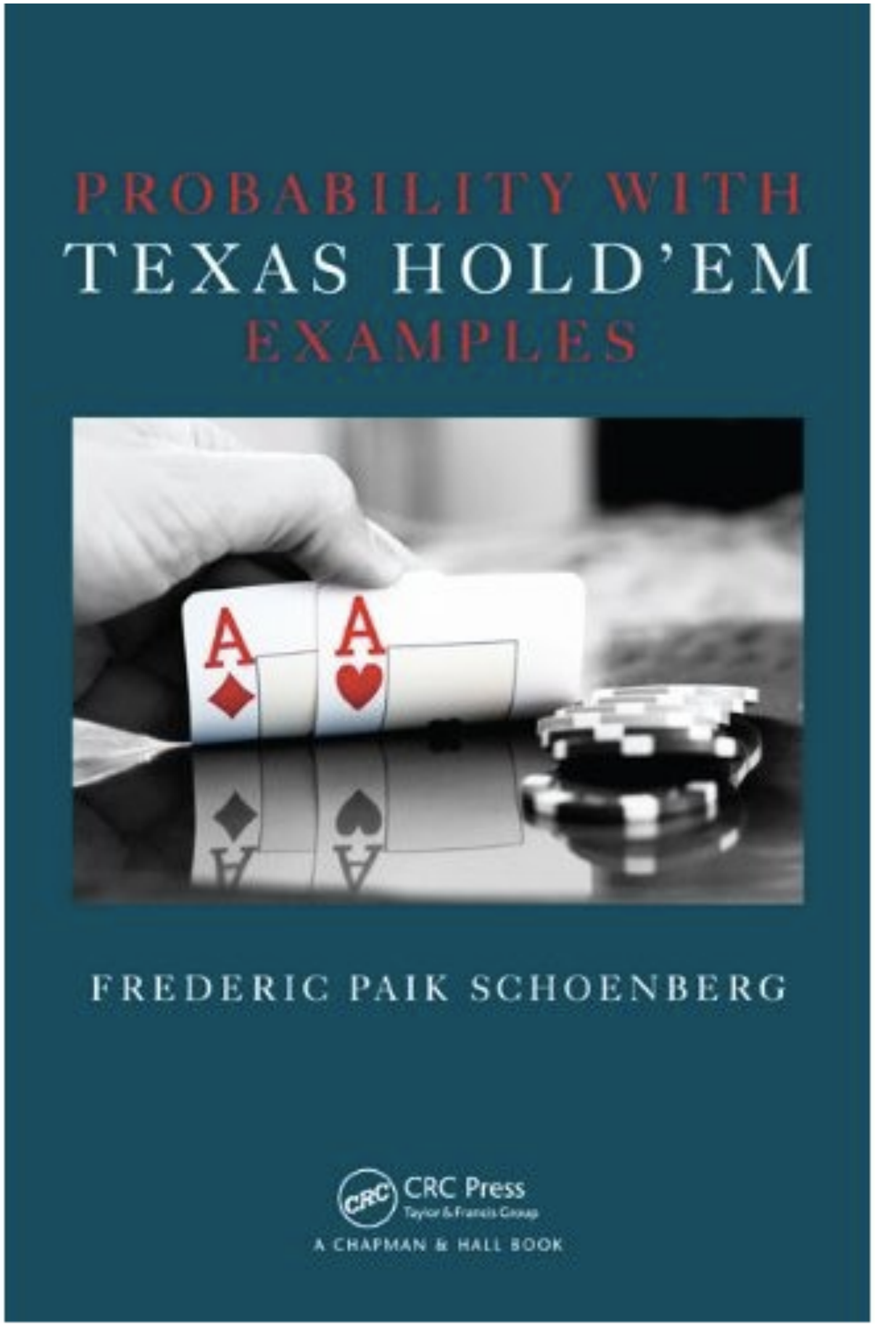

- Undergrad probability course (not a poker strategy guide nor an endorsement of gambling). Standard undergrad topics + random walks, arcsine laws, and a few other specialized topics.

- Instead of balls and urns, the examples involve Texas Hold'em.

- <u>Real</u> examples.

- Luck vs. skill.

