Stat 19, Probability and Poker. Rick Paik Schoenberg

Outline for the day:

1. Discuss Addiction.

2. *R*.

3. Ly vs. Negreanu.

4. Counting and combinations.

5. P(A♠ after first ace).

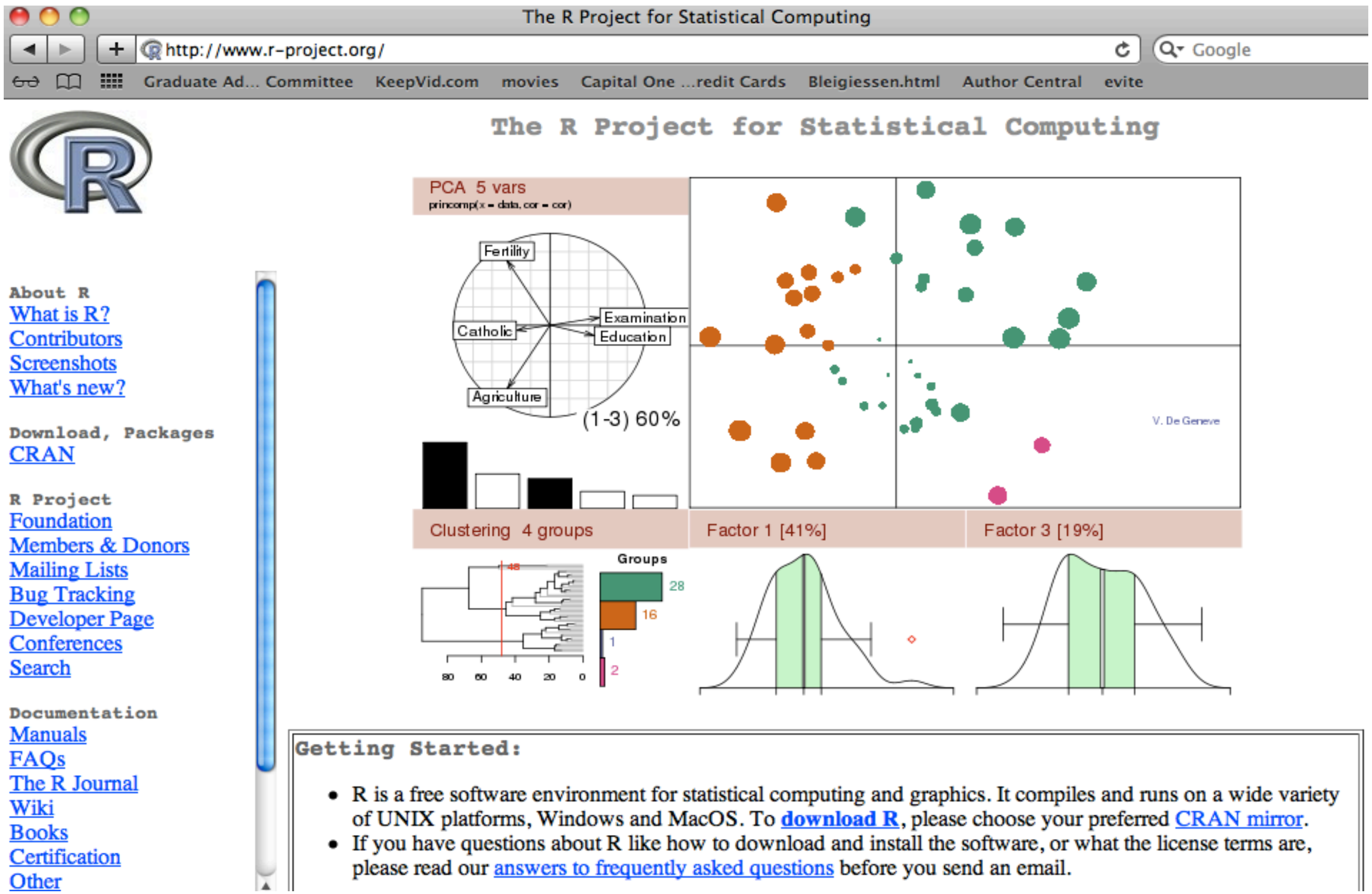
Read harrington1.pdf for next time.

