?

	Good Year	Bad Year	Total
Positive response	15	4	19
Negative response	3	8	11
Total	18	12	30

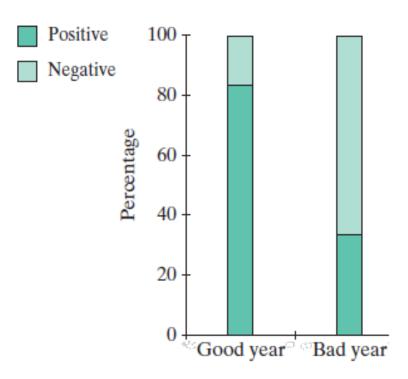
#### Two-Way Tables

- Conditional proportions will help us better determine if there is an association between the question asked and the type of response.
- We can see that the subjects with the positive question were *more likely* to respond positively.

	Good Year	Bad Year	Total
Positive response	<b>15/18</b> ≈ <b>0.83</b>	<b>4/12</b> ≈ <b>0.33</b>	19
Negative response	3	8	11
Total	18	12	30

### Segmented Bar Graphs

 We can also use segmented bar graphs to see this association between the "good year" question and a positive response.



#### **Statistic**

 The statistic we will mainly use to summarize this table is the difference in proportions of positive responses is 0.83 – 0.33 = 0.50.

	Good Year	Bad Year	Total
Positive response	15 (83%)	4 (33%)	19
Negative response	3	8	11
Total	18	12	30

#### **Another Statistic**

- Another statistic that is often used, called relative risk, is the ratio of the proportions: 0.83/0.33 = 2.5.
- We can say that those who were given the good year question were 2.5 times as likely to give a positive response.

	Good Year	Bad Year	Total
Positive response	15 (83%)	4 (33%)	19
Negative response	3	8	11
Total	18	12	30

Comparing two proportions with CIs and testing using simulation, dolphin example.

Section 5.2

Example 5.2

Is swimming with dolphins therapeutic for patients suffering from clinical depression?

- Researchers Antonioli and Reveley (2005), in British Medical Journal, recruited 30 subjects aged 18-65 with a clinical diagnosis of mild to moderate depression
- Discontinued antidepressants and psychotherapy 4 weeks prior to and throughout the experiment
- 30 subjects went to an island near Honduras where they were randomly assigned to two treatment groups

- Both groups engaged in one hour of swimming and snorkeling each day
- One group swam in the presence of dolphins and the other group did not
- Participants in both groups had identical conditions except for the dolphins
- After two weeks, each subjects' level of depression was evaluated, as it had been at the beginning of the study
- The response variable is whether or not the subject achieved substantial reduction in depression

Null hypothesis: Dolphins do not help.

 Swimming with dolphins is not associated with substantial improvement in depression

Alternative hypothesis: Dolphins help.

 Swimming with dolphins increases the probability of substantial improvement in depression symptoms

- The parameter is the (long-run) difference between the probability of improving when receiving dolphin therapy and the prob. of improving with the control ( $\pi_{\text{dolphins}}$   $\pi_{\text{control}}$ )
- So we can write our hypotheses as:

```
\begin{aligned} &\mathbf{H_0:} \ \pi_{\text{dolphins}} - \pi_{\text{control}} = 0. \\ &\mathbf{H_a:} \ \pi_{\text{dolphins}} - \pi_{\text{control}} > 0. \\ &\mathbf{or} \\ &\mathbf{H_0:} \ \pi_{\text{dolphins}} = \pi_{\text{control}} \\ &\mathbf{H_a:} \ \pi_{\text{dolphins}} > \pi_{\text{control}} \end{aligned}
```

(Note: we are not saying our parameters equal any certain number.)

