

# Stat 19, Probability and Poker. Rick Paik Schoenberg

## Outline for the day:

1. Discuss Addiction.
2. *R*.
3. Affleck Duhamel.
4. Counting and combinations.
5.  $P(A\spadesuit \text{ after first ace})$ .

Read harrington1.pdf for next time.

Think of 1-2 questions or comments for next time.

The course website is <http://www.stat.ucla.edu/~frederic/19/S20> .

**R**. To download and install R, go directly to [cran.stat.ucla.edu](http://cran.stat.ucla.edu), or you can start at [www.r-project.org](http://www.r-project.org), in which case you click on “download R”, scroll down to UCLA, and click on [cran.stat.ucla.edu](http://cran.stat.ucla.edu).

From there, click on “download R for ...”, and then get the latest version.

The R Project for Statistical Computing

PCA 5 vars  
princomp(x = data, cor = cor)

Fertility  
Catholic  
Examination  
Education  
Agriculture  
(1-3) 60%

Clustering 4 groups

Factor 1 [41%]  
Factor 3 [19%]

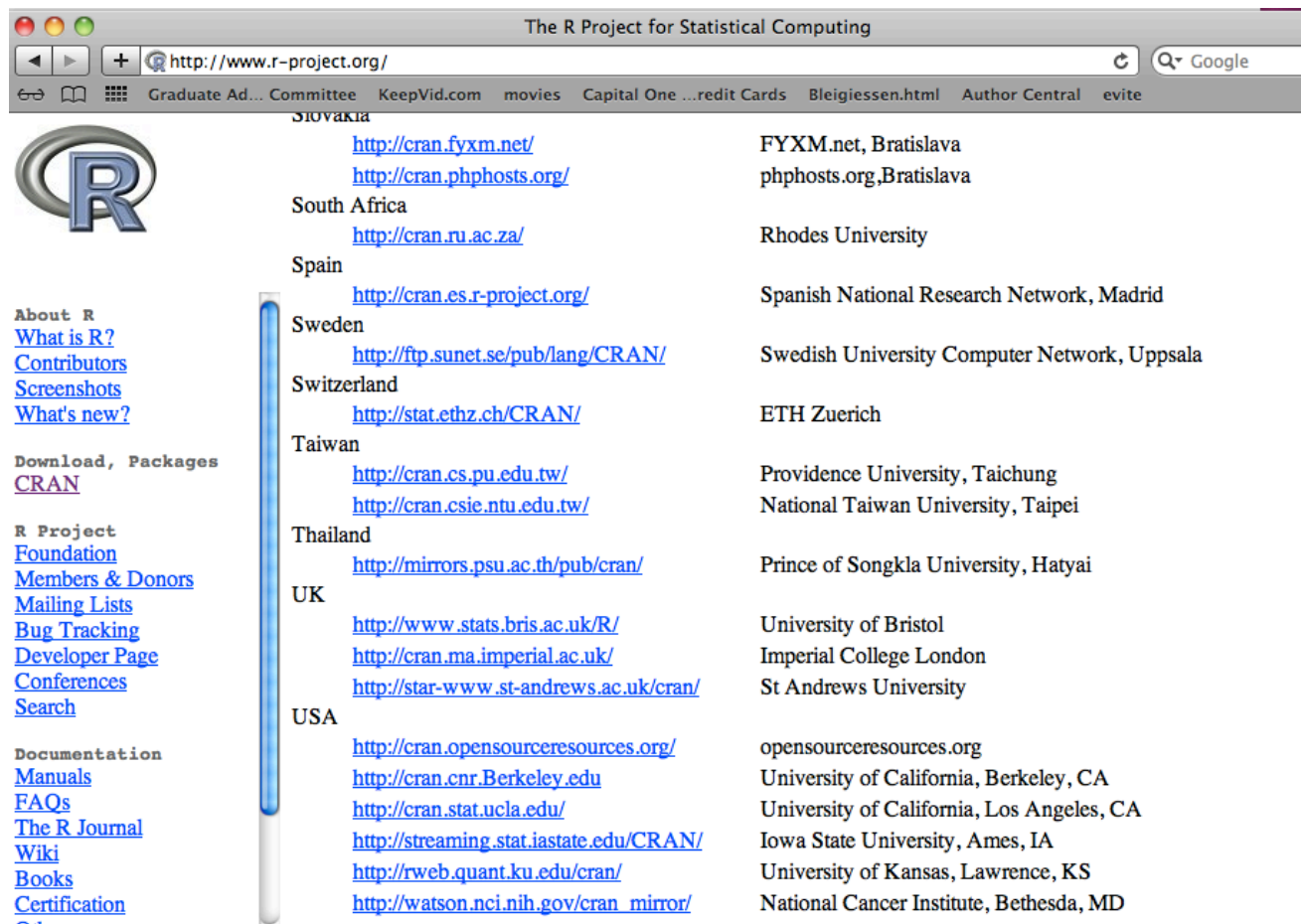
Groups  
28  
16  
1  
2

Getting Started:

- R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To **download R**, please choose your preferred [CRAN mirror](#).
- If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.

To download and install *R*, go directly to [cran.stat.ucla.edu](http://cran.stat.ucla.edu), or you can start at [www.r-project.org](http://www.r-project.org), in which case you click on “download *R*”, scroll down to UCLA, and click on [cran.stat.ucla.edu](http://cran.stat.ucla.edu).

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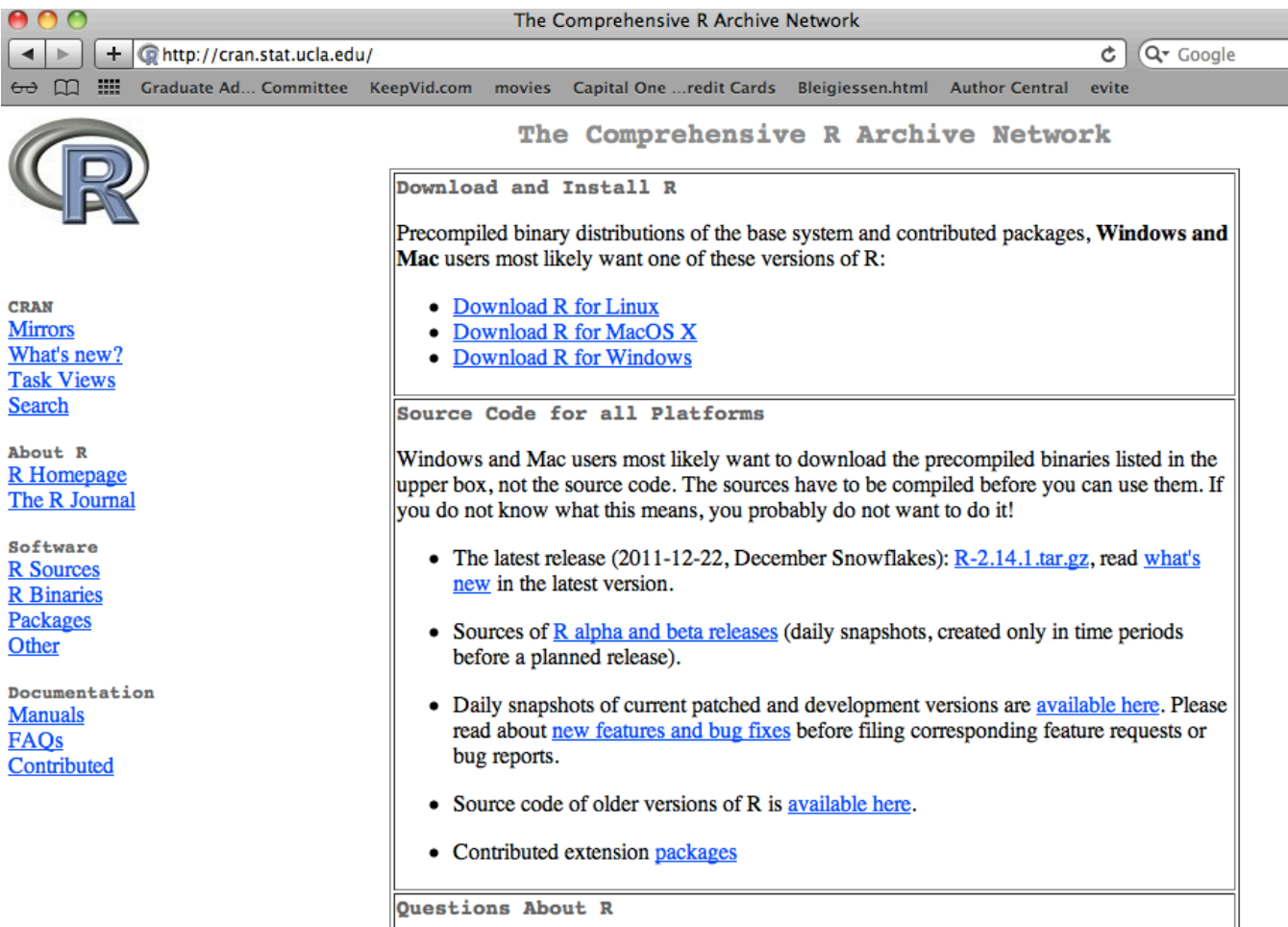


