## Stat 19, Probability and Poker. Rick Paik Schoenberg

## Outline for the day:

1.**R**.

- 2. Discuss harrington1.pdf.
- 3. Gus and Daniel.
- 4. Counting and combinations.
- 5. P(A♠ after first ace).

Read harrington2.pdf for next time.

Think of 1-2 questions or comments for next time.

The course website is <u>http://www.stat.ucla.edu/</u>~frederic/19/F21.



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# Ly vs. Negreanu.

Ex. Suppose you have two \$s, and there are exactly two \$s on the flop. Given this info, what is P(at least one more \$ on turn or river)?

Answer: 52-5 = 47 cards left (nine  $\clubsuit$ s, 38 others).

So n = choose(47,2) = 1081 combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

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Two- $\clubsuit$  combos: choose(9,2) = 36. One- $\clubsuit$  combos: 9 x 38 = 342. Total = 378. So answer is 378/1081 = 35.0%.

Answer #2: Use the addition rule...

#### ADDITION RULE, revisited.....

Axioms (initial assumptions/rules) of probability:

- 1)  $P(A) \ge 0$ .
- 2)  $P(A) + P(A^c) = 1$ .
- 3) Addition rule:

If  $A_1, A_2, A_3, \dots$  are mutually exclusive, then  $P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$ 



As a result, even if A and B might not be mutually exclusive, P(A or B) = P(A) + P(B) - P(A and B). Ex. You have two \$s, and there are exactly two \$s on the flop. Given this info, what is P(at least one more \$ on turn or river)? <u>Answer #1:</u> 52-5 = 47 cards left (nine \$s, 38 others). So n = choose(47,2) = 1081 combinations for next 2 cards. Each equally likely (and obviously mutually exclusive). Two- \$ combos: choose(9,2) = 36. One-\$ combos: 9 x 38 = 342. Total = 378. So answer is 378/1081 = 35.0%.

<u>Answer #2:</u> Use the addition rule.

 $P(\geq 1 \text{ more } \clubsuit) = P(\clubsuit \text{ on turn OR river})$ 

 $= P(\clubsuit \text{ on turn}) + P(\clubsuit \text{ on river}) - P(both)$ 

= 9/47 + 9/47 - choose(9,2)/choose(47,2)

= 19.15% + 19.15% - 3.3% = 35.0%.

Counting.

<u>Fact:</u> If  $A_1, A_2, ..., A_n$  are equally likely & mutually exclusive, and if  $P(A_1 \text{ or } A_2 \text{ or } ... \text{ or } A_n) = 1$ , then  $P(A_k) = 1/n$ . [So, you can *count*:  $P(A_1 \text{ or } A_2 \text{ or } ... \text{ or } A_k) = k/n$ .]

<u>Ex.</u> You have 76, and the board is KQ54. P(straight)? [52-2-4=46.] P(straight) = P(8 on river OR 3 on river) = P(8 on river) + P(3 on river) = 4/46 + 4/46. If there are  $a_1$  distinct possible outcomes on experiment #1, and for each of them, there are  $a_2$  distinct possible outcomes on experiment #2, then there are  $a_1 \ge a_2$  distinct possible ordered outcomes on both.

In general, with j experiments, each with  $a_i$  possibilities, the # of distinct outcomes *where order matters* is  $a_1 \times a_2 \times \ldots \times a_j$ .

### Permutations and combinations.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities? 52 x 51. [ordered:  $(A \clubsuit, K \blacklozenge) \neq (K \diamondsuit, A \clubsuit)$ .]

Each such outcome, where order matters, is called a *permutation*. Number of permutations of the deck?  $52 \times 51 \times ... \times 1 = 52!$ ~ 8.1 x 10<sup>67</sup>

- A <u>combination</u> is a collection of outcomes, where order *doesn't* matter.
  e.g. in hold'em, how many *distinct* 2-card hands are possible?
  52 x 51 if order matters, but then you'd be double-counting each
  [ since now (A♣, K♦) = (K♦, A♣) ].
- So, the number of *distinct* hands where *order doesn't matter* is  $52 \ge 51/2$ .
- In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

"n choose k" = 
$$\binom{n}{k}$$
 = choose(n,k) = n!  
k! (n-k)!

 $k! = 1 \times 2 \times ... \times k.$  [convention: 0! = 1.]

Deal til first ace appears. Let X = the *next* card after the ace. P(X = A $\blacklozenge$ )? P(X = 2 $\clubsuit$ )?