#### **Results:**

	Dolphin group	Control group	Total
Improved	10 (66.7%)	3 (20%)	13
Did Not Improve	5	12	17
Total	15	15	30

The difference in proportions of improvers is:

$$\hat{p}_d - \hat{p}_c = 0.667 - 0.20 = 0.467.$$

- There are two possible explanations for an observed difference of 0.467.
  - A tendency to be more likely to improve with dolphins (alternative hypothesis)
  - The 13 subjects were going to show improvement with or without dolphins and random chance assigned more improvers to the dolphins (null hypothesis)

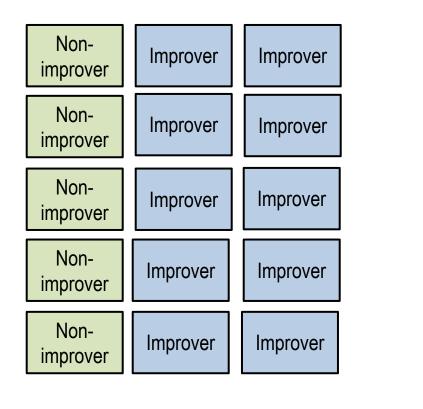
- If the null hypothesis is true (no association between dolphin therapy and improvement) we would have 13 improvers and 17 non-improvers regardless of the group to which they were assigned.
- Hence the assignment doesn't matter and we can just randomly assign the subjects' results to the two groups to see what would happen under a true null hypothesis.

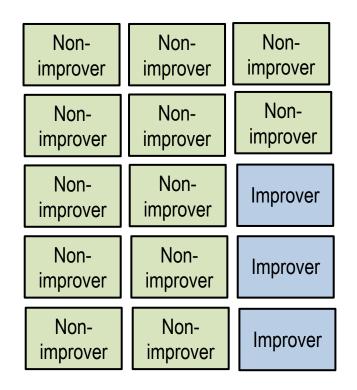
- We can simulate this with cards
  - 13 cards to represent the improvers
  - 17 cards represent the non-improvers
- Shuffle the cards
  - put 15 in one pile (dolphin therapy)
  - put 15 in another (control group)

- Compute the proportion of improvers in the Dolphin Therapy group
- Compute the proportion of improvers in the Control group
- The difference in these two proportions is what could just as well have happened under the assumption there is no association between swimming with dolphins and substantial improvement in depression.