The screenshot shows a web browser window titled "The R Project for Statistical Computing" with the address bar at <http://www.r-project.org/>. The page features the R logo on the left and a list of mirrors on the right. The mirrors are organized by country, with each country name followed by one or more mirror URLs and their respective locations.

Country	Mirror URL	Location
Slovakia	<a href="http://cran.fyxm.net/">http://cran.fyxm.net/</a>	FYXM.net, Bratislava
	<a href="http://cran.phphosts.org/">http://cran.phphosts.org/</a>	phphosts.org, Bratislava
South Africa	<a href="http://cran.ru.ac.za/">http://cran.ru.ac.za/</a>	Rhodes University
Spain	<a href="http://cran.es.r-project.org/">http://cran.es.r-project.org/</a>	Spanish National Research Network, Madrid
Sweden	<a href="http://ftp.sunet.se/pub/lang/CRAN/">http://ftp.sunet.se/pub/lang/CRAN/</a>	Swedish University Computer Network, Uppsala
Switzerland	<a href="http://stat.ethz.ch/CRAN/">http://stat.ethz.ch/CRAN/</a>	ETH Zuerich
Taiwan	<a href="http://cran.cs.pu.edu.tw/">http://cran.cs.pu.edu.tw/</a>	Providence University, Taichung
	<a href="http://cran.csie.ntu.edu.tw/">http://cran.csie.ntu.edu.tw/</a>	National Taiwan University, Taipei
Thailand	<a href="http://mirrors.psu.ac.th/pub/cran/">http://mirrors.psu.ac.th/pub/cran/</a>	Prince of Songkla University, Hatyai
UK	<a href="http://www.stats.bris.ac.uk/R/">http://www.stats.bris.ac.uk/R/</a>	University of Bristol
	<a href="http://cran.ma.imperial.ac.uk/">http://cran.ma.imperial.ac.uk/</a>	Imperial College London
	<a href="http://star-www.st-andrews.ac.uk/cran/">http://star-www.st-andrews.ac.uk/cran/</a>	St Andrews University
USA	<a href="http://cran.opensourceresources.org/">http://cran.opensourceresources.org/</a>	opensourceresources.org
	<a href="http://cran.cnr.Berkeley.edu">http://cran.cnr.Berkeley.edu</a>	University of California, Berkeley, CA
	<a href="http://cran.stat.ucla.edu/">http://cran.stat.ucla.edu/</a>	University of California, Los Angeles, CA
	<a href="http://streaming.stat.iastate.edu/CRAN/">http://streaming.stat.iastate.edu/CRAN/</a>	Iowa State University, Ames, IA
	<a href="http://rweb.quant.ku.edu/cran/">http://rweb.quant.ku.edu/cran/</a>	University of Kansas, Lawrence, KS
	<a href="http://watson.nci.nih.gov/cran_mirror/">http://watson.nci.nih.gov/cran_mirror/</a>	National Cancer Institute, Bethesda, MD

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From there, click on “download *R* for ...”, and then get the latest version.



The screenshot shows a web browser window titled "The Comprehensive R Archive Network" with the address bar displaying "http://cran.stat.ucla.edu/". The browser's address bar also shows a search engine (Google) and a list of bookmarks including "Graduate Ad... Committee", "KeepVid.com", "movies", "Capital One ...redit Cards", "Bleigiessen.html", "Author Central", and "evite".

