Statistics 222, Spatial Statistics.

<u>Outline for the day:</u>

1. Syllabus, etc.
2. Examples of spatial-temporal point processes.
3. Characterizations of point processes.
4. Integration.

Read reinhart18.pdf .

Note that I will be putting course materials not on the Canvas or CCLE websites but on the main course website.

The course website is http://www.stat.ucla.edu/~frederic/222/F23 .

You can always ask me questions by email. Only one question is off limits, and it is "What did we do in class?" If you need to ask this question, please ask one of your fellow students.

1. Syllabus.

2. Examples of spatial-temporal point processes.

A point process is a random collection of points falling in some metric space. For a spatial-temporal point process, the metric space is a portion of space-time, $S = R^d \times R$. Often d = 2, but sometimes 3.

Examples include incidence of disease, sightings or births of a species, occurrences of fires, earthquakes, lightning strikes, tsunamis, or volcanic eruptions.

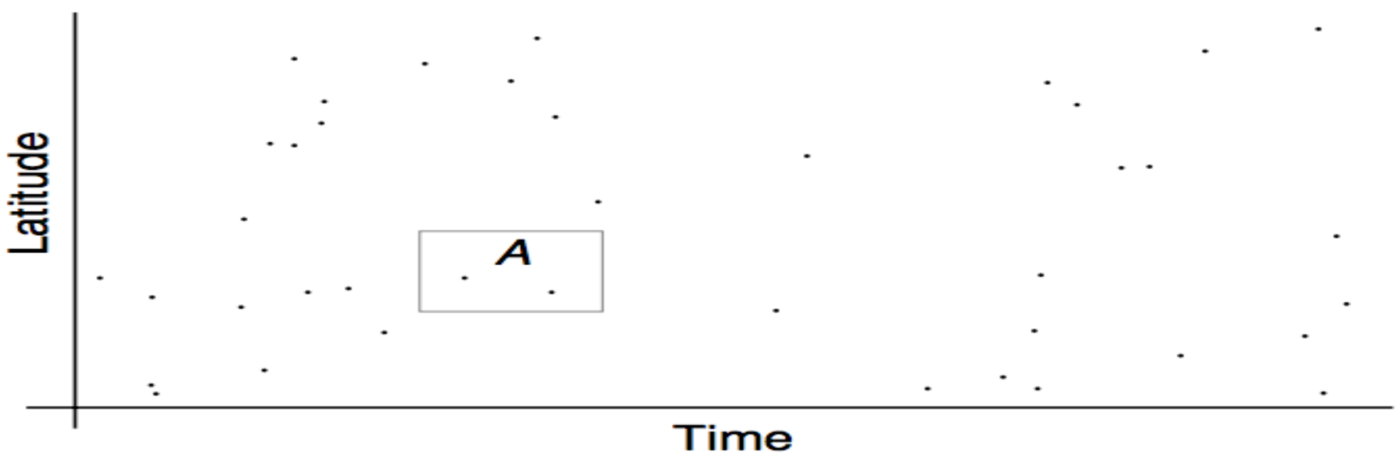
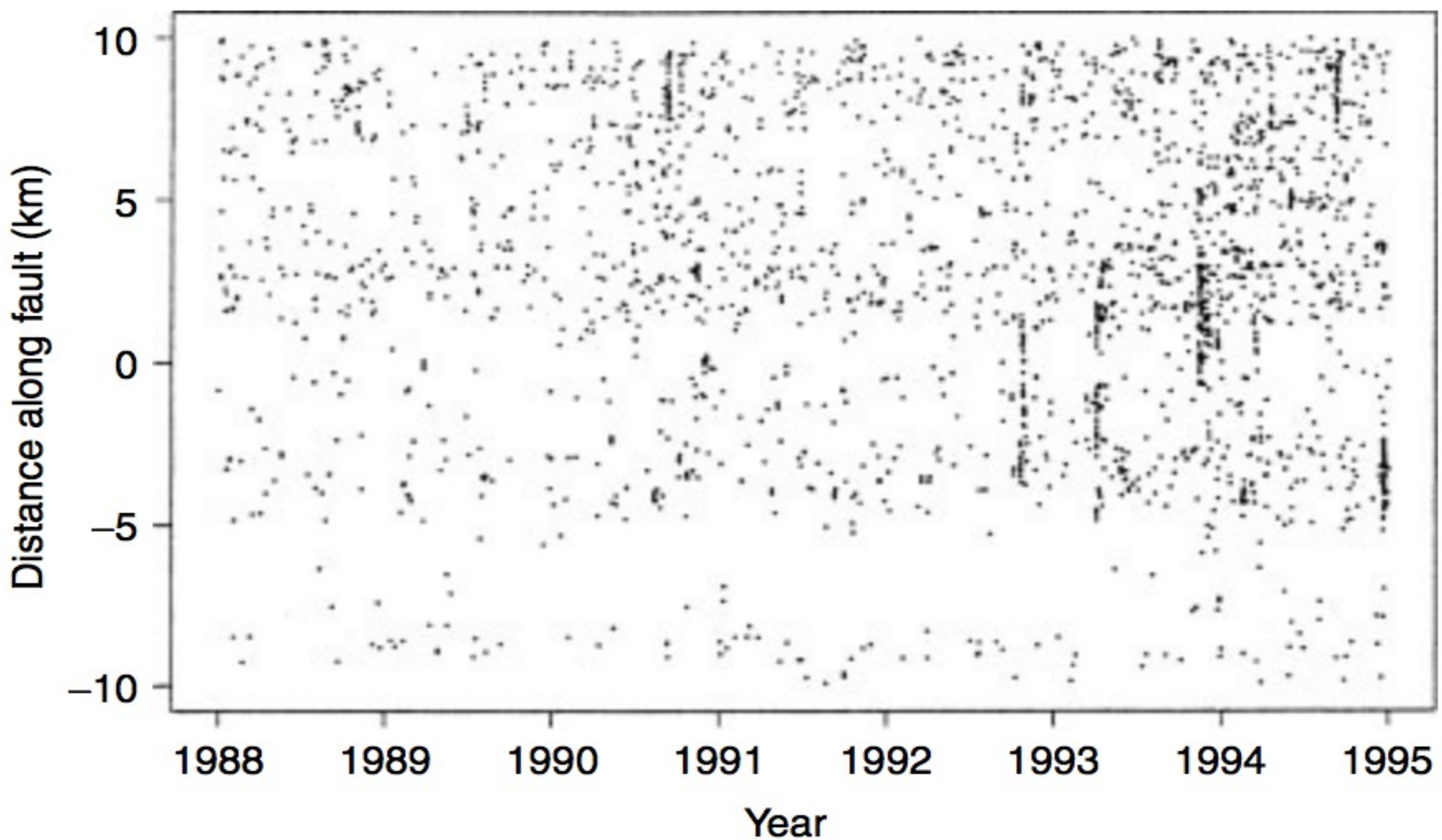