# **Dolphin Therapy**

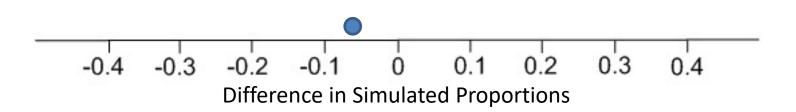
#### Control





60.0% Improvers

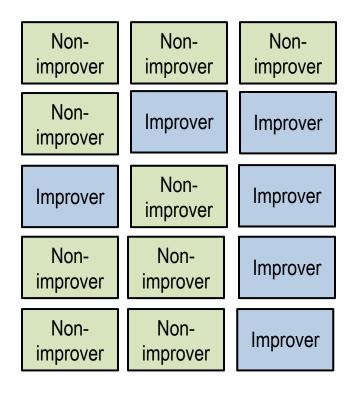
**20.0%** Improvers

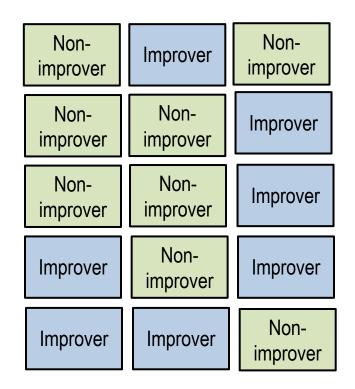


0.400 - 0.467 = -0.067

# **Dolphin Therapy**

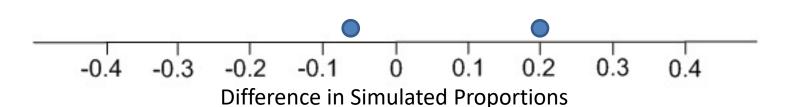
#### Control







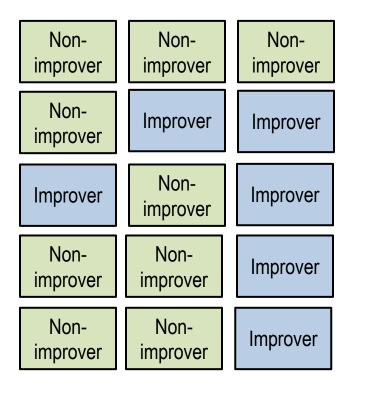
38.3% Improvers

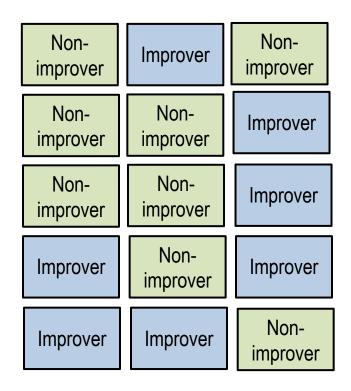


0.533 - 0.333 = 0.200

# **Dolphin Therapy**

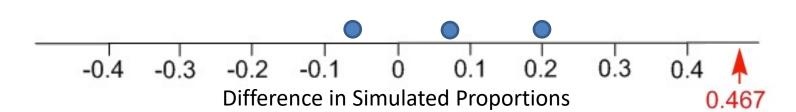
#### Control





**\$B.3%** Improvers

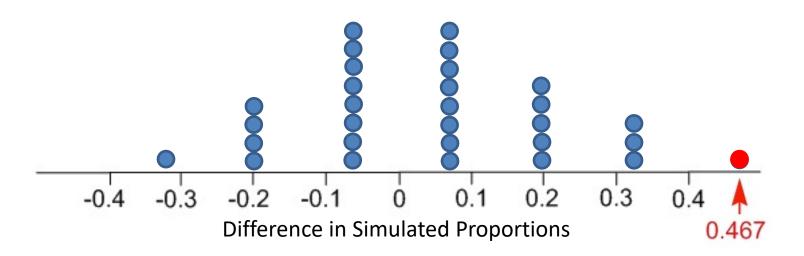
ቆፀ.ፀ% Improvers



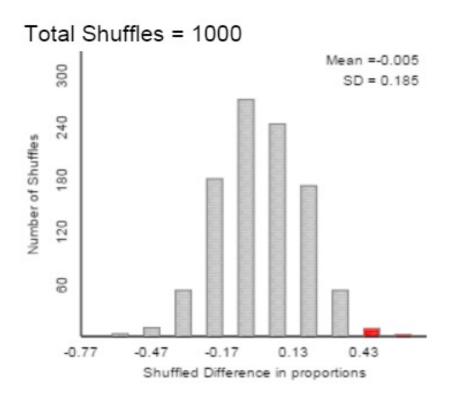
0.467 - 0.400 = 0.067

#### **More Simulations**

Only one simulated statistics out of 30 was as large or larger than our observed difference in proportions of 0.467, hence our p-value for 0.200 this null distribution is  $1/30 \approx 0.037$ 



We did 1000 repetitions to develop a null distribution.



- 13 out of 1000 results had a difference of 0.467 or higher (p-value = 0.013).
- 0.467 is  $\frac{0.467-0}{0.185} \approx 2.52$  SE above zero.
  - Using either the p-value or standardized statistic, we have strong evidence against the null and can conclude that the improvement due to swimming with dolphins was statistically significant.

- A 95% confidence interval for the difference in the probability using the standard error from the simulations is  $0.467 \pm 1.96(0.185) = 0.467 \pm 0.363$ , or (.104, .830).
- We are 95% confident that when allowed to swim with dolphins, the probability of improving is between 0.104 and 0.830 higher than when no dolphins are present.
- How does this interval back up our conclusion from the test of significance?

- Can we say that the presence of dolphins caused this improvement?
  - Since this was a randomized experiment, and assuming everything was identical between the groups, we have strong evidence that dolphins were the cause
- Can we generalize to a larger population?
  - Maybe mild to moderately depressed 18-65 year old patients willing to volunteer for this study
  - We have no evidence that random selection was used to find the 30 subjects. "Outpatients, recruited through announcements on the internet, radio, newspapers, and hospitals."

Comparing two proportions: Theory-Based Approach, and smoking and gender example.

Section 5.3

#### Introduction

- Just as with a single proportion, we can often predict results of a simulation using a theory-based approach.
- The theory-based approach also gives a simpler way to generate a confidence intervals.
- The main new mathematical fact to use is the SE for the difference between two proportions is