Think of 2 questions or comments for next time.

The course website is http://www.stat.ucla.edu/~frederic/19/F19 .

***R*.** To download and install *R*, go directly to cran.stat.ucla.edu, or you can start at www.r-project.org, in which case you click on "download *R*", scroll down to UCLA, and click on cran.stat.ucla.edu.

From there, click on "download R for …", and then get the latest version.



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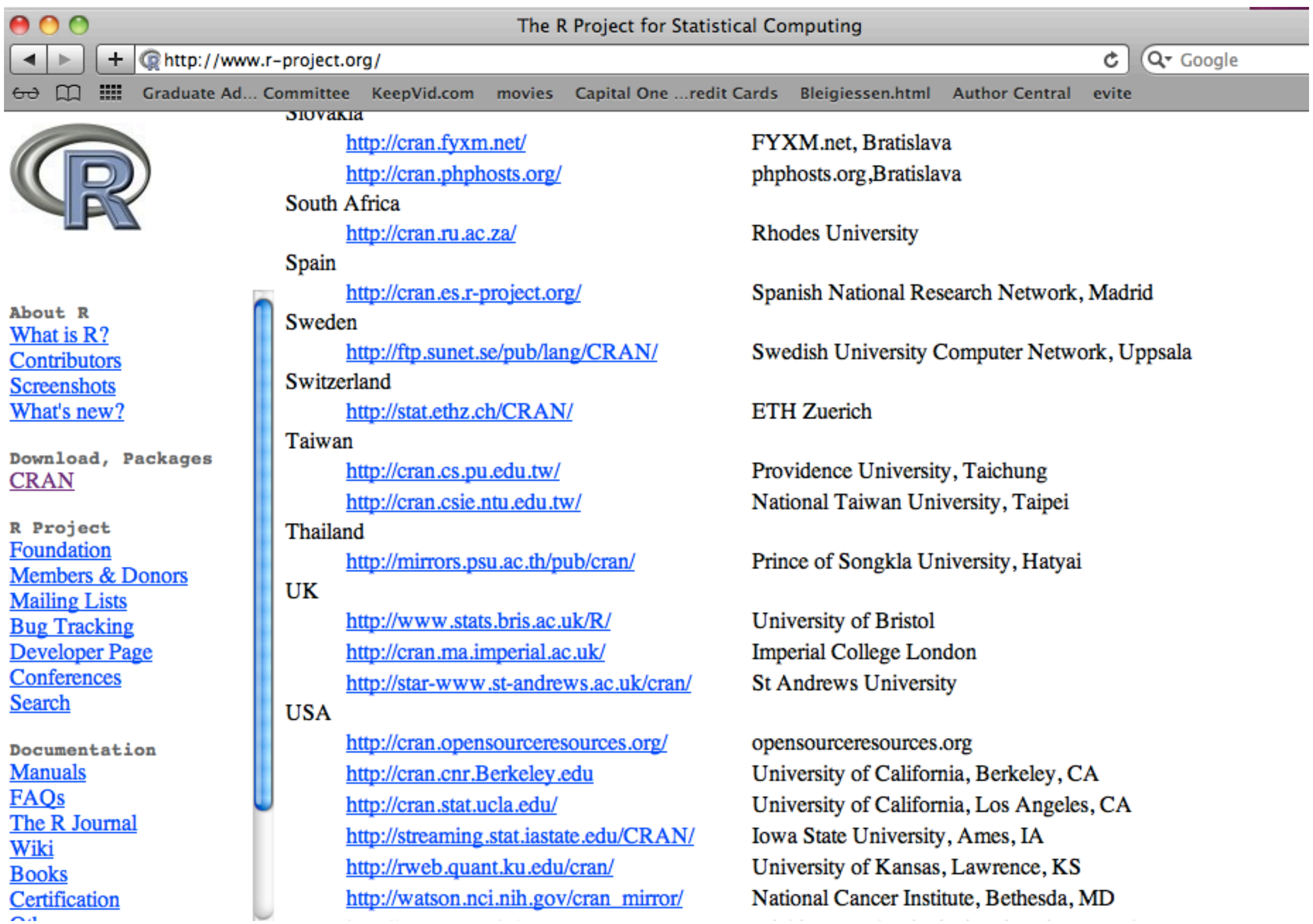