Figure 1 Spatial–temporal point process.

Microearthquake origin times and epicenters in Parkfield, CA, recorded by the US High-Resolution Seismographic Station Network.

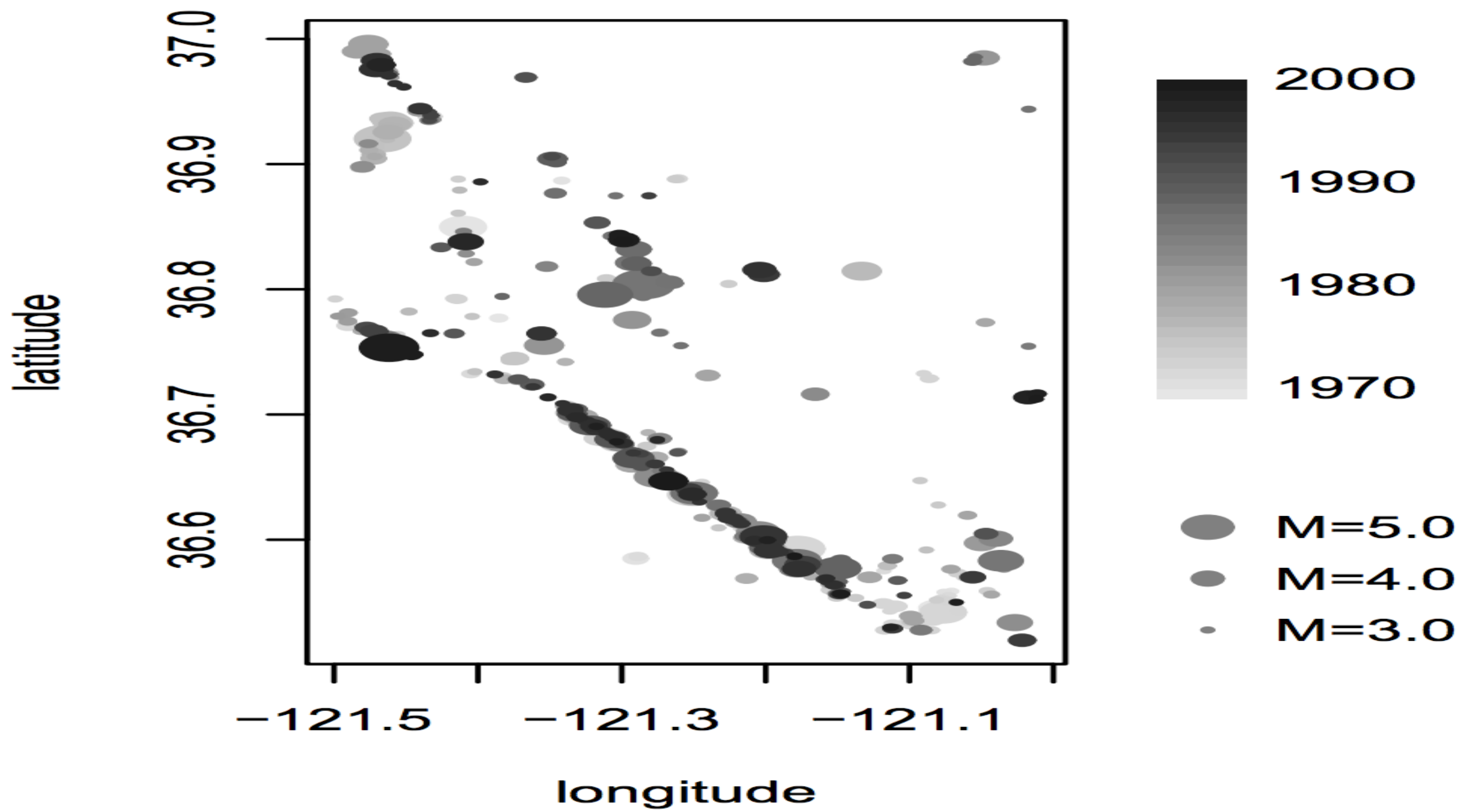


Wildfire centroids between 1876 and 1996 in Los Angeles County, CA, recorded by the Los Angeles County Department of Public Works

Figure 3 Centroids of recorded Los Angeles County wildfires, 1878–1996.

For a marked point process, each point has some mark or random variable associated with it.

Locations, times and magnitudes of moderate-sized ($M \geq 3.5$) earthquakes in Bear Valley, CA, between 1970 and 2000.



3. Characterizations of point processes.

Point processes have been characterized various ways.

Daley and Vere-Jones (2003) detail the history originating from life tables, dating back to the 1700s.

3a. STOCHASTIC PROCESS.

Point processes on the line were originally characterized as examples of stochastic processes on the line, that are:

* Non-decreasing,
* Z+ valued.

So it is tempting to define a point process as any non-decreasing, Z+ valued stochastic process. On the line, this definition works just fine.

