Luck versus skill in poker, and Minieri versus Lederer on Poker After Dark.

A topic that commonly arises in discussing poker and other games involving cards or dice, spinners, etc., is how much of the game is luck and how much is skill. Luck and skill are incredibly hard to define, however, and attempts at rigorous definitions are elusive in books and journal articles on game theory. Some articles have defined skill in terms of the standard deviation of results among different players, with the idea that players should perform more similarly if a game is mostly based on luck, but their results might differ more substantially if the game is based on skill. Others have defined skill in terms of the standard deviation of a given player's results, since a lower standard deviation would indicate that fewer repetitions are necessary in order to determine the statistical significance of one's long-term edge in the game, and hence the sooner one can establish that one's results are primarily due to skill rather than luck.

These definitions above are obviously extremely problematic for various reasons. One is that both definitions rely on the game in question being played repeatedly before even a crude assessment of luck or skill could be made. More importantly, there are many contests of skill wherein the differences between players are small, or where one's results vary wildly. For instance, in Olympic trials of the 100-meter sprints, the differences between finishers are typically quite small, often just hundredths of a second. That hardly implies that the results are based on luck. There are also sporting events where an individual's results might vary a lot from one day to another (e.g. pitching in baseball), but that hardly means that luck plays a major role.

A possibility for the game of poker might be to define luck as equity gained when cards are dealt by the dealer, and skill as equity gained by one's actions during the betting rounds. That is, there are several reasons you might gain equity in a hand:

* The cards dealt by the dealer (whether the players' hole cards or the flop, turn, or river) give you a greater chance of winning the hand in a showdown,

* The size of the pot is increased while your chance to win the hand in a showdown is better than that of your opponent(s).

* By betting, you win a pot that you might otherwise have lost.

Certainly, anyone would characterize the first case as luck, unless perhaps one believes in ESP or time travel. And for reasons why time travel is impossible, see Craig's blog craigshappyplace.blogspot.com. My suggestion is that estimates of the skill in poker can be obtained by looking at the second and third cases above. That is, we may look at one's skill as being comprised of the equity that one gains during the betting rounds, whereas luck is equity gained by the deal of the cards. The nice thing about this is that it is easily quantifiable, and one may dissect a particular poker game and analyze how much equity each player gained due to luck or skill.

There are obvious objections to this. First, why equity? One's equity (or express equity) in a pot is typically defined as one's expected return from the pot given no future betting, and the assumption of no future betting seems absurdly simplistic and unrealistic. On the other hand, unlike implied equity which would account for betting on future streets, express equity is unambiguously defined and easy to compute. Second, situations can occur where one would expect a terrible player to gain equity during the betting rounds against even the greatest player in the world, so to attribute such equity gains to "skill" might be objectionable. For instance, in heads up hold'em, if the two players are dealt AA and KK, one would expect the player with KK to put a great deal of chips in while way behind, and this situation seems more like bad luck for the player with KK than any deficit in skill. Again, as stated above, my response would be that

skill is difficult to define, and in fact most poker players, probably due to their huge and fragile egos, tend to chalk up losses for virtually any reason as being solely due to bad luck. In some sense, anything can be attributed to luck if one has a general enough definition of the word. Even if a player does an amazingly skillful poker play, such as folding a very strong hand because of an observed tell or betting pattern, one could argue that it was lucky that the player happened to observe that tell, or even that the player was lucky to have been born with the ability to discern the tell. On the other hand, situations like the AA versus KK example truly do seem like bad luck. It is difficult to think of any remedy to this problem. It may be that the word "skill" is too strong a word, and that while it may be of interest to analyze hands in terms of equity, one should instead use the term equity gained during the betting round rather than skill.

There is some ambiguity in the definition of equity before the flop, since the small and big blind have put in different amounts of chips. One definition I will use is that preflop equity is the expected profit (equity one would have in the pot after calling minus cost), assuming the big blind and small blind call as well, or the equity one would have by folding, whichever happens to be greater. For example, in heads up hold'em with blinds of 800 and 1600, the preflop equity for the big blind is $p*2*b - 1600$, and $\max\{p*2*b - 1600, -800\}$ for the small blind, where p is the probability of winning the pot in a showdown, and b is the amount of the big blind. Define increases in the size of the pot as relative to the big blind, i.e. increasing the pot size by calling preflop does not count as skill. In what follows the probability of a tie is divided equally among the two players, in determining p.