$$\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}.$$

# Parents' Smoking Status and their Babies' Gender

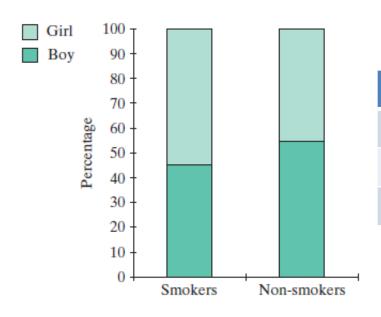
Example 5.3

#### **Smoking and Gender**

- How does parents' behavior affect the gender of their children?
- Fukuda et al. (2002) found the following in Japan.
  - Out of 565 births where both parents smoked more than a pack a day, 255 were boys. This is 45.1% boys.
  - Out of 3602 births where both parents did not smoke,
     1975 were boys. This 54.8% boys.
  - In total, out of 4170 births, 2230 were boys, which is 53.5%.
- Other studies have shown a reduced male to female birth ratio where high concentrations of other environmental chemicals are present (e.g. industrial pollution, pesticides)

# **Smoking and Gender**

- A segmented bar graph and 2-way table
- Let's compare the proportions to see if the difference is statistically significantly.



	<b>Both Smoked</b>	Neither Smoked
Boy	255 (45.1%)	1,975 (54.8%)
Girl	310	1,627
Total	565	3,602

#### **Null Hypothesis:**

- There is no association between smoking status of parents and sex of child.
- The probability of having a boy is the same for parents who smoke and don't smoke.
- $\pi_{\text{smoking}}$   $\pi_{\text{nonsmoking}}$  = 0

#### **Alternative Hypothesis:**

- There is an association between smoking status of parents and sex of child.
- The probability of having a boy is not the same for parents who smoke and don't smoke
- $\pi_{\text{smoking}} \pi_{\text{nonsmoking}} \neq 0$

- What are the observational units in the study?
- What are the variables in this study?
- Which variable should be considered the explanatory variable and which the response variable?
- What is the parameter of interest?
- Can you draw cause-and-effect conclusions for this study?

Using the 3S Strategy to asses the strength

#### 1. Statistic:

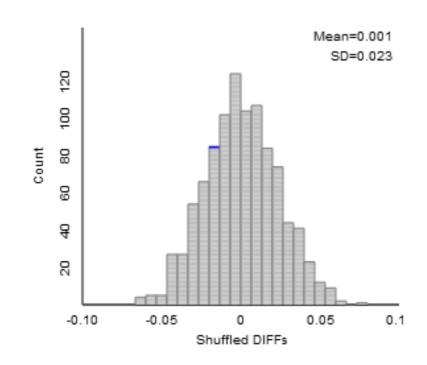
• The proportion of boys born to nonsmokers minus the proportion of boys born to smokers is 0.548 - 0.451 = 0.097.

#### 2. Simulate:

- Many repetitions of shuffling the 2230 boys and 1937 girls to the 565 smoking and 3602 nonsmoking parents
- Calculate the difference in proportions of boys between the groups for each repetition.
- Shuffling simulates the null hypothesis of no association

#### 3. Strength of evidence:

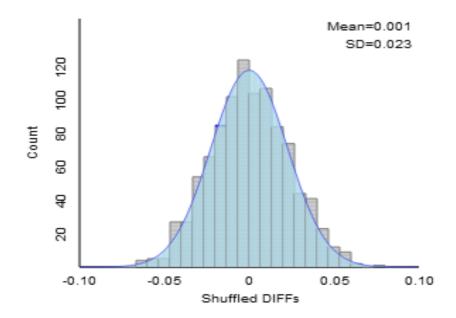
- Nothing as extreme as our observed statistic (≥ 0.097 or ≤ -0.097) occurred in 5000 repetitions,
- How many SEs is 0.097
   above the mean?
   Z = 0.097/0.023 = 4.22
   using simulations. What about using the theory-based approach?



Count Samples Beyond .097

Count = 0/1000 (0.0000)

- Notice the null distribution is centered at zero and is bell-shaped.
- This can be approximated by the normal distribution.



## **Formulas**

• The theory-based approach yields z = 4.30.

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

• Here 
$$z = \frac{.548 - .451}{\sqrt{.535 (1 - .535) \left(\frac{1}{3602} + \frac{1}{565}\right)}} = 4.30.$$

• p-value is 2\*(1-pnorm(4.30)) = 0.00171%.

- Fukuda et al. (2002) found the following in Japan.
  - Out of 3602 births where both parents did not smoke,
     1975 were boys. This 54.8% boys.
  - Out of 565 births where both parents smoked more than a pack a day, 255 were boys. This is 45.1% boys.
  - In total, out of 4170 births, 2230 were boys, which is 53.5% boys.

