Stat 221, Time Series Analysis. Day 2.

Outline for the day:

1. Syllabus, etc.

2. More examples of time series.

3. Stochastic processes.

4. White Noise (WN).

5. Moving Averages (MA).

6. AutoRegressive (AR) processes.

7. Suggested homework problems to look at on your own, not collected.

Again, note that the CCLE and Canvas websites for this course are not maintained.

The course website is http://www.stat.ucla.edu/~frederic/221/F22 .

Only one question is off limits, and it is "What did we do in class?"

We discussed various examples of time series, including quarterly financial time series, global and local temperature data, and speech.

Now we will review other examples.

Brain imaging.

As with fish and SOI data, in the brain imaging FMRI data from Figure 1.6, we might be interested in whether and how the two variables are related to one another, and whether the two areas of the brain might be responding to the stimulus in different ways.

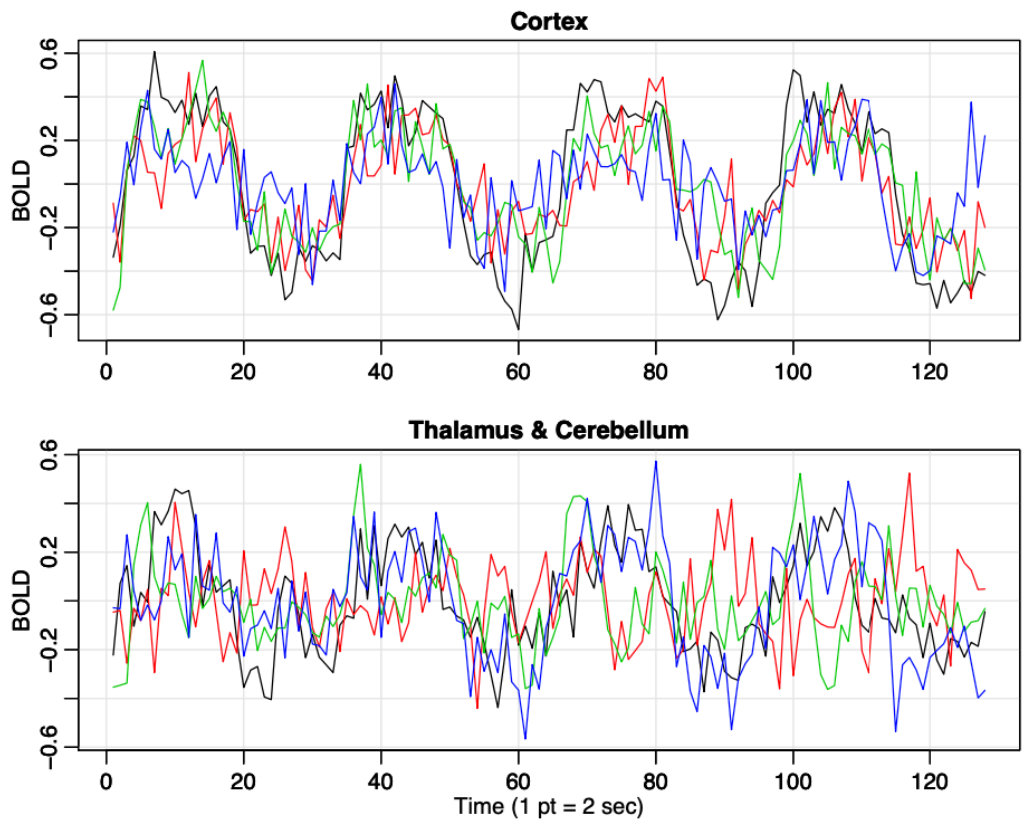

Fig. 1.6. *fMRI data from various locations in the cortex, thalamus, and cerebellum; $n = 128$ points, one observation taken every 2 seconds.*