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   0      1    2  3           4     5      6   7
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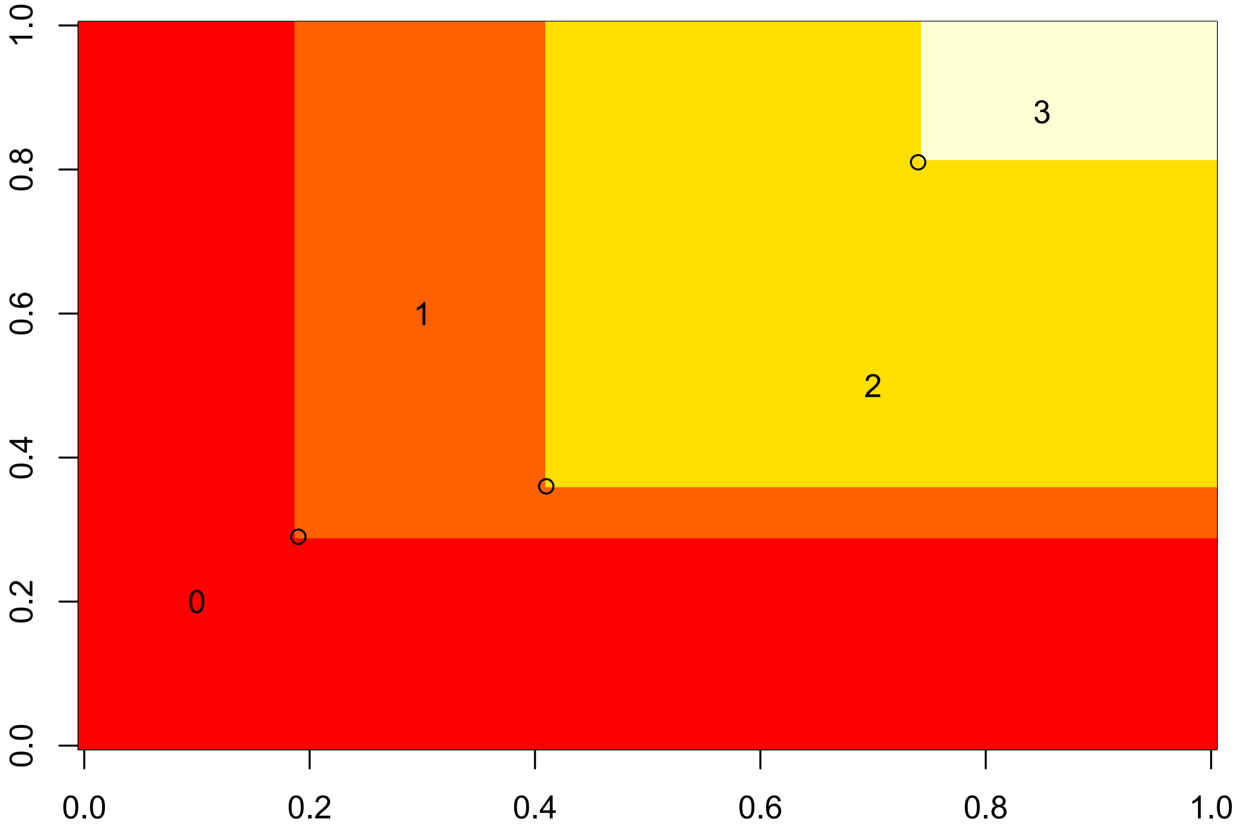
3a continued.

Non-decreasing, Z+ valued stochastic process.

