

4.4. Luck and skill in Texas Hold'em.

The determination of whether Texas Hold'em is primarily a game of luck or skill has recently become the subject of intense legal debate. Complicating things is the fact that the terms *luck* and *skill* are extremely difficult to define. Surprisingly, rigorous definitions of these terms appear to have eluded most books and journal articles on game theory. A few articles have defined skill in terms of the variance in results among different players, with the idea that players should perform more similarly if a game is mostly based on luck, but their results might differ more substantially if the game is based on skill.

Another definition of skill is the extent to which players can improve, and it has been shown that poker does indeed possess a significant amount of this feature (e.g. Dedonno and Detterman, 2008). Others have defined skill in terms of the variation in a given player's results, since less variation would indicate that fewer repetitions are necessary in order to determine the statistical significance of one's long-term edge in the game, and hence the sooner one can establish that one's average profits or losses are primarily due to skill rather than short term luck.

These definitions above are obviously extremely problematic for various reasons. One is that both definitions rely on the game in question being played repeatedly before even a crude assessment of luck or skill could be made. More importantly, there are many contests of skill wherein the differences between players are small, or where one's results vary wildly. For instance, in Olympic trials of the 100-meter sprints, the differences between finishers are typically quite small, often just hundredths of a second. This hardly

implies that the results are based on luck. There are also sporting events where an individual's results might vary widely from one day to another, e.g. pitching in baseball, but that hardly means that luck plays a major role.

Regarding the quantification of the amount of luck or skill in a particular game of poker, a possibility might be to define luck as equity gained when cards are dealt by the dealer, and skill as equity gained by one's actions during the betting rounds. (Recall that *equity* was defined in Section 4.3 as the product *cp*.) That is, there are several reasons you might gain equity during a hand:

- * The cards dealt by the dealer (whether the players' hole cards or the flop, turn, or river) give you a greater chance of winning the hand in a showdown,
- * The size of the pot is increased while your chance to win the hand in a showdown is better than that of your opponent(s).
- * By betting, you get others to fold and thus win a pot that you might otherwise have lost.

Certainly, anyone would characterize the first case as luck, unless perhaps one believes in ESP or time travel. Thus, a possible way to estimate the skill in poker can be obtained by looking at the second and third cases above. That is, we may view one's skill as being comprised of the equity that one gains during the betting rounds, whereas luck is equity gained by the deal of the cards. The nice thing about this is that it is easily quantifiable, and one may dissect a particular poker game and analyze how much equity each player gained due to luck or skill.

There are obvious objections to this. First, why equity? One's equity (which is sometimes called *express equity*) in a pot is defined as one's expected return from the pot given no

future betting, and the assumption of no future betting may seem absurdly simplistic and unrealistic. On the other hand, unlike *implied equity* which would account for betting on future betting rounds, express equity is unambiguously defined and easy to compute. Second, situations can occur where one would expect a terrible player to gain equity during the betting rounds against even the greatest player in the world, so to attribute such equity gains to *skill* might be objectionable. For instance, in heads up Texas Hold'em, if the two players are dealt AA and KK, one would expect the player with KK to put a great deal of chips in while way behind, and this situation seems more like bad luck for the player with KK than any deficit in skill. One possible response to this objection is that skill is difficult to define, and in fact most poker players, probably due to their huge and fragile egos, tend to chalk up losses for virtually any reason as being solely due to bad luck. In some sense, anything can be attributed to luck if one has a general enough definition of the word. Even if a player does an amazingly skillful poker play, such as folding a very strong hand because of an observed tell or betting pattern, one could argue that it was lucky that the player happened to observe that tell, or even that the player was lucky to have been born with the ability to discern the tell. On the other hand, situations like the AA versus KK example truly do seem like bad luck. It is difficult to think of any remedy to this problem. It may be that the word *skill* is too strong a word, and that while it may be of interest to analyze hands in terms of equity, one should instead use the term *equity gained during the betting rounds* rather than skill in what follows.

Below is an extended example intended to illustrate the division of luck and skill in a given game of Texas Hold'em. The example involves the end of a tournament on Poker

After Dark televised during the first week of October 2009. Dario Minieri and Howard Lederer were the final two players. Since this portion of the tournament involves only these two players, and since nearly all hands were televised, this example provides us with an opportunity to attempt to parse out how much of Lederer's win was due to skill and how much was due to luck.