## **Formulas**

 How do we find the margin of error for the difference in proportions?

Multiplier 
$$\times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- The multiplier is from the normal distribution and is dependent upon the confidence level.
  - 1.645 for 90% confidence
  - 1.96 for 95% confidence
  - 2.576 for 99% confidence
- We can write the confidence interval in the form:
  - statistic ± margin of error.

- Our statistic is the observed sample difference in proportions, 0.097.
  - Plugging in 1.96 ×  $\sqrt{\left(\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}\right)} = 0.044$ , we get 0.097 ± 0.044 as our 95% CI.
  - We could also write this interval as (0.053, 0.141).
- We are 95% confident that the probability of a boy baby where neither family smokes minus the probability of a boy baby where both parents smoke is between 0.053 and 0.141.

## A clarification on the formulas

The margin of error for the difference in proportions is

Multiplier × SE, where SE = 
$$\sqrt{\left(\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}\right)}$$

In testing, the null hypothesis is no difference between the two groups, so we used the SE

$$\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}\right)}$$

where  $\hat{p}$  is the proportion in both groups combined. But in

Cls, we use the formula  $\sqrt{\left(\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}\right)}$  because we are not assuming  $\hat{p}_1=\hat{p}_2$  with CIs.

 How would the interval change if the confidence level was 99%?

• The SE = 
$$\sqrt{\left(\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}\right)} = .0224.$$

- Previously, for a 95% CI, it was  $0.097 \pm 1.96 \times .0224 = 0.097 \pm 0.044$ .
- For a 99% CI, it is  $0.097 \pm 2.576 \times .0224$  =  $0.097 \pm 0.058$ .

 Written as the statistic ± margin of error, the 99% CI for the difference between the two proportions is

 $0.097 \pm 0.058$ .

- Margin of error
  - 0.058 for the 99% confidence interval
  - 0.044 for the 95% confidence interval

How would the 95% confidence interval change if we were estimating

$$\pi_{\rm smoker} - \pi_{\rm nonsmoker}$$

instead of

$$\pi_{\text{nonsmoker}} - \pi_{\text{smoker}}$$
?

- (-0.141, -0.053) or  $-0.097 \pm 0.044$  instead of
- (0.053, 0.141) or  $0.097 \pm 0.044$ .

 The negative signs indicate the probability of a boy born to smoking parents is lower than that for nonsmoking parents.

#### **Validity Conditions of Theory-Based**

- Same as with a single proportion.
- Should have at least 10 observations in each of the cells of the 2 x 2 table.

	Smoking Parents	Non- smoking Parents	Total
Male	255	1975	2230
Female	310	1627	1937
Total	565	3602	4167

- The strong significant result in this study yielded quite a bit of press when it came out.
- Soon other studies came out which found no relationship between smoking and gender (Parazinni et al. 2004, Obel et al. 2003).
- James (2004) argued that confounding variables like social factors, diet, environmental exposure or stress were the reason for the association between smoking and gender of the baby. These are all confounded since it was an observational study. Different studies could easily have had different levels of these confounding factors.

# Five number summary, IQR, and geysers.

6.1: Comparing Two Groups: Quantitative Response

6.2: Comparing Two Means: Simulation-Based Approach

6.3: Comparing Two Means: Theory-Based Approach

# **Exploring Quantitative Data**

Section 6.1

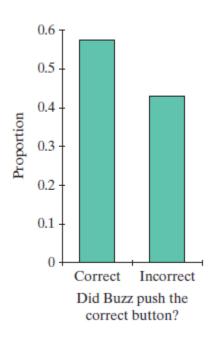
## Quantitative vs. Categorical Variables

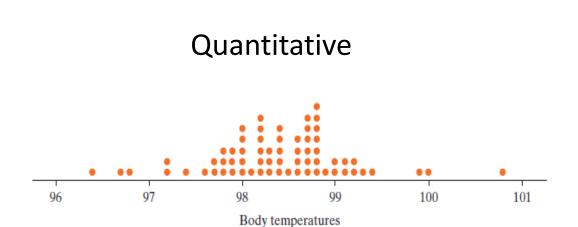
- Categorical
  - Values for which arithmetic does not make sense.
  - Gender, ethnicity, eye color...

