Stat 19, Probability and Poker. Rick Paik Schoenberg

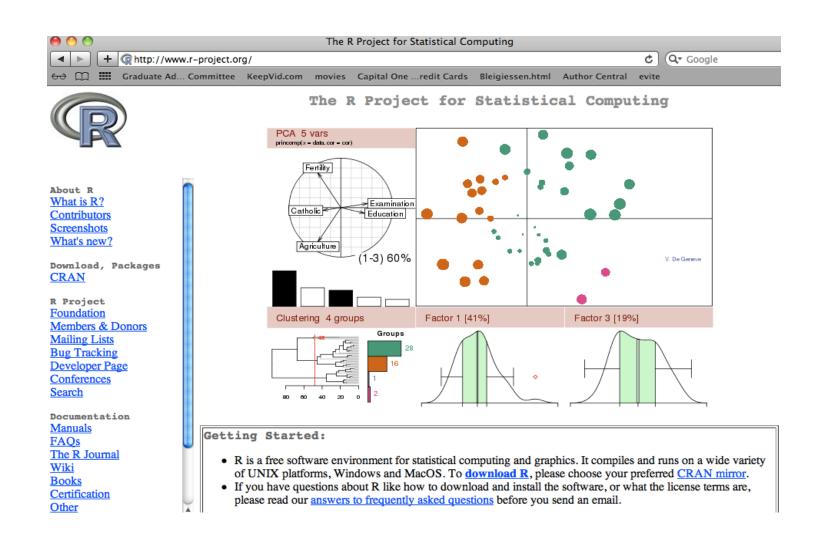
Outline for the day:

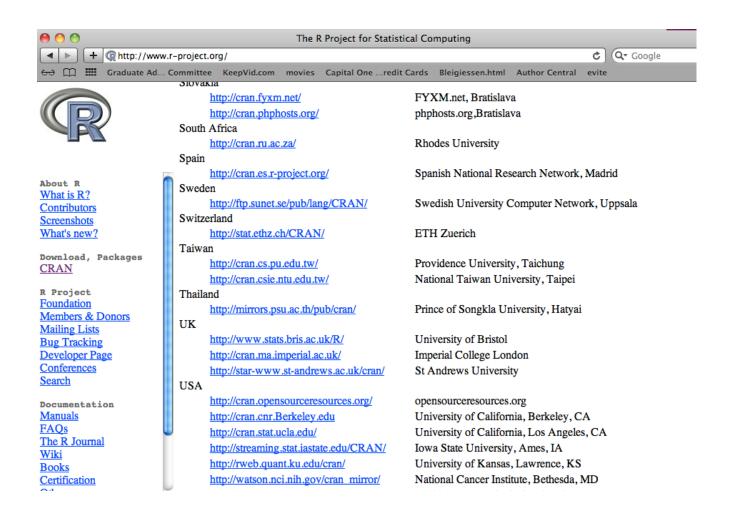
- 1. Discuss Addiction.
- 2. *R*.
- 3. Greenstein and Farha.
- 4. Axioms of probability.
- 5. Counting and combinations.
- 6. P(A♠ after first ace).

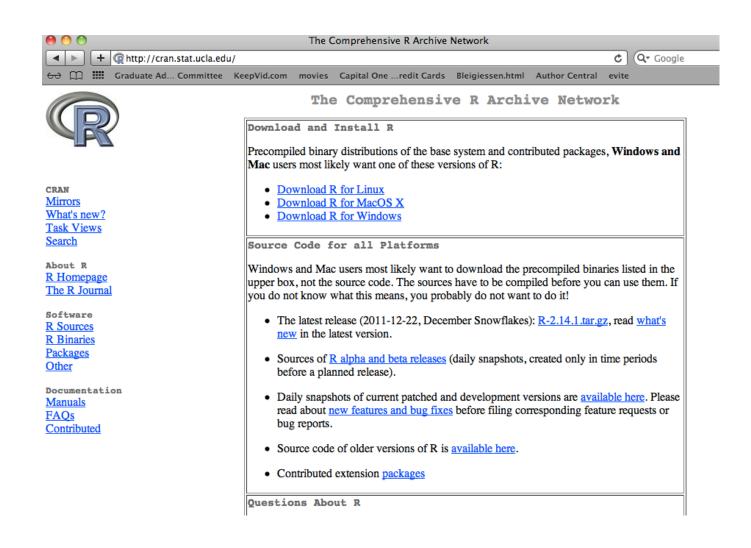
Read harrington1.pdf for next time.