The main content area of the website is titled "The Comprehensive R Archive Network" and is divided into several sections:

- Download and Install R**

Precompiled binary distributions of the base system and contributed packages, **Windows and Mac** users most likely want one of these versions of R:

  - [Download R for Linux](#)
  - [Download R for MacOS X](#)
  - [Download R for Windows](#)
- Source Code for all Platforms**

Windows and Mac users most likely want to download the precompiled binaries listed in the upper box, not the source code. The sources have to be compiled before you can use them. If you do not know what this means, you probably do not want to do it!

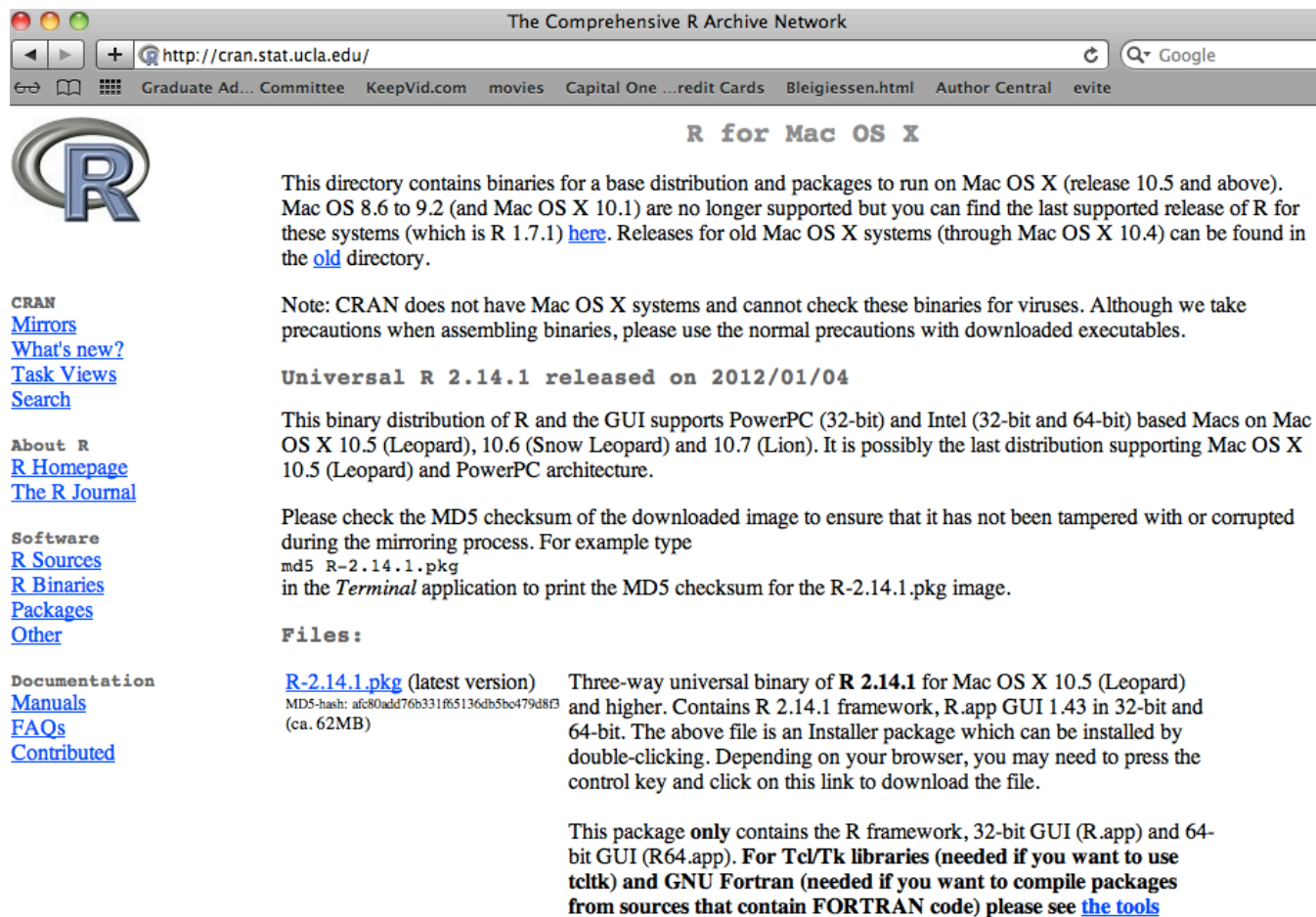
  - The latest release (2011-12-22, December Snowflakes): [R-2.14.1.tar.gz](#), read [what's new](#) in the latest version.
  - Sources of [R alpha and beta releases](#) (daily snapshots, created only in time periods before a planned release).
  - Daily snapshots of current patched and development versions are [available here](#). Please read about [new features and bug fixes](#) before filing corresponding feature requests or bug reports.
  - Source code of older versions of R is [available here](#).
  - Contributed extension [packages](#)
- Questions About R**