Below are the 27 hands shown on Poker After Dark last week between Dario Minieri and Howard Lederer in the Heads Up portion of the tournament, with each hand's equity gains and losses broken down as luck or skill. I will just analyze everything from Minieri's perspective, i.e. "skill -100" will refer to 100 chips of equity gained by Lederer during a betting round.

For example. look at hand 4.

a) Preflop dealing (luck). Minieri +642.08.
Minieri is dealt a 70.065% probability of winning the pot, so his increase in equity is 70.065 * 3200 - 1600= 642.08.
Lederer is dealt 29.935%, so his increase in equity is 29.935% * 3200 - 1600 = -642.08.

b) Preflop betting (skill). Minieri +1083.51.
The pot gets increased to 8600. 8600-3200=5400.
Minieri has paid an additional 2700 but has 70.065% * 5400 = 3783.51 additional equity, so Minieri's profit due to betting is 3783.51 - 2700 = 1083.51.
Correspondingly, Lederer's profit due to betting is -1083.51, since 29.935% * 5400 - 2700 = -1083.51.

c) Flop dealing (luck). Minieri +1362.67.
On the flop, Minieri's equity went from 70.065% of the 8600 pot to 85.91% of it. So by luck Minieri increased his equity by (85.91% - 70.065%)*8600 = +1362.67

d) Flop betting (skill). Minieri +1211.74.
Because of the betting on the flop, Minieri's equity went from 85.91% of the 8600 pot to 100% of it, so Minieri increased his equity by (100% - 85.91%)*8600 = 1211.74.

So, by luck, Minieri got 642.08 + 1362.67 = 2004.75.
By skill, Minieri got 1083.51 + 1211.74 = 2295.25.

Notice that the total = 2004.75 + 2295.25 = 4300, which is the number of chips Minieri won from Lederer in the hand.

Before the heads-up battle began, they said that Minieri had 72k, Lederer 48k. Either I made some small mistakes below or Minieri must have won some chips in the hands they didn't show because the grand total has Minieri losing about 74,500 chips.

Blinds 800/1600.

1. Lederer Ac 7s, Minieri 6s 6d. Lederer 43.535%, Minieri 56.465%.
Lederer raises to 4300. Minieri raises to 47800. Lederer folds.
Luck +206.88.
Skill +4093.12.

2. Minieri 4s 2d, Lederer Ks 7h. Minieri 34.36%, Lederer 65.64%.
Minieri raises to 4300, Lederer all in for 43500, Minieri folds.
Luck -500.48.
Skill -3799.52.

3. Lederer 6h 3d, Minieri Ad 9c. Lederer 34.965%, Minieri 65.035%.
Lederer folds.
Luck +481.12.
Skill +318.88.

4. Minieri Ac Jc, Lederer Ah 9h. Minieri 70.065%, Lederer 29.935%.
Minieri raises to 4300, Lederer calls 2700.
Flop 6c Ts Tc. Minieri 85.91%, Lederer 14.09%.
Lederer checks, Minieri 6500, Lederer folds.
Luck +2004.75.
Skill +2295.25.

5. Lederer 5s 3h, Minieri 7d 6s. Lederer 35.765%, Minieri 64.235%.
Lederer folds.
Luck +455.52.
Skill +344.48.

6. Minieri Kh Tc, Lederer 5d 2d.
Minieri 61.41%, Lederer 38.59%
Minieri raises to 3200, Lederer raises to 9700, Minieri folds.
Luck +365.12.
Skill -3565.12

7. Minieri Td 7s, Lederer Qc 2h.
Minieri 43.57%, Lederer 56.43%.
Minieri raises to 3200, Lederer calls 1600.
Flop 8s 2s Qh.
Minieri 7.27%, Lederer 92.73%.

Lederer checks, Minieri bets 3200, Lederer calls.
Turn 4d.
Minieri 0%, Lederer 100%.
Lederer checks, Minieri bets 10,000, Lederer calls.
River Ah.
Lederer checks, Minieri checks.
Luck -205.76 - 2323.20 - 930.56 = -3459.52.
Skill -205.76 - 2734.72 - 10000 = -12940.48.