Technical side note: Before we begin, we need to clarify a couple of potential ambiguities. There is some ambiguity in the definition of equity before the flop, since the small and big blind have put in different amounts of chips. The definition used here is that preflop equity is the expected profit (equity one would have in the pot after calling minus cost), assuming the big blind and small blind call as well, or the equity one would have by folding, whichever happens to be greater. For example, in heads up Texas Hold'em with blinds of 800 and 1600, the preflop equity for the big blind is $2bp - 1600$, and $\max\{2bp - 1600, -800\}$ for the small blind, where p is the probability of winning the pot in a showdown, and b is the amount of the big blind. Define increases in the size of the pot as relative to the big blind, i.e. increasing the pot size by calling preflop does not count as skill. The probability p of winning the hand in a showdown was obtained using the odds calculator at cardplayer.com, and the probability of a tie is divided equally among the two players in determining p .

Example 4.4.1. Below are summaries of all 27 hands shown on Poker After Dark in October 2009 between Dario Minieri and Howard Lederer in the Heads Up portion of the tournament, with each hand's equity gains and losses broken down as luck or skill. Each hand is analyzed from Minieri's perspective, i.e. "skill -100" refers to 100 chips of equity

gained by Lederer during a betting round. The question we seek to address is: how much of Lederer's win was due to skill, and how much of it was due to luck?

For example, here is a detailed breakdown of hand 4, where the blinds were 800/1600, Minieri was dealt $A\clubsuit J\clubsuit$, Lederer had $A\heartsuit 9\heartsuit$, Minieri raised to 4300 and Lederer called. The flop was $6\clubsuit 10\spadesuit 10\clubsuit$, Lederer checked, Minieri bet 6500, and Lederer folded.

a) Preflop dealing (luck): Minieri +642.08. Minieri was dealt a 70.065% probability of winning the pot in a showdown, so Minieri's increase in equity is $70.065\% \times 3200 - 1600 = 642.08$ chips. Lederer was dealt a 29.935% probability to win the pot in a showdown, so his increase in equity is $29.935\% \times 3200 - 1600 = -642.08$ chips.

b) Preflop betting (skill): Minieri +1083.51. The pot was increased to 8600. $8600 - 3200 = 5400$. Minieri paid an additional 2700 but had $70.065\% \times 5400 = 3783.51$ additional equity, so Minieri's expected profit due to betting was $3783.51 - 2700 = 1083.51$ chips. Correspondingly, Lederer's expected profit due to betting was -1083.51 chips, since $29.935\% \times 5400 - 2700 = -1083.51$.

c) Flop dealing (luck): Minieri +1362.67. After the flop was dealt, Minieri's probability of winning the 8600 chip pot in a showdown went from 70.065% to 85.91%. So by luck, Minieri increased his equity by $(85.91\% - 70.065\%) \times 8600 = +1362.67$ chips.

d) Flop betting (skill): Minieri +1211.74. Because of the betting on the flop, Minieri's equity went from 85.91% of the 8600 pot to 100% of the pot, so Minieri increased his equity by $(100\% - 85.91\%) \times 8600 = 1211.74$ chips.

So during the hand, by luck, Minieri increased his equity by $642.08 + 1362.67 = 2004.75$ chips. By skill, Minieri increased his equity by $1083.51 + 1211.74 = 2295.25$ chips.

Notice that the total = $2004.75 + 2295.25 = 4300$, which is the number of chips Minieri won from Lederer in the hand.

Note that before the heads-up battle began, the broadcast reported that Minieri had 72,000 chips, and Lederer 48,000. Minieri must have won some chips in hands they did not televise, because the grand total has Minieri losing about 74,500 chips.

(Blinds 800 and 1600.)

Hand 1. Lederer $A\clubsuit 7\spadesuit$, Minieri $6\spadesuit 6\diamondsuit$. Lederer 43.535%, Minieri 56.465%. Lederer raises to 4300. Minieri raises to 47800. Lederer folds.

Luck +206.88. Skill +4093.12.

Hand 2. Minieri $4\spadesuit 2\diamondsuit$, Lederer $K\spadesuit 7\heartsuit$. Minieri 34.36%, Lederer 65.64%. Minieri raises to 4300, Lederer raises all in for 43500, Minieri folds.

Luck -500.48. Skill -3799.52.

Hand 3. Lederer $6\heartsuit 3\diamondsuit$, Minieri $A\diamondsuit 9\clubsuit$. Lederer 34.965%, Minieri 65.035%. Lederer folds in the small blind.

Luck +481.12. Skill +318.88.