On the left side of the page, there is a navigation menu with the following links:

- CRAN
  - [Mirrors](#)
  - [What's new?](#)
  - [Task Views](#)
  - [Search](#)
- About R
  - [R Homepage](#)
  - [The R Journal](#)
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  - [R Binaries](#)
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- Documentation
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  - [FAQs](#)
  - [Contributed](#)

To download and install *R*, go directly to [cran.stat.ucla.edu](http://cran.stat.ucla.edu), or you can start at [www.r-project.org](http://www.r-project.org), in which case you click on “download *R*”, scroll down to UCLA, and click on [cran.stat.ucla.edu](http://cran.stat.ucla.edu).

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The Comprehensive R Archive Network

<http://cran.stat.ucla.edu/> Google

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## R for Mac OS X

This directory contains binaries for a base distribution and packages to run on Mac OS X (release 10.5 and above). Mac OS 8.6 to 9.2 (and Mac OS X 10.1) are no longer supported but you can find the last supported release of R for these systems (which is R 1.7.1) [here](#). Releases for old Mac OS X systems (through Mac OS X 10.4) can be found in the [old](#) directory.

Note: CRAN does not have Mac OS X systems and cannot check these binaries for viruses. Although we take precautions when assembling binaries, please use the normal precautions with downloaded executables.

### Universal R 2.14.1 released on 2012/01/04

This binary distribution of R and the GUI supports PowerPC (32-bit) and Intel (32-bit and 64-bit) based Macs on Mac OS X 10.5 (Leopard), 10.6 (Snow Leopard) and 10.7 (Lion). It is possibly the last distribution supporting Mac OS X 10.5 (Leopard) and PowerPC architecture.

Please check the MD5 checksum of the downloaded image to ensure that it has not been tampered with or corrupted during the mirroring process. For example type

```
md5 R-2.14.1.pkg
```

in the *Terminal* application to print the MD5 checksum for the R-2.14.1.pkg image.

**Files:**

<a href="#">R-2.14.1.pkg</a> (latest version) <small>MD5-hash: afc80add76b331f65136db5bc479d8f3 (ca. 62MB)</small>	Three-way universal binary of <b>R 2.14.1</b> for Mac OS X 10.5 (Leopard) and higher. Contains R 2.14.1 framework, R.app GUI 1.43 in 32-bit and 64-bit. The above file is an Installer package which can be installed by double-clicking. Depending on your browser, you may need to press the control key and click on this link to download the file.
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This package **only** contains the R framework, 32-bit GUI (R.app) and 64-bit GUI (R64.app). **For Tcl/Tk libraries (needed if you want to use tcltk) and GNU Fortran (needed if you want to compile packages from sources that contain FORTRAN code) please see [the tools](#)**

**CRAN**  
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## Ly vs. Negreanu.

Ex. Suppose you have two ♣s, and there are exactly two ♣s on the flop. Given this info, what is  $P(\text{at least one more } \clubsuit \text{ on turn or river})$ ?

Answer:  $52-5 = 47$  cards left (nine ♣s, 38 others).

So  $n = \text{choose}(47,2) = 1081$  combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two-♣ combos:  $\text{choose}(9,2) = 36$ . One-♣ combos:  $9 \times 38 = 342$ .

Total = 378. So answer is  $378/1081 = 35.0\%$ .

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Answer #2: Use the addition rule...

