

Homework 3. Stat 202a. Due Thu, Nov 10, 11:59pm.

You must work on the homework INDEPENDENTLY! Collaborating on this homework will be considered cheating. **Late homeworks will not be accepted!** Your homework solution should be a single PDF document. The first pages should be your *output* from the problems. After that, on subsequent pages, include all your *code* for these problems. Email your homework to statgrader@stat.ucla.edu.

1. Approximation of an infinite series in C.

It is well known that $1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 + \dots = \ln(2)$.

Write a C function called *alt2(n)* that computes the first n terms in this series, as a function of n . Call your C function from R to evaluate *alt2(n)* for various n . Using R, plot *alt2(n)* vs. n , for n ranging from some small number up to 1 million. You may set up your range of the y-axis in a way that you feel is appropriate. You do not need to show *alt2(n)* for all values of n and should not plot *alt2* for very small values of n if they are off the plot.

2. Kernel density estimation in C and plotted in R.

Write a C function to compute a Gaussian kernel density estimate for univariate data. The inputs to the function should be two integers, m and n , a vector g of m gridpoints at which to calculate the estimates, a vector x consisting of the n observed data points, and a vector y of length m which will contain the resulting density estimates.

Gather data on all earthquakes of magnitude at least 3.0 in the longitude range -122.0 to -118.0 and latitude range 34.0 to 38.0, from Jan 1, 1960 to Oct 1 2022, from http://service.scedc.caltech.edu/eq-catalogs/date_mag_loc.php. Input the data into R. Use minimum magnitude = 3.0, maximum magnitude = 9.0, min depth = -5km, max depth = 100km, event type = earthquake, geographic type = local. Take this vector of earthquake magnitudes, and use your C function to make a kernel density estimate of the earthquake magnitudes, using a Gaussian kernel with bandwidth selected using the rule of thumb suggested by Scott (1992). You may calculate this bandwidth in R. Let $\{m_1, m_2, \dots, m_{100}\}$ be a vector of 100 equally spaced magnitudes spanning the observed range of magnitudes in your dataset, compute your kernel estimates on this grid using the C function, and plot your kernel density estimates $\hat{f}(m_1), \hat{f}(m_2), \dots, \hat{f}(m_{100})$.

3. Approximation of an integral in C.

Consider the integral from 0 to x_{\max} of the shifted Pareto density, $f(x) = (p-1) c^{p-1} (x+c)^{-p}$, for $x \geq 0$, and $f(x) = 0$ otherwise, where $c > 0$ and $p > 1$ are parameters.

Let $c = 3$ and $p = 2$. Write a C function called *paretoint(xmax,c,p)* that approximates this integral over a grid of 1 million values ranging from $x = 0$ to x_{\max} . Note that technically *paretoint()* is not only going to be a function of x_{\max} , c , and p , but will also have another input variable which will store the result. Call your C function from R to evaluate *paretoint(xmax,c,p)* for various choices of x_{\max} between 10 and 1000 (you do not need to calculate *paretoint* for every integer between 10 and 1000, but choose

around 10-15 numbers between 10 and 1000), and for $c = 3$ and $p = 2$ each time. Using R, plot $\text{paretoint}(xmax, 3, 2)$ vs. $xmax$, for $xmax$ ranging from 10 up to 1000. You may set up your range of the y-axis in a way that you feel is appropriate.

Repeat the above, but now using $c = 12$ and $p = 3.5$.

Output: Your output for this assignment should be a pdf document containing the following, in this order.

Figure 1. A plot of $\text{alt2}(n)$ versus n , for several values of n ranging up to 1 million.

Figure 2. A plot of your kernel density estimates $\hat{f}(m_1)$, $\hat{f}(m_2)$, ..., $\hat{f}(m_{100})$ versus m .

Figure 3. A plot of $\text{paretoint}(xmax, 3, 2)$ vs. $xmax$, for $xmax$ ranging from 10 to 1000.

Figure 4. A plot of $\text{paretoint}(xmax, 12, 3.5)$ vs. $xmax$, for $xmax$ between 10 and 1000.

All of your code, at the end.