

Homework 1. Stat 202a. Due Thu, Oct 15, 11:00am. **Late hws will not be accepted!**

You must work on the homework INDEPENDENTLY! Collaborating on this homework will be considered cheating. Submit your homework by email to stat202a@stat.ucla.edu. Your homework solution should be a single PDF document. The first pages should be your *output* from the problems below. This preferably means you typing out your answers and explaining them briefly in a clear, readable way, rather than simply cutting and pasting your R output. After that, on subsequent pages, include all your *code* for these problems. Your functions `pi2` and `pi3`, for example, should be included in the code section.

1. Assessing estimates of the 90th percentile of 100 iid uniform(0,1) random variables.

The R function `quantile()` implements a somewhat complex interpolation method in order to estimate a particular quantile, such as the 90th percentile. We will compare the estimate in `quantile()` with simpler estimates.

a) Write a function that takes as input a vector of length 100 and outputs the 90th of the 100 values sorted from smallest to largest. Note that the input vector might not be sorted.

b) Write a function to find the 91st of the sorted vector of 100 values.

c) Write a function that outputs the average of the 90th and 91st of the sorted vector of 100 values.

d) For each of your functions in parts a-c, as well as the function `quantile(x,0.9)`, do the following:

(i) Generate 100 iid uniform (0,1) random variables, and calculate your estimate of the 90th percentile.

(ii) Repeat step (i) 100,000 times.

(iii) Plot the sample mean of the first m of your estimates, as a function of m . (continued on next page)

e) Report the ultimate sample mean of your 100,000 estimates, for each of the four estimates. In 1-2 sentences, indicate which of the 4 estimates appears to be the best, and why.

2. Functions to approximate π .

a) Write a function called `pi2(n)` that approximates π as a function of n , using the approximation $\pi = \lim_{n \rightarrow \infty} \sqrt{6 \sum_{k=1}^n k^{-2}}$. Evaluate `pi2(10j)` for $j = 0, 1, 2, \dots, 6$.

b) Write a function `pi3(n)` that approximates π as a function of n , by simulating random points in the square with vertices $(-1,-1)$, $(-1,1)$, $(1,1)$, and $(1,-1)$, seeing what fraction of them are in the unit circle [the circle with radius 1 centered at the origin], and then converting this fraction into an estimate of π . Evaluate `pi3(10j)` for $j = 0, 1, 2, \dots, 6$. For $j=6$, plot your simulated points, using different plotting symbols for simulated points inside and outside the unit circle. There is no need for you to plot the unit circle also.