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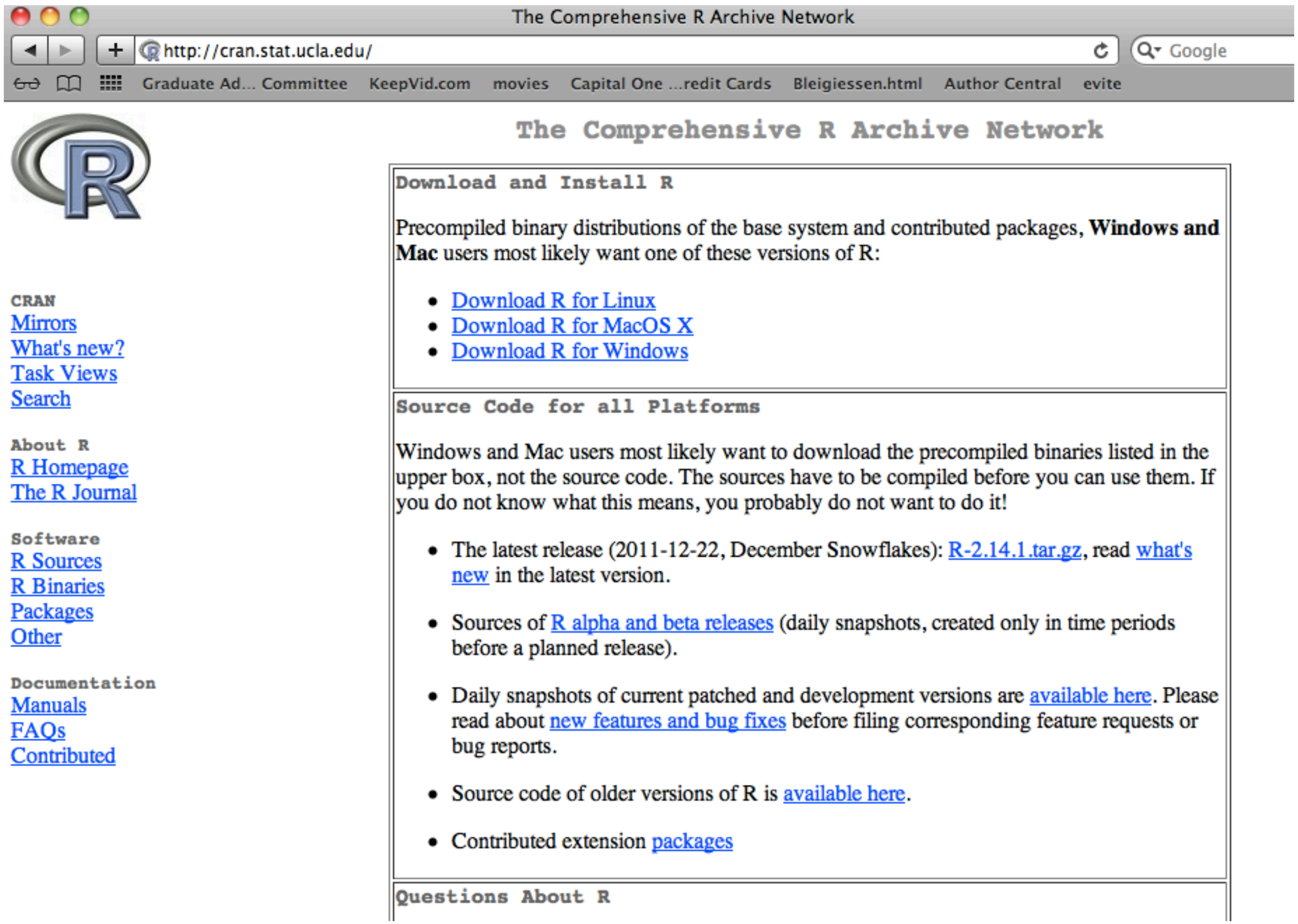