- Quantitative
  - You can add or subtract the values, etc.
  - Age, height, weight, distance, time...

## Graphs for a Single Variable

#### Categorical

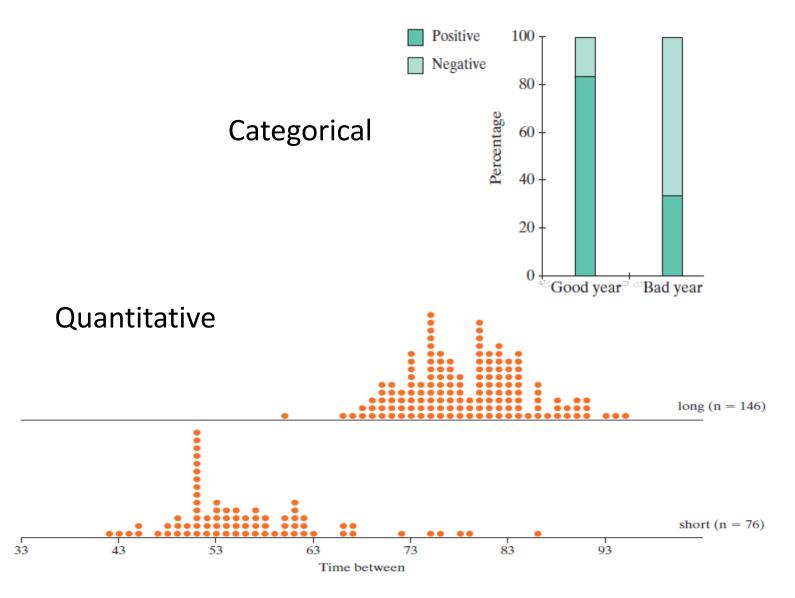




Bar Graph

**Dot Plot** 

## Comparing Two Groups Graphically



#### **Notation Check**

#### **Statistics**

- $\bar{x}$  Sample mean
- $\hat{p}$  Sample proportion.

#### **Parameters**

- $\mu$  Population mean
- $\pi$  Population proportion or probability.

Statistics summarize a sample and parameters summarize a population

## Quartiles

- Suppose 25% of the observations lie below a certain value x. Then x is called the *lower quartile* (or 25<sup>th</sup> percentile).
- Similarly, if 25% of the observations are greater than x, then x is called the *upper quartile* (or 75<sup>th</sup> percentile).
- The lower quartile can be calculated by finding the median, and then determining the median of the values below the overall median. Similarly the upper quartile is median{x<sub>i</sub>: x<sub>i</sub> > overall median}.

# IQR and Five-Number Summary

- The difference between the quartiles is called the *inter-quartile range* (IQR), another measure of variability along with standard deviation.
- The five-number summary for the distribution of a quantitative variable consists of the minimum, lower quartile, median, upper quartile, and maximum.
- Technically the IQR is not the interval (25th percentile, 75<sup>th</sup> percentile), but the difference 75<sup>th</sup> percentile 25<sup>th</sup>.
- Different software use different conventions, but we will use the convention that, if there is a range of possible quantiles, you take the middle of that range.
- For example, suppose data are 1, 3, 7, 7, 8, 9, 12, 14.
- $M = 7.5, 25^{th}$  percentile = 5,  $75^{th}$  percentile = 10.5. IQR = 5.5.

# IQR and Five-Number Summary

- For medians and quartiles, we will use the convention, if there is a range of possibilities, take the middle of the range.
- In R, this is type = 2. type = 1 means take the minimum.
- x = c(1, 3, 7, 7, 8, 9, 12, 14)
- quantile(x,.25, type=2) ## 5.5
- IQR(x,type=2) ## 5.5
- IQR(x,type=1) ## 6. Can you see why?