- Computation using the *holdem* R package.

Example 8.3. On one interesting hand from Season 2 of High Stakes Poker, Corey Zeidman (9♥ 9♣) called $800, Doyle Brunson (Q♠ 10♠) raised to $6200, and Eli Elezra (10♥ 10♦), Daniel Negreanu (K♠ J♠), and Zeidman all called. Use *R* to calculate the probability that after the 3 flop cards are dealt, Elezra would be ahead, in terms of the best *current* 5-card poker hand, of his 3 opponents.

Answer. With 8 cards belonging to the 4 players removed from the deck, there are C(44,3) = 13,244 possible flop combinations, each equally likely to occur. One can use *R* to imitate dealing each of these flops, and seeing if Elezra is leading on the flop for each. After loading the functions in holdem, one may use the code below to solve this problem.

```
n = choose(44,3); result = rep(0,n); a1 = c(8,22,34,35,48,49,50,51)
a2 = c(1:52)[-a1]; i= 0
for(i1 in 1:42){for(i2 in ((i1+1):43)){for(i3 in ((i2+1):44)){
            flop1 = c(a2[i1],a2[i2],a2[i3]); flop2 = switch2(flop1)
            b1 = handeval(c(10,10,flop2$num),c(3,2,flop2$st))
            b2 = handeval(c(9,9,flop2$num),c(3,1,flop2$st))
            b3 = handeval(c(12,10,flop2$num),c(4,4,flop2$st))
            b4 = handeval(c(13,11,flop2$num),c(4,4,flop2$st))
             i = i+1;  if(b1 > max(b2,b3,b4)) result[i] = 1}}
cat(i1)}
sum(result > 0.5)
```

This code loops through all 13,244 possible flops and finds that Elezra is ahead on 5,785 out of 13,244 flops. Thus the probability is *5,785/13,244 ~ 43.68%.*

Incidentally, in the actual hand, the flop was 6♦ 9♦ 4♥, Zeidman went all in for $41,700, Elezra called, and Zeidman won after the turn and river were the uneventful 2♠ and 2♦.

Example 8.4. On High Stakes Poker, Daniel Negreanu has been on the losing side of some unfortunate situations where both he and his opponent had extremely powerful hands. For instance, in one hand against Gus Hansen, Negreanu had 6♠ 6♥, Hansen had 5♦ 5♣, and the board came 9♣ 6♦ 5♥ 5♠ 8♠. On another hand, against Erick Lindgren, Negreanu had 10♥ 9♥, Lindgren had 8♠ 8♣, and the board came Q♣ 8♥ J♦ 8♦ A♥. Such hands are sometimes called *coolers*. If, for simplicity, we assume that two players are both all-in heads up before their cards are dealt and we define a cooler as any hand where the two players both have straights or better, then what is the probability of a cooler? Perform 10,000 simulations in *R* to approximate the answer.

Answer. After loading the *holdem* package, the code below may be used to approximate a solution to this problem.

```
n = 10000; a = rep(0,n)
for(i in 1:n){
   x1 = deal1(2)
   b1 = handeval(c(x1$plnum1[1,],x1$brdnum1), c(x1$plsuit1[1,],x1$brdsuit1))
   b2 = handeval(c(x1$plnum1[2,],x1$brdnum1), c(x1$plsuit1[2,],x1$brdsuit1))
   if(min(b1,b2) > 4000000) a[i] = 1
   if(i/1000 == floor(i/1000)) cat(i)
}
sum(a>.5)
```

One run of this code resulted in 2,505 of the 100,000 simulated hands being coolers. (Different runs of the same code will yield slightly different results.) Thus, we estimate the probability as 2,505/100,000 = 2.505%. Using the central limit theorem, a 95% confidence interval for the true probability of a cooler is thus

2.505% ± √(2.505% 97.495% ÷ 100,000)
~ 2.505% ± 0.097%.

```
name1 = c("gravity","tommy","ursula","timemachine","vera","william","xena")
  decision1 = list(gravity, tommy, ursula, timemachine, vera, william, xena)
  tourn1(name1, decision1, myfast1 = 2)
  # tourn1(name1, decision1, myfast1 = 0, t1=0, t2=0.5)
```