Hand 4. Minieri $A\clubsuit J\clubsuit$, Lederer $A\heartsuit 9\heartsuit$. Minieri 70.065%, Lederer 29.935%. Minieri raises to 4300, Lederer calls 2700. Flop $6\clubsuit 10\spadesuit 10\clubsuit$. Minieri 85.91%, Lederer 14.09%.

Lederer checks, Minieri bets 6500, Lederer folds.

Luck +2004.75. Skill +2295.25.

Hand 5. Lederer 5♠ 3♥, Minieri 7♦ 6♠. Lederer 35.765%, Minieri 64.235%. Lederer folds in the small blind.

Luck +455.52. Skill +344.48.

Hand 6. Minieri K♥ 10♣, Lederer 5♣ 2♣. Minieri 61.41%, Lederer 38.59% Minieri raises to 3200, Lederer raises to 9700, Minieri folds.

Luck +365.12. Skill -3565.12

Hand 7. Minieri 10♦ 7♠, Lederer Q♣ 2♥. Minieri 43.57%, Lederer 56.43%. Minieri raises to 3200, Lederer calls 1600. Flop 8♠ 2♠ Q♥. Minieri 7.27%, Lederer 92.73%. Lederer checks, Minieri bets 3200, Lederer calls. Turn 4♦. Minieri 0%, Lederer 100%. Lederer checks, Minieri bets 10,000, Lederer calls. River A♥. Lederer checks, Minieri checks.

Luck -205.76 - 2323.20 - 930.56 = -3459.52.

Skill -205.76 - 2734.72 - 10000 = -12940.48.

Hand 8. Lederer 7♣ 2♦, Minieri 9♣ 4♦. Minieri 64.28%, Lederer 35.72%. Lederer folds.

Luck +456.96. Skill +343.04.

Hand 9. Minieri 4♠ 2♣, Lederer 8♥ 7♦. Minieri 34.345%, Lederer 65.655%. Minieri raises to 3200, Lederer calls 1600. Flop 3♦ 9♥ J♥. Minieri 22.025%, Lederer 77.975%.

Lederer checks, Minieri bets 4800, Lederer folds.

Luck $-500.96 - 788.48 = -1289.44$. Skill $-500.96 + 4990.40 = +4489.44$.

Hand 10. Lederer K♠ 5♠, Minieri K♥ 7♣. Minieri 59.15%, Lederer 40.85%. Lederer calls 800, Minieri raises to 6400, Lederer folds.

Luck $+292.80$. Skill $+1307.20$.

Hand 11. Minieri A♥ 8♥, Lederer 6♥ 3♠. Minieri 66.85%, Lederer 33.15%. Minieri raises to 3200. Lederer folds.

Luck $+539.20$. Skill $+1060.80$.

Hand 12. Lederer A♦ 4♦. Minieri 7♦ 3♥. Minieri 34.655%, Lederer 65.345%. Lederer raises to 4300, Minieri raises to 11500, Lederer folds.

Luck -491.04 . Skill $+4791.04$.

Hand 13. Minieri 6♣ 3♣, Lederer K♠ 6♠. Minieri 29.825%, Lederer 70.175%. Minieri raises to 4800, Lederer calls 3200. Flop 5♥ J♣ 5♣. Minieri 47.425%, Lederer 52.575%.

Lederer checks, Minieri bets 6000, Lederer folds.

Luck $-645.60 + 1689.60 = +1044$. Skill $-1291.20 + 5047.20 = +3756$.

Hand 14. Lederer 7♦ 5♠, Minieri 8♦ 5♦. Minieri 69.44%, Lederer 30.56%. Lederer

calls 800, Minieri checks. Flop $K♥ 10♠ 8♣$. Minieri 94.395%, Lederer 5.605%. Minieri checks, Lederer bets 1800, Minieri calls. Turn $7♠$. Minieri 95.45%, Lederer 4.55%.

Minieri checks, Lederer checks. River $6♥$. Check, check.

Luck $+622.08 + 798.56 + 71.74 + 309.40 = 1801.78$. Skill $0 + 1598.22 + 0 + 0 = 1598.22$.

Blinds 1000/2000.

Hand 15. Minieri $9♦ 5♠$, Lederer $A♥ 5♦$. Minieri 26.755%, Lederer 73.245%. Minieri calls 1000, Lederer raises to 7000, Minieri raises to 14000, Lederer calls 7000. Flop $10♠ Q♦ 6♥$. Minieri 15.35%, Lederer 84.65%. Lederer checks, Minieri bets 14000, Lederer folds.