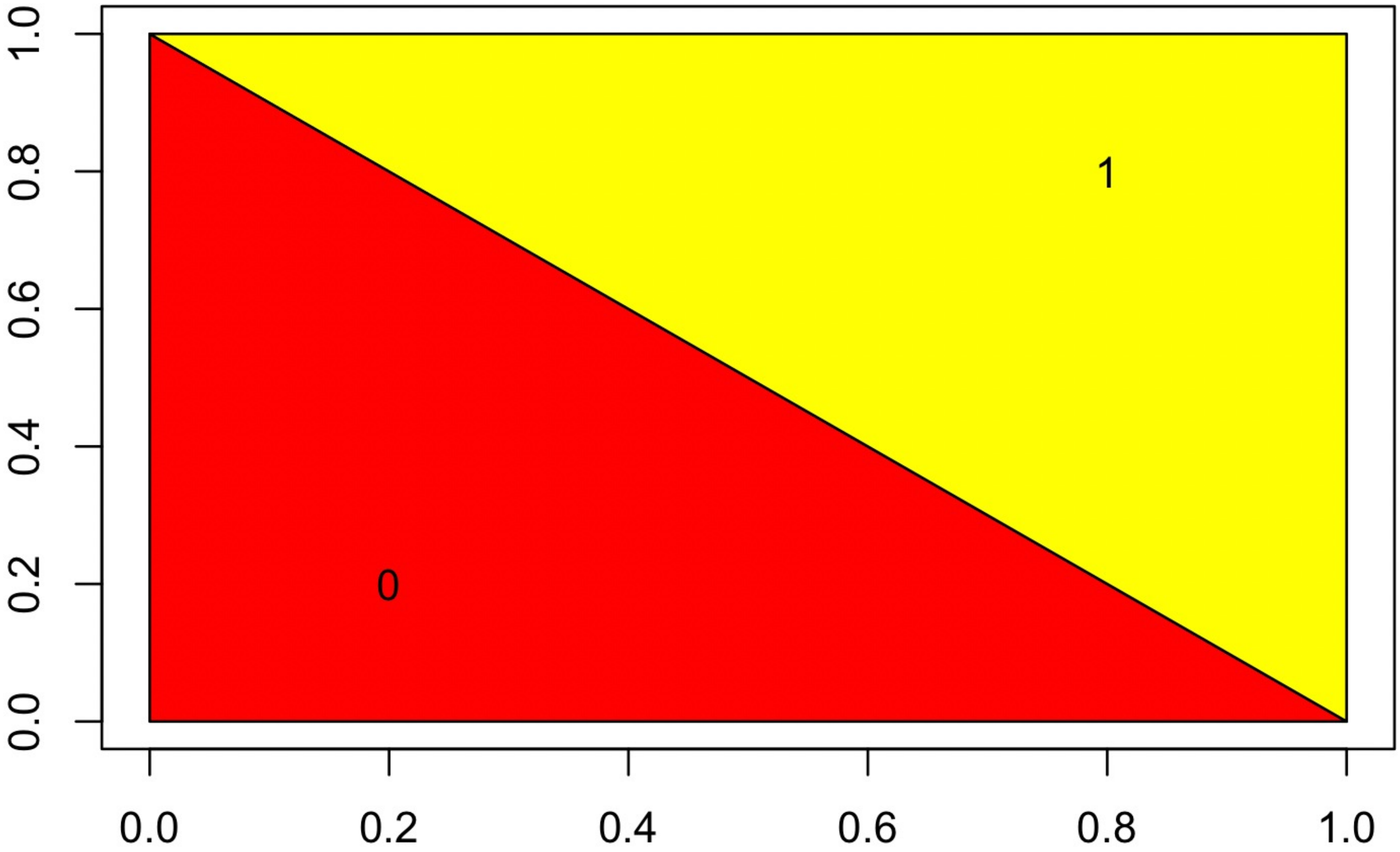
This has problems extending to spatial-temporal processes.

Let N(x,y) = the number of points (x_1,y_1) with $x_1 \leq x$ and $y_1 \leq y$.



3a continued.

It is very unclear what points this nondecreasing Z+ valued process corresponds to though.



3b. A LIST OF POINTS.

For any finite collection of points, one could represent it simply as a finite list of points, $N = \{x_1, x_2, ...\}$.