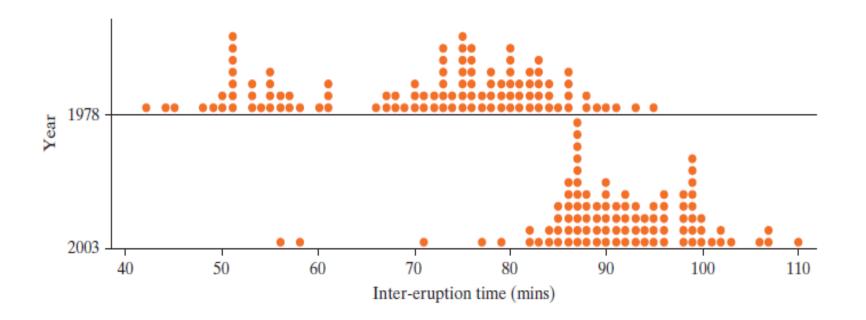
- For example, suppose data are 1, 3, 7, 7, 8, 9, 12, 14.
- $M = 7.5, 25^{th}$  percentile = 5,  $75^{th}$  percentile = 10.5. IQR = 5.5.

# **Geyser Eruptions**

Example 6.1

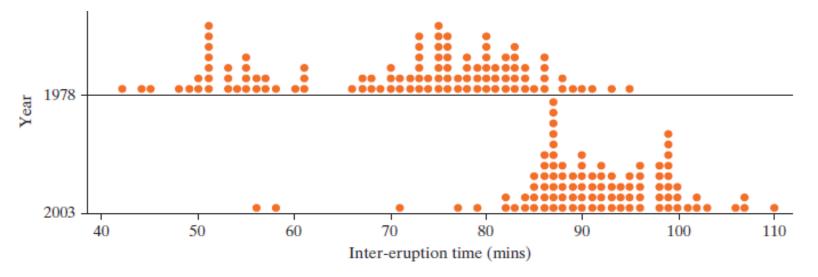
## Old Faithful Inter-Eruption Times

 How do the five-number summary and IQR differ for inter-eruption times between 1978 and 2003?



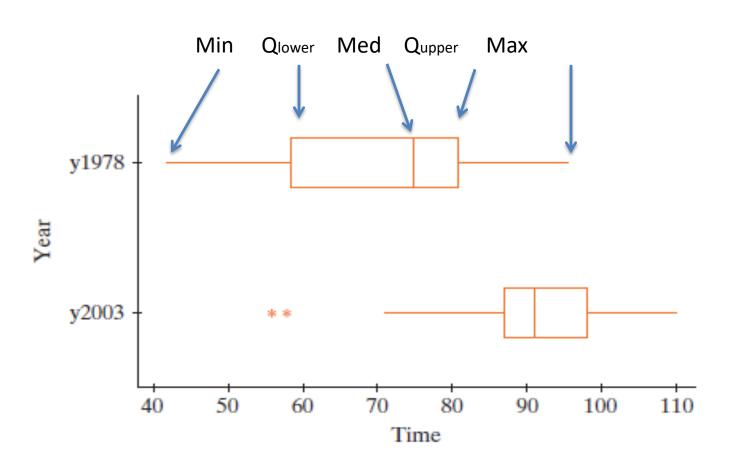
## Old Faithful Inter-Eruption Times

	Minimum	Lower quartile	Median	Upper quartile	Maximum		
1978 times	42	58	75	81	95		
2003 times	56	87	91	98	110		



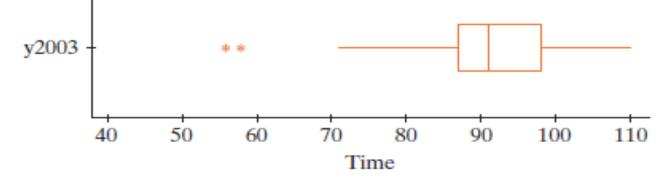
- 1978 IQR = 81 58 = 23
- 2003 IQR = 98 87 = 11

# **Boxplots**



# **Boxplots** (Outliers)

- A data value that is more than 1.5 × IQR above the upper quartile or below the lower quartile is considered an outlier.
- When these occur, the whiskers on a boxplot extend out to the farthest value not considered an outlier and outliers are represented by a dot or an asterisk.



## Cancer Pamphlet Reading Levels

- Short et al. (1995) compared reading levels of cancer patients and readability levels of cancer pamphlets. What is the:
  - Median reading level?
  - Mean reading level?
- Are the data skewed one way or the other?

Pamphlets' readability levels	6	7	8	9	10	11	12	13	14	15	16	Total
Count (number of pamphlets)	3	3	8	4	1	1	4	2	1	2	1	30

- Skewed a bit to the right
- Mean to the right of median