Luck $-929.80 - 3193.40 = -4123.20$. Skill $-5578.80 + 23702 = 18123.20$.

Hand 16. Lederer $5♠ 5♥$, Minieri $A♣ J♦$. Minieri 46.085%, Lederer 53.915%. Lederer calls 1000, Minieri raises to 26800, Lederer calls all in. The board is $3♠ 9♠ K♠ 10♦ 9♦$.

Luck $-156.60 - 24701.56 = -24858.16$. Skill -1941.84 .

Hand 17. Minieri $K♣ 10♣$, Lederer $7♦ 5♦$. Minieri 62.22%, Lederer 37.78%. Minieri raises to 5000, Lederer calls 3000. Flop $J♠ J♦ 4♠$. Minieri 69.90%, Lederer 30.10%.

Check check. Turn $8♠$. Minieri 77.27%, Lederer 22.73%. Lederer bets 6000, Minieri folds.

Luck $+488.80 + 768 + 737 = 1993.80$. Skill $+733.20 + 0 - 7727 = -6993.80$.

Hand 18. Lederer 5♠ 5♣, Minieri 10♠ 6♥. Minieri 46.12%, Lederer 53.88%. Lederer calls 1000, Minieri checks. Flop 7♣ 8♣ Q♥. Minieri 38.235%, Lederer 61.765%. Minieri checks, Lederer bets 2000, Minieri calls. Turn J♥. Minieri 22.73%, Lederer 77.27%. Minieri bets 4000, Lederer folds.

Luck $-155.20 - 315.40 - 1240.40 = -1711$. Skill $0 - 470.60 + 6181.60 = +5711$.

Hand 19. Lederer K♥ 5♠, Minieri K♣ 10♦. Minieri 73.175%, Lederer 26.825%. Lederer raises to 5000, Minieri calls 3000. Flop J♦ 8♥ 10♥. Minieri 92.575%, Lederer 7.425%. Check, check. Turn 5♦. Minieri 95.45%, Lederer 4.55%. Minieri bets 6000, Lederer folds.

Luck $+927 + 1940 + 287.50 = 3154.50$. Skill $+1390.50 + 0 + 455 = 1845.50$.

Hand 20. Minieri 7♣ 2♠, Lederer Q♠ 9♠. Minieri 30.205%, Lederer 69.795%. Minieri raises to 6000. Lederer calls 4000. Flop A♦ A♠ Q♦. Minieri 1.165%, Lederer 98.835%. Lederer checks, Minieri bets 6000, Lederer calls. Turn J♣. Minieri 0%, Lederer 100%. Lederer checks, Minieri bets 14000, Lederer raises to 35800, Minieri folds. Luck $-791.80 - 3484.80 - 279.60 = -4556.20$. Skill $-1583.60 - 5860.20 - 14000 = -21443.80$.

Hand 21. Minieri 10♥ 3♦, Lederer Q♥ J♠. Minieri 30.00%, Lederer 70.00%. Minieri calls 1000, Lederer checks. Flop 8♠ 4♥ J♣. Minieri 4.34%, Lederer 95.66%. Lederer checks, Minieri bets 2000, Lederer raises to 7500, Minieri raises to 18500, Lederer raises all-in, Minieri folds. Luck $-800 - 1026.40 = -1826.40$. Skill $0 - 18673.60 = -18673.60$.

Hand 22. Lederer $A\spadesuit 2\diamondsuit$, Minieri $5\clubsuit 3\heartsuit$. Minieri 42.345%, Lederer 57.655%. Lederer calls 1000. Minieri checks. Flop $K\spadesuit 10\clubsuit 3\spadesuit$. Minieri 80.10%, Lederer 19.90%. Check check. Turn $Q\spadesuit$. Minieri 65.91%, Lederer 34.09%. Check, Lederer bets 2000, Minieri folds.

Luck $-306.20 + 1510.20 - 567.60 = 636.40$. Skill $0 + 0 - 2636.40 = -2636.40$.

(Blinds 1500/3000.)

Hand 23. Minieri $7\heartsuit 7\clubsuit$, Lederer $8\diamondsuit 3\diamondsuit$. Minieri 68.175%, Lederer 31.825%. Minieri all-in for 21,700, Lederer folds.

Luck $+1090.50$. Skill $+1909.50$.

Hand 24. Minieri $Q\heartsuit 5\heartsuit$, Lederer $8\diamondsuit 5\diamondsuit$. Minieri 68.37%, Lederer 31.63%. Minieri all-in for 26,200, Lederer folds.

