

Stat 19, Probability and Poker. Rick Paik Schoenberg

Outline for the day:

1. Discuss Harrington handouts.
2. Ly vs. Negreanu.
3. Addition rule.
4. $P(A\spadesuit \text{ after first ace})$.

Read harrington2.pdf for next time.

Think of at least 2 questions or comments, for each class.

The course website is <http://www.stat.ucla.edu/~frederic/19/S24> .

Ly vs. Negreanu.

Ex. Suppose you have two ♣s, and there are exactly two ♣s on the flop. Given this info, what is $P(\text{at least one more } \spadesuit \text{ on turn or river})$?

Answer: $52-5 = 47$ cards left (nine ♣s, 38 others).

So $n = \text{choose}(47,2) = 1081$ combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two-♣ combos: $\text{choose}(9,2) = 36$. One-♣ combos: $9 \times 38 = 342$.

Total = 378. So answer is $378/1081 = 35.0\%$.

Answer #2: Use the addition rule...

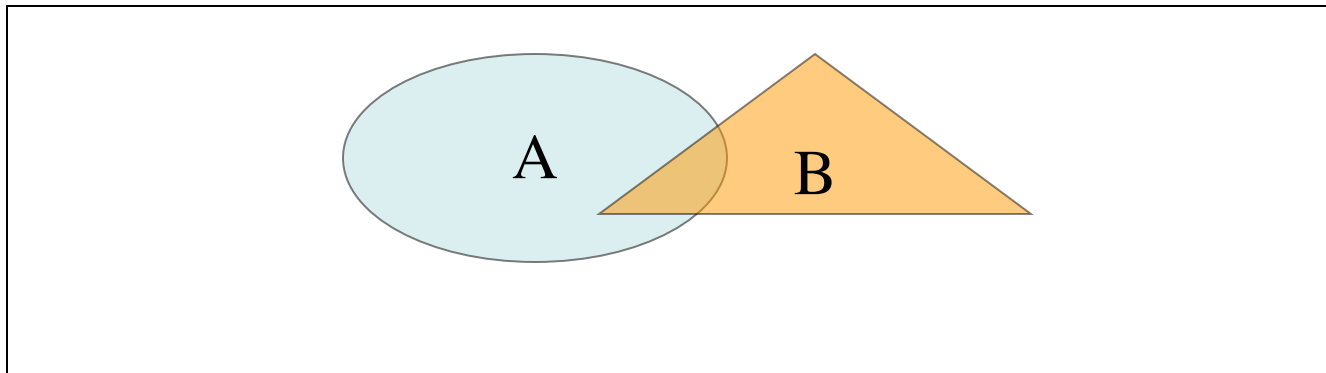
ADDITION RULE, revisited.....

Axioms (initial assumptions/rules) of probability:

- 1) $P(A) \geq 0$.
- 2) $P(A) + P(A^c) = 1$.
- 3) Addition rule:

If A_1, A_2, A_3, \dots are mutually exclusive, then

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$



As a result, even if A and B might not be mutually exclusive,
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Ex. You have two ♣s, and there are exactly two ♣s on the flop.
Given this info, what is $P(\text{at least one more } \clubsuit \text{ on turn or river})$?

Answer #1: $52-5 = 47$ cards left (nine ♣s, 38 others).

So $n = \text{choose}(47,2) = 1081$ combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two- ♣ combos: $\text{choose}(9,2) = 36$. One-♣ combos: $9 \times 38 = 342$.

Total = 378. So answer is $378/1081 = 35.0\%$.

Answer #2: Use the addition rule.

$$\begin{aligned} P(\geq 1 \text{ more } \clubsuit) &= P(\clubsuit \text{ on turn OR river}) \\ &= P(\clubsuit \text{ on turn}) + P(\clubsuit \text{ on river}) - P(\text{both}) \\ &= 9/47 + 9/47 - \text{choose}(9,2)/\text{choose}(47,2) \\ &= 19.15\% + 19.15\% - 3.3\% = 35.0\%. \end{aligned}$$

4. Deal til first ace appears. Let X = the *next* card after the ace.

$P(X = A\spadesuit)$? $P(X = 2\clubsuit)$?

(a) How many permutations of the 52 cards are there?

52!

(b) How many of these perms. have $A\spadesuit$ right after the 1st ace?

(i) How many perms of the *other* 51 cards are there?

51!

(ii) For *each* of these, imagine putting the $A\spadesuit$ right after the 1st ace.

1:1 correspondence between permutations of the other 51 cards & permutations of 52 cards such that $A\spadesuit$ is right after 1st ace.

So, the answer to question (b) is 51!.

Answer to the overall question is $51! / 52! = 1/52$.

Obviously, same goes for $2\clubsuit$.