J. Møller and R. Waagepetersen (2004), *Statistical Inference and Simulation for Spatial Point Processes*, CRC, Boca Raton uses this formulation.

If $S = R^d \times R$ is the domain where the points occur, then N is a vector in $\emptyset \cup S \cup S^2 \cup S^3 \cup ....$

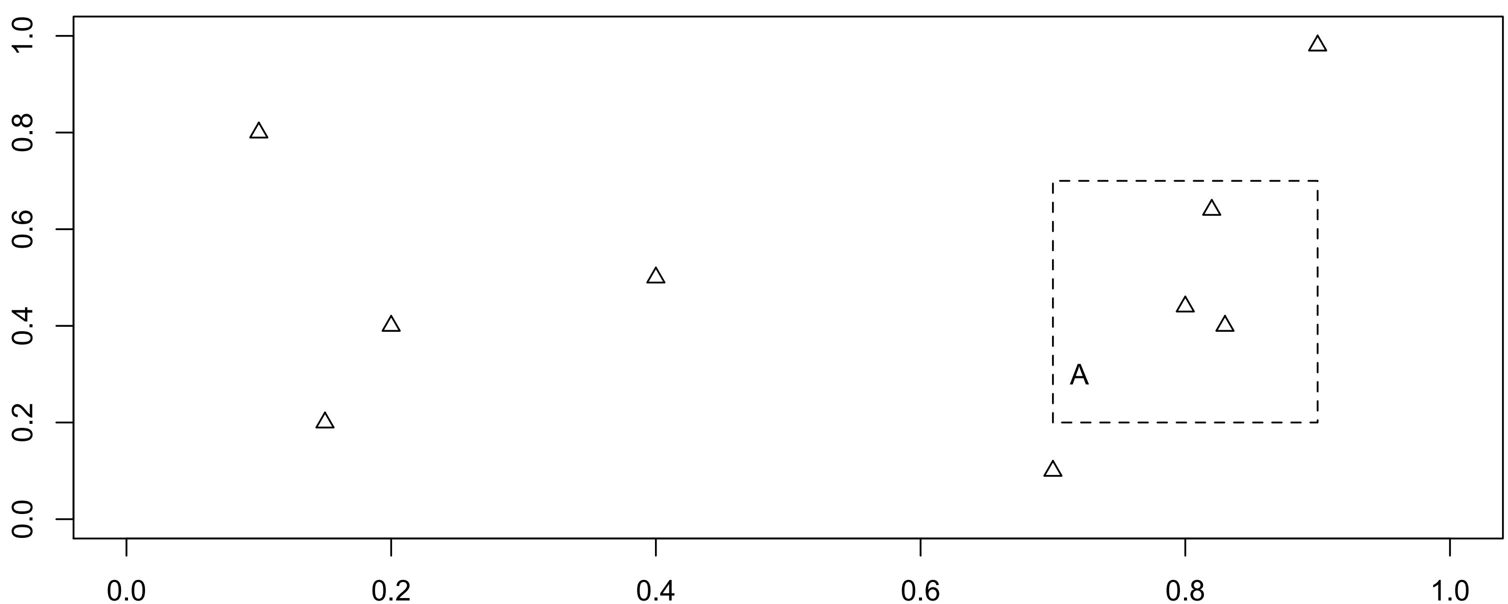
This works just fine for finite point processes, and point processes with a countable number of points. But you can imagine wanting to consider the point process with points everywhere, or points at all irrational times, or some other uncountable collection of points.

You could restrict your definition to include only point processes with points on a set of measure zero, but then what about if N has points at all times in the Cantor set?

3c. RANDOM MEASURE.

A Z+ valued random measure is now the preferred mathematical definition, as it includes a wide range of processes on the line and extends readily to space-time.

The measure N(A) represents the number of points falling in the region A of space-time. For this set A, for example, the value of N(A) is 3. Attention is sometimes restricted to processes with only a finite number of points in any compact subset of S.



3c, continued.