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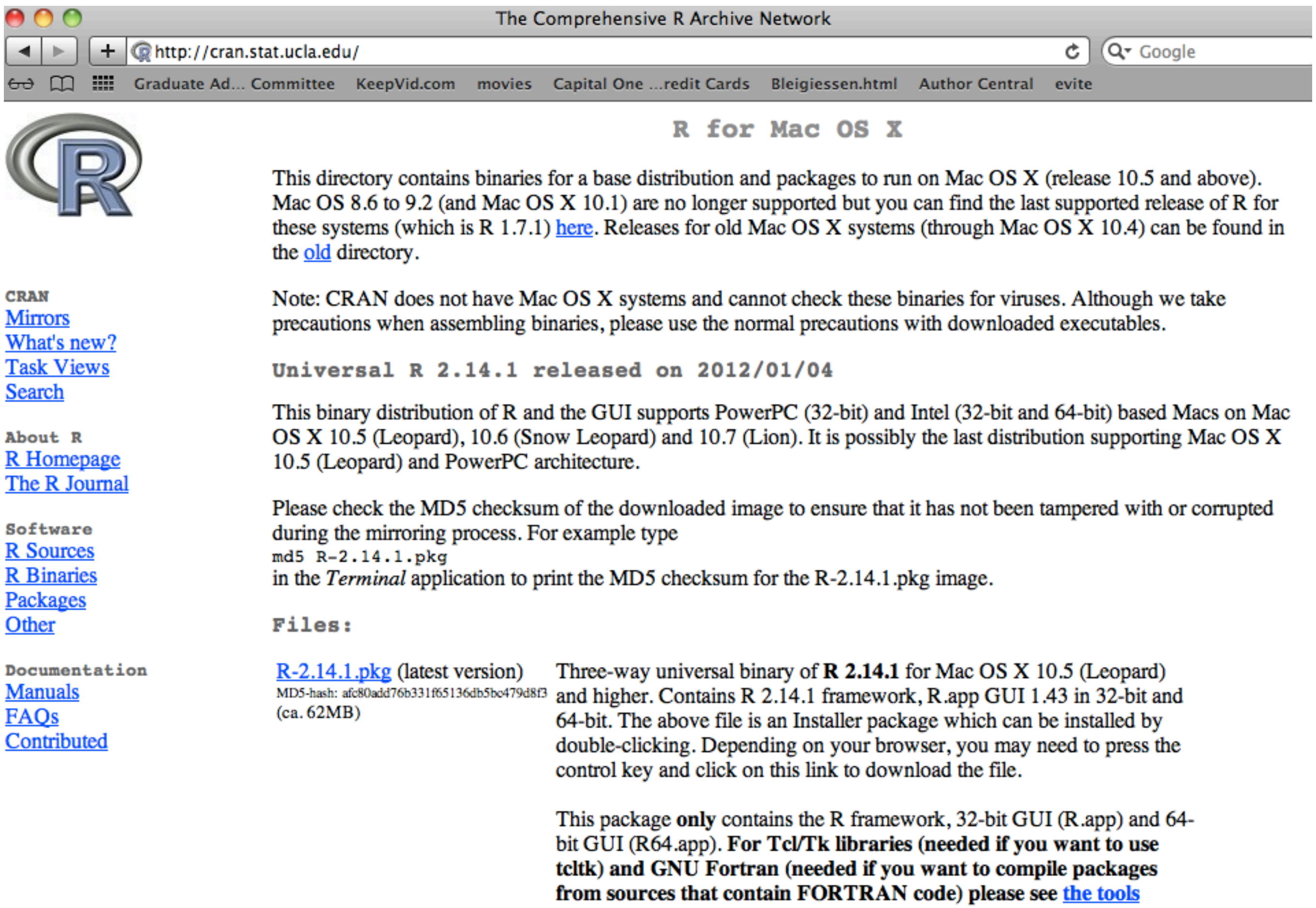


Ly vs. Negreanu.

Ex. Suppose you have two ♣s, and there are exactly two ♣s on the flop. Given this info, what is P(at least one more ♣ on turn or river)?

Answer: 52-5 = 47 cards left (nine ♣s, 38 others).

So n = choose(47,2) = 1081 combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two-♣ combos: choose(9,2) = 36. One-♣ combos: 9 x 38 = 342.
Total = 378. So answer is 378/1081 = 35.0%.