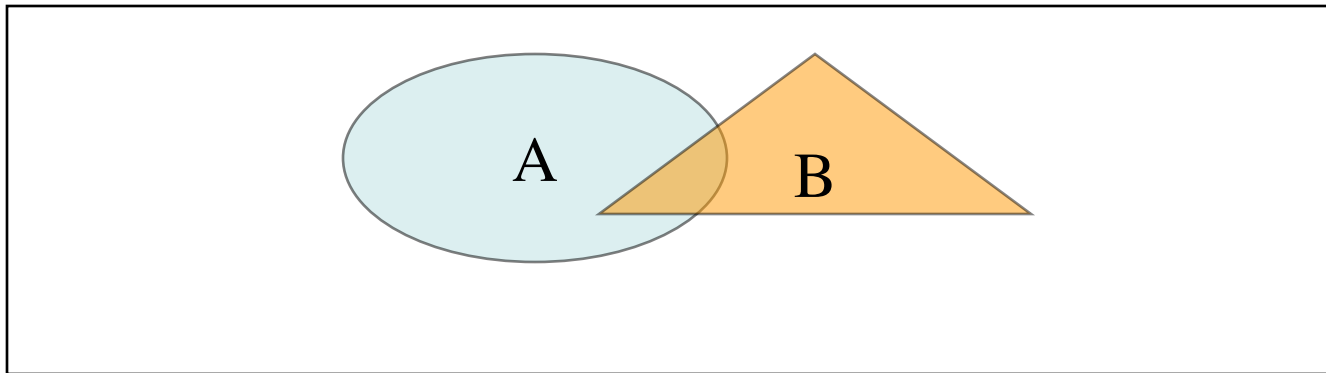
## ADDITION RULE, revisited.....

Axioms (initial assumptions/rules) of probability:

- 1)  $P(A) \geq 0$ .
- 2)  $P(A) + P(A^c) = 1$ .
- 3) Addition rule:

If  $A_1, A_2, A_3, \dots$  are mutually exclusive, then

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$



As a result, even if A and B might not be mutually exclusive,  
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ .

Ex. You have two ♣s, and there are exactly two ♣s on the flop.  
Given this info, what is  $P(\text{at least one more } \clubsuit \text{ on turn or river})?$

Answer #1:  $52-5 = 47$  cards left (nine ♣s, 38 others).

So  $n = \text{choose}(47,2) = 1081$  combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two- ♣ combos:  $\text{choose}(9,2) = 36$ . One-♣ combos:  $9 \times 38 = 342$ .

Total = 378. So answer is  $378/1081 = 35.0\%$ .

-----  
Answer #2: Use the addition rule.

$$\begin{aligned} P(\geq 1 \text{ more } \clubsuit) &= P(\clubsuit \text{ on turn OR river}) \\ &= P(\clubsuit \text{ on turn}) + P(\clubsuit \text{ on river}) - P(\text{both}) \\ &= 9/47 + 9/47 - \text{choose}(9,2)/\text{choose}(47,2) \\ &= 19.15\% + 19.15\% - 3.3\% = 35.0\%. \end{aligned}$$



## Counting.

Fact: If  $A_1, A_2, \dots, A_n$  are equally likely & mutually exclusive,  
and if  $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n) = 1$ ,  
then  $P(A_k) = 1/n$ .

[So, you can *count*:  $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = k/n$ .]

Ex. You have 76, and the board is KQ54. P(straight)?

[52-2-4=46.]  $P(\text{straight}) = P(8 \text{ on river OR } 3 \text{ on river})$

$$= P(8 \text{ on river}) + P(3 \text{ on river}) = 4/46 + 4/46.$$

If there are  $a_1$  distinct possible outcomes on experiment #1, and for each of them, there are  $a_2$  distinct possible outcomes on experiment #2, then there are  $a_1 \times a_2$  distinct possible *ordered* outcomes on both.

In general, with  $j$  experiments, each with  $a_i$  possibilities, the # of distinct outcomes *where order matters* is  $a_1 \times a_2 \times \dots \times a_j$ .

## Permutations and combinations.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?  
 $52 \times 51$ . [ordered: (A♣, K♦)  $\neq$  (K♦, A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck?  $52 \times 51 \times \dots \times 1 = 52!$

$$\sim 8.1 \times 10^{67}$$

A combination is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[ since now (A♣, K♦) = (K♦, A♣) ].

So, the number of *distinct* hands where *order doesn't matter* is

$$52 \times 51 / 2.$$

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

$$\text{"n choose k"} = \binom{n}{k} = \text{choose}(n,k) = \frac{n!}{k! (n-k)!} .$$

$$k! = 1 \times 2 \times \dots \times k. \quad [\text{convention: } 0! = 1.]$$

Deal til first ace appears. Let  $X$  = the *next* card after the ace.

$P(X = A\spadesuit)$ ?  $P(X = 2\clubsuit)$ ?