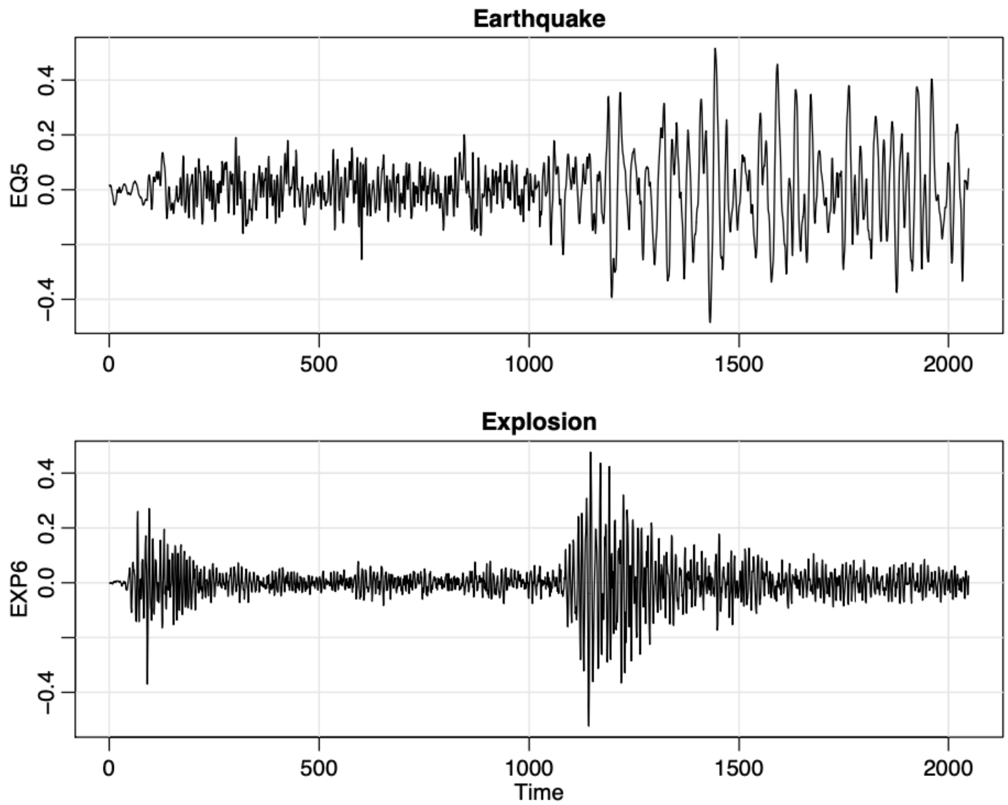

Fig. 1.7. *Arrival phases from an earthquake (top) and explosion (bottom) at 40 points per second.*

Earthquakes and explosions.

Our final example of time series data is the earthquake and explosion data in Figure 1.7. The data come from recording instruments in Scandinavia observing earthquakes and mining explosions. The question here is how to tell the difference between an earthquake and a mining explosion based on these seismograms. Seismologists have noticed that the Primary wave (the P-wave) and the Secondary wave (S-wave) seem to extend to heights that are more comparable in explosions than in earthquakes, where the height in the S-wave is much greater. These *heights* of the corresponding waves are called *amplitudes*. Here, the ratio of the amplitude of the S-wave to the amplitude of the P-wave seems to be about 2 for the earthquake data, whereas for the explosion data it is close to 1.

3. Stochastic processes and time series.

A collection of random variables $\{x_t\}$ indexed by time is called a *stochastic process*. An observation of a stochastic process is called a *realization* of the stochastic process.

Generally for a stochastic process, a value of x_t exists for any possible t. In this course, we will analyze time series, which may be viewed as stochastic processes but where we only observe x_t at times 0, 1, 2, 3, etc., or maybe also at times -1, -2, -3, etc., but not at all times. The key idea is that often it is useful to *imagine* that x_t exists at all times but we are only observing it at the integer times.

A basic question with any time series is how smooth it is. For many time series, if the value is large at time 100, then the value is probably also large at time 101. As a result, many of the basic models for time series try to characterize this relationship between x_t and x_{t-1}, especially in terms of the correlation between x_t and x_{t-1}.

4. White Noise.

If the time series consists of uncorrelated random variables with mean 0 and finite variance $\sigma_W{}^2$, then we call the series white noise (WN), and use the notation w_t for such a series. $w_t = wn(0, \sigma_W{}^2)$.

Often when we fit a more complicated model, the residuals from the model are posited to be WN, just as in regression, the errors are typically modelled as iid with mean 0 and constant variance.

The name white noise comes from the fact that with white light, all possible periodic cyclical components are equally strong. As we will see, in time series, the process WN, when decomposed into different cycles of different frequencies, also has equal strength among these cycles. Put another way, as we will see when we discuss chapter 4, the spectrum of WN is flat.

We will typically assume WN is independent and identically distributed (iid), though sometimes we do not need this assumption and it is sometimes enough only to assume the values are uncorrelated.

An example of WN is shown in the top of Figure 1.8, which shows 500 values where $\sigma_w^2 = 1$.