Answer #2: Use the addition rule…

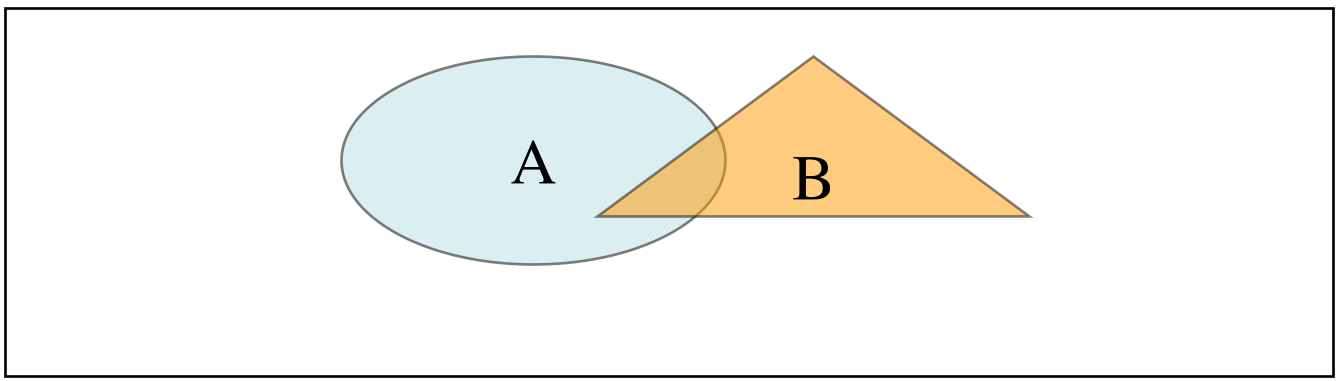
<u>ADDITION RULE, revisited…..</u>

Axioms (initial assumptions/rules) of probability:

1) $P(A) \geq 0$.
2) $P(A) + P(A^c) = 1$.
3) Addition rule:

If $A_1, A_2, A_3, \ldots$ are mutually exclusive, then

$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$



As a result, even if A and B might not be mutually exclusive,

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

<u>Ex.</u> You have two ♣s, and there are exactly two ♣s on the flop.
Given this info, what is P(at least one more ♣ on turn or river)?

<u>Answer #1:</u> 52-5 = 47 cards left (nine ♣s, 38 others).
So n = choose(47,2) = 1081 combinations for next 2 cards.
Each equally likely (and obviously mutually exclusive).
Two- ♣ combos: choose(9,2) = 36. One-♣ combos: 9 x 38 = 342.
Total = 378. So answer is 378/1081 = 35.0%.

<u>Answer #2:</u> Use the addition rule.

P(≥ 1 more ♣) = P(♣ on turn OR river)
= P(♣ on turn) + P(♣ on river) - P(both)
= 9/47 + 9/47 - choose(9,2)/choose(47,2)
= 19.15% + 19.15% - 3.3% = 35.0%.

Counting.

Fact: If $A_1, A_2, \ldots, A_n$ are equally likely & mutually exclusive,
and if $P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_n) = 1$,
then $P(A_k) = 1/n$.
[So, you can *count*: $P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_k) = k/n$.]

Ex. You have 76, and the board is KQ54. P(straight)?
[52-2-4=46.] P(straight) = P(8 on river OR 3 on river)
= P(8 on river) + P(3 on river) = 4/46 + 4/46.

If there are a_1 distinct possible outcomes on experiment #1, and for each of them, there are a_2 distinct possible outcomes on experiment #2, then there are $a_1 \times a_2$ distinct possible *ordered* outcomes on both.

In general, with j experiments, each with a_i possibilities,
the # of distinct outcomes *where order matters* is $a_1 \times a_2 \times \ldots \times a_j$.

Permutations and combinations.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?
52 x 51. [ordered: (A♣ , K♦) ≠ (K♦, A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck? 52 x 51 x … x 1 = 52!

$\sim 8.1 \times 10^{67}$

A <u>*combination*</u> is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[since now (A♣, K♦) = (K♦, A♣)].

So, the number of *distinct* hands where *order doesn't matter* is

52 x 51 / 2.

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

"n choose k" = $\binom{n}{k}$ = choose(n,k) = $\frac{n!}{k!\,(n-k)!}$.

k! = 1 x 2 x … x k. [convention: 0! = 1.]

Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?