8. Lederer 7c 2d, Minieri 9c 4d.
Minieri 64.28%, Lederer 35.72%.
Lederer folds.
Luck +456.96.
Skill +343.04.

9. Minieri 4s 2c, Lederer 8h 7d.
Minieri 34.345%, Lederer 65.655%.
Minieri raises to 3200, Lederer calls 1600.
Flop 3d 9h Jh.
Minieri 22.025%, Lederer 77.975%.
Lederer checks, Minieri bets 4800, Lederer folds.
Luck -500.96 - 788.48 = -1289.44
Skill -500.96 + 4990.40 = +4489.44.

10. Lederer Ks 5s, Minieri Kh 7c.
Minieri 59.15%, Lederer 40.85%.
Lederer calls 800, Minieri raises to 6400, Lederer folds.
Luck +292.80.
Skill +1307.20.

11. Minieri Ah 8h, Lederer 6h 3s.
Minieri 66.85%, Lederer 33.15%.
Minieri raises to 3200. Lederer folds.
Luck +539.20.
Skill +1060.80.

12. Lederer Ad 4d. Minieri 7d 3h.
Minieri 34.655%, Lederer 65.345%.
Lederer raises to 4300, Minieri raises to 11500, Lederer folds.
Luck -491.04.
Skill +4791.04.

13. Minieri 6c 3c, Lederer Ks 6s.
Minieri 29.825%, Lederer 70.175%.
Minieri raises to 4800, Lederer calls 3200.
Flop 5h Jc 5c.

Minieri 47.425%, Lederer 52.575%.
Lederer checks, Minieri bets 6000, Lederer folds.
Luck -645.60 + 1689.60 = +1044.
Skill -1291.20 + 5047.20 = +3756.

14. Lederer 7d 5s, Minieri 8d 5d.
Minieri 69.44%, Lederer 30.56%.
Lederer calls 800, Minieri checks.
Flop Kh Ts 8c.
Minieri 94.395%, Lederer 5.605%.
Minieri checks, Lederer bets 1800, Minieri calls.
Turn 7s.
Minieri 95.45%, Lederer 4.55%.
Minieri checks, Lederer checks.
River 6h.
Check, check.
Luck +622.08 + 798.56 + 71.74 + 309.40 = 1801.78.
Skill 0 + 1598.22 + 0 + 0 = 1598.22.

Blinds 1000/2000.

15. Minieri 9d 5s, Lederer Ah 5d.
Minieri 26.755%, Lederer 73.245%.
Minieri calls 1000, Lederer raises to 7000, Minieri raises to 14000, Lederer calls 7000.
Flop Ts Qd 6h.
Minieri 15.35%, Lederer 84.65%.
Lederer checks, Minieri bets 14000, Lederer folds.
Luck -929.80 - 3193.40 = -4123.20.
Skill -5578.80 + 23702 = 18123.20.

16. Lederer 5s 5h, Minieri Ac Jd.
Minieri 46.085%, Lederer 53.915%.
Lederer calls 1000, Minieri raises to 26800, Lederer calls all in.
Board 3s 9s Ks Td 9d.
Luck -156.60 - 24701.56 = -24858.16.
Skill -1941.84.

17. Minieri Kc Tc, Lederer 7d 5d.
Minieri 62.22%, Lederer 37.78%.
Minieri raises to 5000, Lederer calls 3000.
Flop Js Jd 4s.
Minieri 69.90%, Lederer 30.10%.
Check check.
Turn 8s.
Minieri 77.27%, Lederer 22.73%.
Lederer bets 6000, Minieri folds.

Luck +488.80 + 768 + 737 = 1993.80.
Skill +733.20 + 0 - 7727 = -6993.80.

18. Lederer 5s 5c, Minieri Ts 6h.
Minieri 46.12%, Lederer 53.88%.
Lederer calls 1000, Minieri checks.
Flop 7c 8c Qh.
Minieri 38.235%, Lederer 61.765%.
Minieri checks, Lederer bets 2000, Minieri calls 2000.
Turn Jh.
Minieri 22.73%, Lederer 77.27%.
Minieri bets 4000, Lederer folds.
Luck -155.20 - 315.40 - 1240.40 = -1711.
Skill 0 - 470.60 + 6181.60 = +5711.