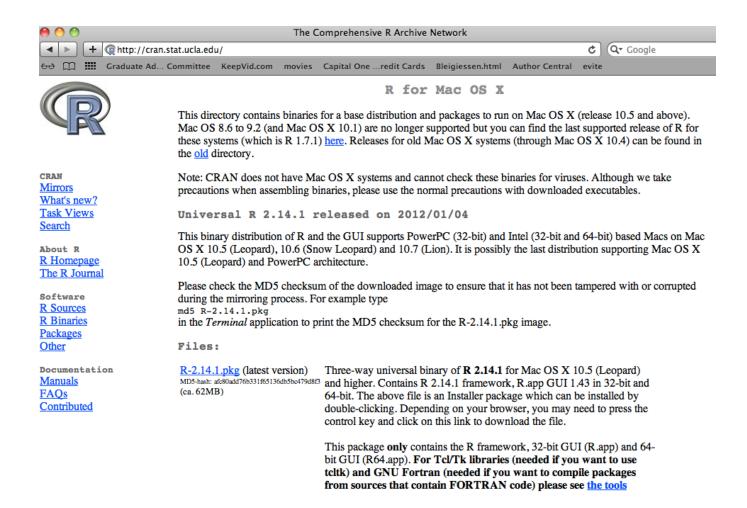
Think of 1-2 questions or comments for next time.

The course website is http://www.stat.ucla.edu/~frederic/19/S24.







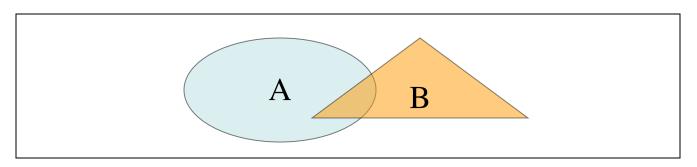


3. Greenstein and Farha.

4. Axioms (initial assumptions/rules) of probability:

- 1) $P(A) \ge 0$.
- 2) $P(A) + P(A^c) = 1$.
- 3) If A_1, A_2, A_3, \dots are mutually exclusive, then $P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

(#3 is sometimes called the *addition rule*)
Probability <=> Area. Measure theory, Venn diagrams



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

Counting.

Fact: If $A_1, A_2, ..., A_n$ are equally likely & mutually exclusive, and if $P(A_1 \text{ or } A_2 \text{ or } ... \text{ or } A_n) = 1$, then $P(A_k) = 1/n$.

[So, you can *count*: $P(A_1 \text{ or } A_2 \text{ or } ... \text{ or } A_k) = k/n$.]

Ex. You have 76, and the board is KQ54. P(straight)? [52-2-4=46.] P(straight) = P(8 on river OR 3 on river) = P(8 on river) + P(3 on river) = 4/46 + 4/46.

If there are a_1 distinct possible outcomes on experiment #1, and for each of them, there are a_2 distinct possible outcomes on experiment #2, then there are $a_1 \times a_2$ distinct possible *ordered* outcomes on both.

In general, with j experiments, each with a_i possibilities, the # of distinct outcomes where order matters is $a_1 \times a_2 \times ... \times a_i$.

Permutations and combinations.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities? 52×51 . [ordered: $(A - K) \neq (K - A)$.]

Each such outcome, where order matters, is called a *permutation*. Number of permutations of the deck? $52 \times 51 \times ... \times 1 = 52!$ $\sim 8.1 \times 10^{67}$ A <u>combination</u> is a collection of outcomes, where order <u>doesn't</u> matter. e.g. in hold'em, how many <u>distinct</u> 2-card hands are possible? 52×51 if order matters, but then you'd be double-counting each [since now $(A \clubsuit, K \spadesuit) = (K \spadesuit, A \clubsuit)$].

So, the number of *distinct* hands where *order doesn't matter* is $52 \times 51 / 2$.

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

"n choose k" =
$$\binom{n}{k}$$
 = choose(n,k) = $\frac{n!}{k! (n-k)!}$.

 $k! = 1 \times 2 \times ... \times k$. [convention: 0! = 1.]

Deal til first ace appears. Let $X = the \ next$ card after the ace.

$$P(X = A \clubsuit)? P(X = 2 \clubsuit)?$$