Luck $+1102.20$. Skill $+1897.80$.

Hand 25. Lederer $9\clubsuit 3\clubsuit$, Minieri $5\diamondsuit 2\diamondsuit$. Minieri 40.63%, Lederer 59.37%. Lederer folds.

Luck -562.20 . Skill $+2060.20$.

Hand 26. Minieri $10\clubsuit 2\spadesuit$, Lederer $7\clubsuit 7\heartsuit$. Minieri 29.04%, Lederer 70.96%. Minieri folds.

Luck -1257.60 . Skill -242.40 .

Hand 27. Lederer $Q\clubsuit 9\clubsuit$, Minieri $A\clubsuit 5\spadesuit$. Minieri 55.37%, Lederer 44.63%. Lederer all-in for 29,200. Minieri calls. Board $7\clubsuit 6\clubsuit 10\spadesuit Q\spadesuit 6\diamondsuit$.

Luck $+322.20 - 32336.08 = -32013.88$. Skill $+2813.88$.

Grand Totals: Luck -61023.59 . Skill -13478.41 .

Overall, although Lederer's gains were primarily (about 81.9%) due to luck, Lederer also gained more equity due to skill than Minieri. On the first 19 hands, Minieri actually gained 20,836.41 in equity due to skill, and it appeared that Minieri was outplaying Lederer. On hands 20 and 21, however, Minieri tried two huge unsuccessful bluffs, both on hands (especially hand 20) where he should probably have strongly suspected that Lederer would be likely to call, and on those two hands combined, Minieri lost 40,117.40 in equity due to skill. Although Minieri played very well on every other hand, all of Minieri's good plays on other hands could not overcome the huge loss of skill equity from just those two hands.

It is important to note that the player who gains the most equity due to skill does not always win. In the first 19 hands of this example, for instance, Minieri gained 20836.41 in equity attributed to skill, but because of bad luck, Minieri actually *lost* a total of 2800 chips over these same 19 hands. The bad luck Minieri suffered on hand 16 negated most of his gains due to skillful play. A common misconception is that one's luck will ultimately balance out, i.e. that one's total good luck will eventually exactly equal one's

total bad luck, but this is not true. Assuming one plays the same game repeatedly and independently, and assuming the expected value of one's equity due to luck is 0 which seems reasonable, then one's *average* equity per hand gained by luck will ultimately converge to zero. This is the law of large numbers, and is discussed further in Section 7.4. It does not imply that one's *total* equity gained by luck will converge to zero, however. Potential misconceptions about the laws of large numbers and arguments about possible overemphasis on equity are discussed in Section 7.4.

To conclude this Section, a nice illustration of the potential pitfalls of analyzing a hand purely based on equity is a recent hand from Season 7 of High Stakes Poker. In this hand, with blinds of \$400 and \$800 plus \$100 antes from each of the 8 players, after Bill Klein straddled for \$1600, Phil Galfond raised to \$3500 with $Q\heartsuit 10\heartsuit$, Robert Croak called in the big blind with $A\clubsuit J\clubsuit$, Klein called with $10\spadesuit 6\spadesuit$, and the other players folded. The flop came $J\spadesuit 9\heartsuit 2\spadesuit$, giving Croak top pair, Klein a flush draw, and Galfond an open ended straight draw. Croak bet \$5500, Klein raised to \$17500, and Galfond and Croak called. At this point, it is tempting to compute Klein's probability of winning the hand by computing the probability of exactly one more spade coming on the turn and river without making a full house for Croak, or the turn and river including two 6s, or a 10 and a 6. This would yield a probability of $[(8 \times 35 - 4 - 4) + C(3,2) + 2 \times 3] \div C(43,2) = 281/903 \sim 31.12\%$, and Klein could also split the pot with a straight if the turn and river were KQ or Q8 or 78, without a spade, which has a probability of $[3 \times 3 + 3 \times 3 + 3 \times 3] \div C(43,2) = 27/903 \sim 2.99\%$. These seem to be the combinations Klein needs, and one would not expect Klein to win the pot with a random turn and river combination not on

this list, and especially not if the turn and river contain a king and a jack with no spades. But look at what actually happened in the hand. The turn was the K♣, giving Galfond a straight, and Croak checked, Klein bet \$28000, Galfond raised to \$67000, Croak folded, and Klein called. The river was the J♥, Klein bluffed \$150000, and Galfond folded, giving Klein the \$348,200 pot!