19. Lederer Kh 5s, Minieri Kc Td.
Minieri 73.175%, Lederer 26.825%.
Lederer raises to 5000, Minieri calls 3000.
Flop Jd 8h Th.
Minieri 92.575%, Lederer 7.425%.
Check check.
Turn 5d.
Minieri 95.45%, Lederer 4.55%.
Minieri bets 6000, Lederer folds.
Luck +927 + 1940 + 287.50 = 3154.50.
Skill +1390.50 + 0 + 455 = 1845.50.

20. Minieri 7c 2s, Lederer Qs 9s.
Minieri 30.205%, Lederer 69.795%.
Minieri raises to 6000. Lederer calls 4000.
Flop Ad As Qd.
Minieri 1.165%, Lederer 98.835%.
Lederer checks, Minieri bets 6000, Lederer calls.
Turn Jc.
Minieri 0%, Lederer 100%.
Lederer checks, Minieri bets 14000, Lederer raises to 35800, Minieri folds.
Luck -791.80 - 3484.80 - 279.60 = -4556.20.
Skill -1583.60 - 5860.20 - 14000 = -21443.80.

21. Minieri Th 3d, Lederer Qh Js.
Minieri 30.00%, Lederer 70.00%.
Minieri calls 1000, Lederer checks.
Flop 8s 4h Jc.
Minieri 4.34%, Lederer 95.66%.
Lederer checks, Minieri bets 2000, Lederer raises to 7500, Minieri raises to 18500, Lederer raises all-in, Minieri folds.
Luck -800 - 1026.40 = -1826.40.

Skill 0 - 18673.60 = -18673.60.

22. Lederer As 2d, Minieri 5c 3h.
Minieri 42.345%, Lederer 57.655%.
Lederer calls 1000. Minieri checks.
Flop Ks Tc 3s.
Minieri 80.10%, Lederer 19.90%.
Check check.
Turn Qs.
Minieri 65.91%, Lederer 34.09%.
Check, Lederer bets 2000, Minieri folds.
Luck -306.20 + 1510.20 - 567.60 = 636.40.
Skill 0 + 0 - 2636.40 = -2636.40.

Blinds 1500/3000.

23. Minieri 7h 7c, Lederer 8d 3d.
Minieri 68.175%, Lederer 31.825%.
Minieri all-in for 21,700, Lederer folds.
Luck +1090.50.
Skill +1909.50.

24. Minieri Qh 5h, Lederer 8d 5d.
Minieri 68.37%, Lederer 31.63%.
Minieri all-in for 26,200, Lederer folds.
Luck +1102.20.
Skill +1897.80.

25. Lederer 9c 3c, Minieri 5d 2d.
Minieri 40.63%, Lederer 59.37%.
Lederer folds.
Luck -562.20.
Skill +2060.20.

26. Minieri Tc 2s, Lederer 7c 7h.
Minieri 29.04%, Lederer 70.96%.
Minieri folds.
Luck -1257.60.
Skill -242.40.

27. Lederer Qc 9c, Minieri Ac 5s.
Minieri 55.37%, Lederer 44.63%.
Lederer all-in for 29,200. Minieri calls.
Board 7c 6c Ts Qs 6d.
Luck +322.20 - 32336.08 = -32013.88.
Skill +2813.88.

GRAND TOTALS.
Luck -61023.59.
Skill -13478.41.

Interesting that overall, although Lederer's gains were primarily due to luck, Lederer also gained slightly more equity due to skill than Minieri. On the first 19 hands, Minieri gained 20,836.41 due to skill, and I think that shaped my view that Minieri was outplaying Lederer. On hands 20 and 21, Minieri tried two huge unsuccessful bluffs, both on hands (especially hand 20) where he should have strongly suspected that Lederer would be likely to call, and on those two hands combined, Minieri lost 40,117.40 in equity due to skill. Although Minieri played very well on every other hand, all of Minieri's good plays on other hands could not overcome the loss of equity due to skill from just those two hands.