In the 1950s-1970s much attention was paid to the idea that there are non-measurable sets. N(A) is technically only defined for measurable A. A collection of points at all times in the Vitali set, for instance, if not considered a valid point process.

With the random measure formulation, the domain *S* can also be some abstract space. Brillinger (1997, 2000) analyzed point processes on the surface of a sphere or on an ellipse, for instance.

Brillinger, D.R. (1997). A particle migrating randomly on a sphere, Journal of Theoretical Probability 10, 429-433.

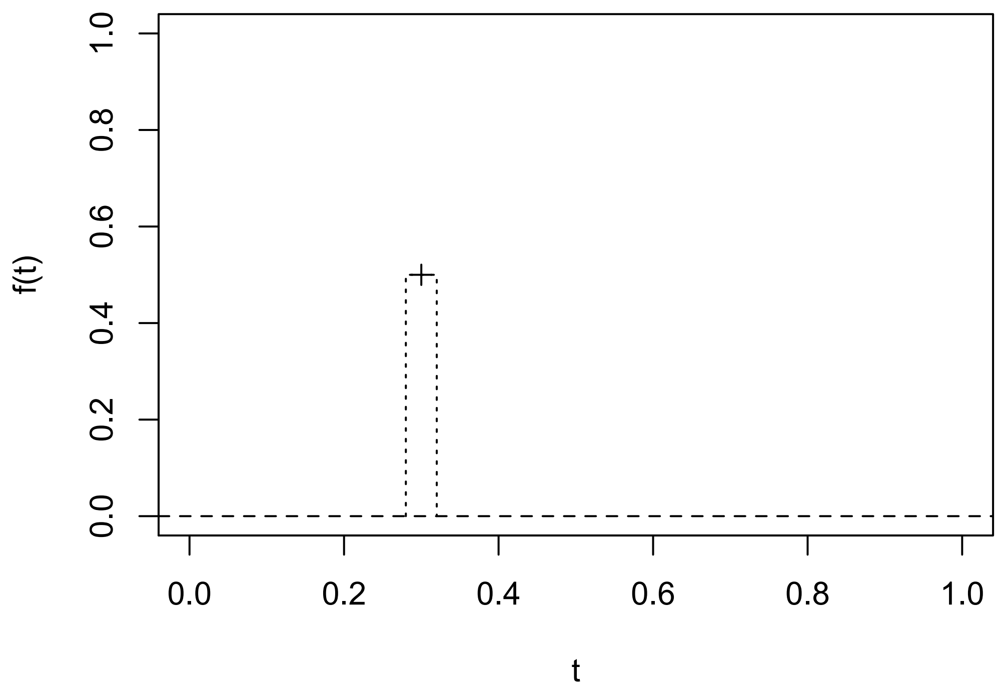
Brillinger, D.R. (2000). Some examples of random process environmental data analysis, in Handbook of Statistics, Vol. 18, C.R. Rao & P.K. Sen, eds, NorthHolland, Amsterdam, pp. 33–56.

4. Integration.

Note that with the measure theory formulation, which we will use throughout, $N(B) = \int_B dN$ is the number of points in B.

What is $\int_t \int_x \int_y f(t,x,y)\, dN$?

It is simply $\sum_i f(t_i,x_i,y_i)$.



Exercises.

1. Suppose the spatial-temporal point process *N* has points at

time 1.2, x=2, y=3.

time 2.4, x=3, y=0.5.

time 8.7, x=2, y=1.

Let *B* = [0,10] (time) x [1.5, 2.5] x [0,5].

What is $\int_B dN$?

Exercises.

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2.

Exercises.

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time 8.7, x=2, y=1.

Let *B* = [0,10] (time) x [0,5] x [0,5].

What is $\int_B (t+x^2y)\, dN$?

Exercises.

2. Suppose the spatial-temporal point process *N* has points at

time 1.2, x=2, y=3.

time 2.4, x=3, y=0.5.

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What is $\int_B (t+x^2y)\, dN$?

13.2 + 6.9 + 12.7 = 32.8.