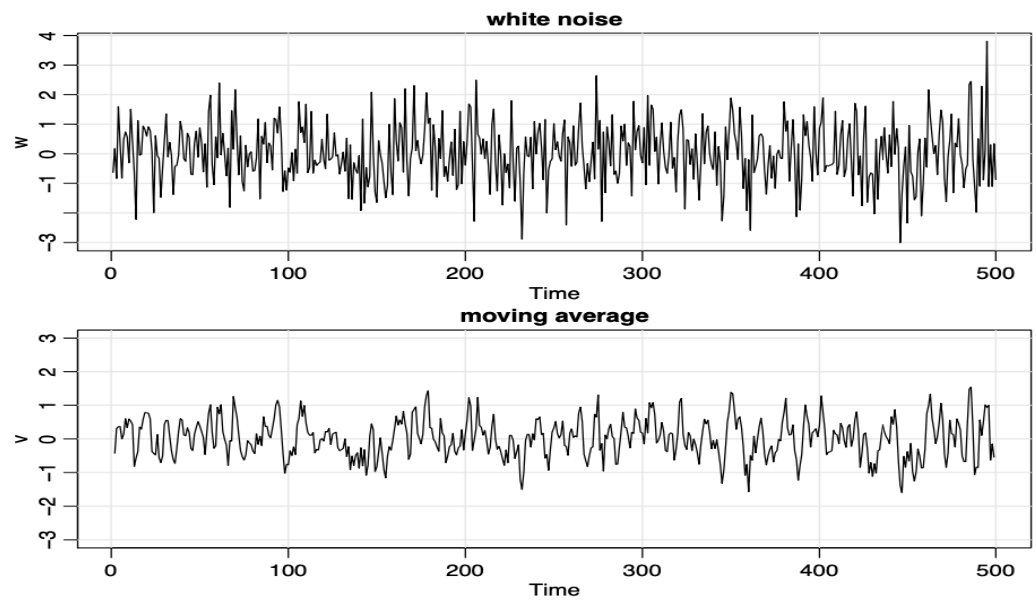

Fig. 1.8. Gaussian white noise series (top) and three-point moving average of the Gaussian white noise series (bottom).

5. Moving Average processes.

If we take WN, w_t, and smooth it out, we obtain a *moving average* (MA) process. For example, suppose we start with WN, w_t, and for each t, we let X_t be the average of w_t, w_{t-1}, and w_{t+1}. In other words, $X_t = 1/3\ (w_{t-1} + w_t + w_{t+1})$. The result is called an MA process and it is shown in the bottom panel of Fig 1.8. We can see that it is much smoother than the WN series. The *R* code

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v = filter(w, sides=2, filter=rep(1/3,3))
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was used to very simply create the vector v = X_t plotted in the figure.

6. AutoRegressive processes.

If we start with w_t = white noise and for each t, we let

$x_t = x_{t-1} - 0.9x_{t-2} + w_t$,

then w_t is called an AutoRegressive (AR) process. It is helpful to imagine starting by letting $x_1 = w_1$ and $x_2 = w_2$, and then generating the values x_3 , x_4 , etc., one at time, up to x_{500}.

For instance, $x_3 = x_2 - 0.9x_1 + w_3$,

and $x_4 = x_3 - 0.9x_2 + w_4$,

and we can easily imagine continuing and generating all 500 values of x_t this way.

The resulting AR process is shown in Figure 1.9. It looks periodic, because if say w_{100} is very large and positive, then this will tend to make x_{100} be large and positive, which will in turn make x_{102} be large in the negative direction, which will in turn cause x_{104} to be large and positive, etc.

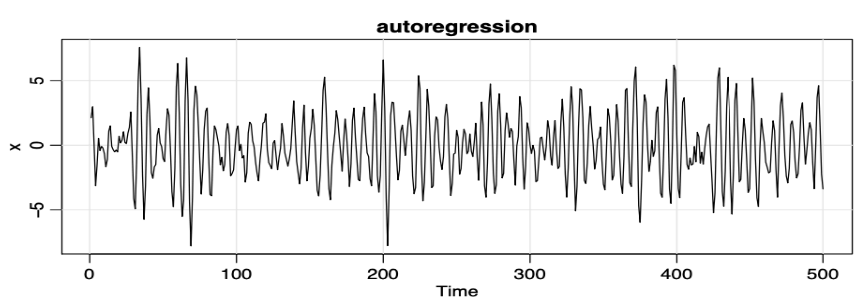

Fig. 1.9. Autoregressive series generated from model (1.2).

7. Suggested but not collected hw problems.

1.3, 1.10, 1.19, 1.20, 1.21, and 1.27. On problem 1.10, where it says "and autocorrelation function g(h) at some time in the future", it should say "and autocovariance function g(h) at some time in the future".
2.9, 3.1, 3.2, 3.4, 3.6, 3.9, 4.5, 4.9, and 4.19.
See tsa4.pdf to find these problems, and see oldhw1notes.pdf for some hints on the first few problems.