**Statistics 222, Spatial Statistics.**

<u>Outline for the day:</u>

1. Marked G and J functions.
2. Weighted K function.
3. van Lieshout p26, 37-38.
4. Project order.
5. Kernel smoothing, summary functions, model fitting, and weighted K function for spatial point processes, unmarked and marked, in R. day07.r.

1. Marked G and J functions.

$G(r) = P_0$(point within r), where $P_0$ means given a pt. at 0.
It is estimated with $\hat{G}(r) = 1/n \sum_i 1(\text{there is } j: |\tau_i - \tau_j| \le r)$.
$= 1/n \sum_i 1(\min_{i \neq j} |\tau_i - \tau_j| \le r)$

One could alternatively compute a *marked* G-function
$1/n_1 \sum_i 1(\min_j |\tau_i - \tau_j| \le r)$
where the sum is over the $n_1$ points $\tau_i$ with mark in some range $M_1$, and the minimum is over the points $\tau_j$ with mark in some range $M_2$.
This is the *marked* or *cross* G-function.
One can similarly define a marked or cross J-function as $J(r) = (1-G(r)) / (1-F(r))$ accordingly, plugging in the corresponding G function.

Marie-Collette van Lieshout

van Lieshout, M.N.M. (2006). A J-function for marked point patterns. *AISM* 58, 235-259.

## 2. Weighted K function.

For a stationary Poisson process with rate $\mu$,
$K(r) = 1/\mu$ E(# of other points within distance *r* of a randomly chosen point).
Estimated via $K_4(r) = 1/(\lambda\hat{}\ n) \ \sum_{i \neq j} (|\tau_i - \tau_j| \le r)\ w(\tau_i, \tau_j)$,
where $\lambda\hat{} = n/|S|$, and $w(\tau_i, \tau_j)$ = 1/proportion of circle centered at i going through j that is in S = border correction term.
If N is inhomogeneous, can instead weight each point by $1/\lambda$,
obtaining $K_w(r) = 1/n \ \sum_{i \neq j} (|\tau_i - \tau_j| \le r)\ w(\tau_i, \tau_j) / \lambda(\tau_i) / \lambda(\tau_j)$.

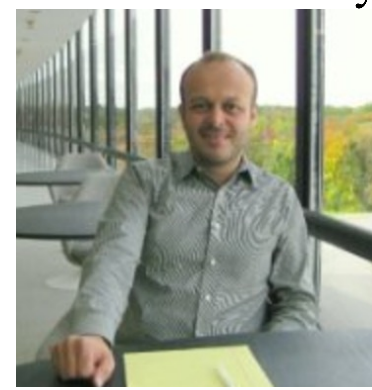
$K_w(r) \sim N(\pi r^2, 2\pi r^2|S| / E(n)^2)$, if $\inf \lambda = 1$.

Baddeley, A., Møller, J., Waagepetersen, R. (2000). Non and semi-parametric estimation of interaction in inhomogeneous point patterns. *Statistica Neerlandica*, 54(3), 329-350.

Veen, A. and Schoenberg, F.P. (2006). Assessing spatial point process models for California earthquakes using weighted K-functions: analysis of California earthquakes, in *Case Studies in Spatial Point Process Models*, Baddeley, A., Gregori, P., Mateu, J., Stoica, R., and Stoyan, D. (eds.), Springer, NY, pp. 293-306.

Adelfio, G. and Schoenberg, F.P. (2009). Point process diagnostics based on weighted second-order statistics and their asymptotic properties. *Annals of the Institute of Statistical Mathematics*, 61(4), 929-948.

Adrian Baddeley



Alejandro Veen

Giada Adelfio

3. $\gamma(t) = \rho(0) - \rho(t)$, for 2nd order stationary processes, from van Lieshout p23. Why?
2nd order stationary = weakly stationary and means
$E(X_t^2) < \infty$, $E(X_0) = E(X_1) = \ldots = E(X_t)$ for all t,
and $Cov(X_0, X_t) = Cov(X_1, X_{t+1}) = Cov(X_2, X_{t+2})$, etc., for any t.
If 2nd order stat., then letting $t = 0$, $Var(X_0) = Var(X_1) = \ldots = Var(X_t)$ for all t.

The semivariogram $\gamma(t) = Var(X_t - X_0)/2$.
The covariogram $\rho(t) = Cov(X_0, X_t)$.

So $\rho(0) - \rho(t) = Cov(X_0, X_0) - Cov(X_0, X_t) = Var(X_0) - Cov(X_0, X_t)$.

$$\begin{aligned}
\gamma(t) &= Var(X_t - X_0)/2 \\
&= Cov(X_t - X_0, X_t - X_0)/2 \\
&= \{Cov(X_t, X_t) + Cov(X_0, X_0) - 2Cov(X_0, X_t)\}/2 \\
&= Var(X_t)/2 + Var(X_0)/2 - Cov(X_0, X_t) \\
&= Var(X_0) - Cov(X_0, X_t) \\
&= \rho(0) - \rho(t).
\end{aligned}$$

4. Presentation times?

5. Kernel smoothing, summary functions, model fitting, and weighted K function for spatial point processes, unmarked and marked, in R. day07.r.