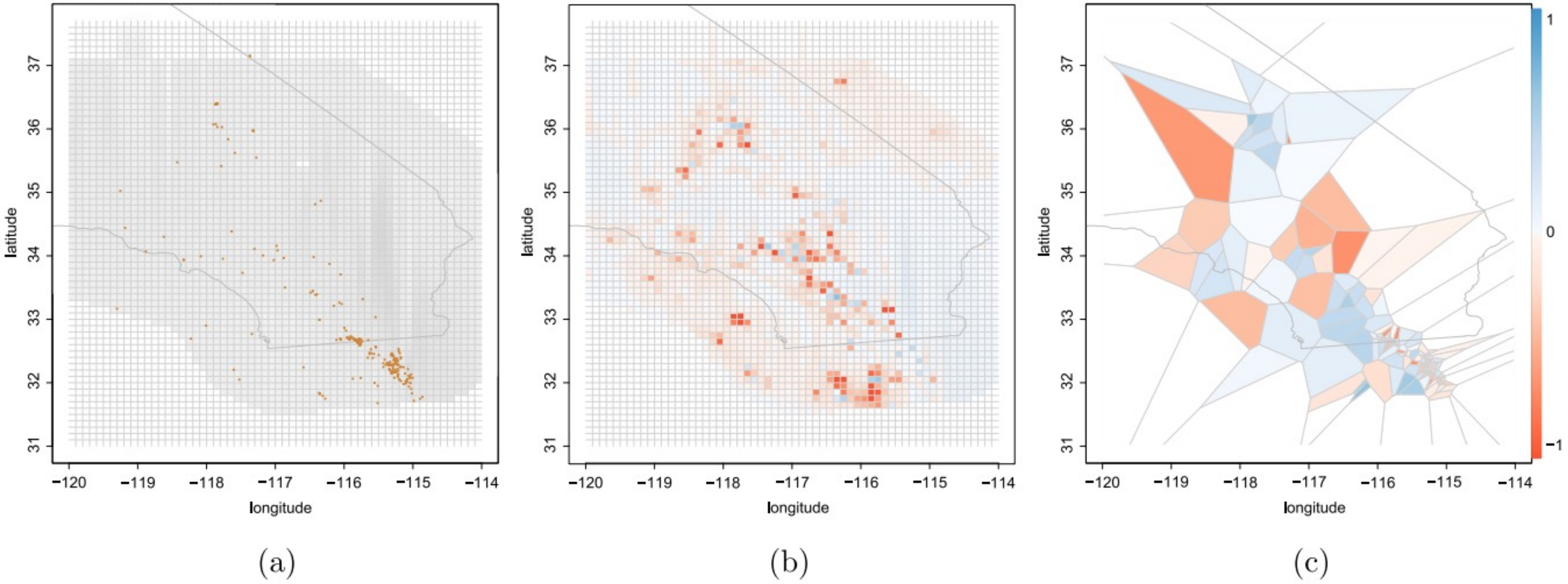
Statistics 222, Spatial Statistics.

Outline for the day:

1. Superthinning.
2. Exercises.
3. van Lieshout, Kriging, CAR, SAR, and ch3 examples.

No class Mon Nov6!

With 2 models, can compare loglikelihoods across pixels or Voronoi cells.



(a) (b) (c)

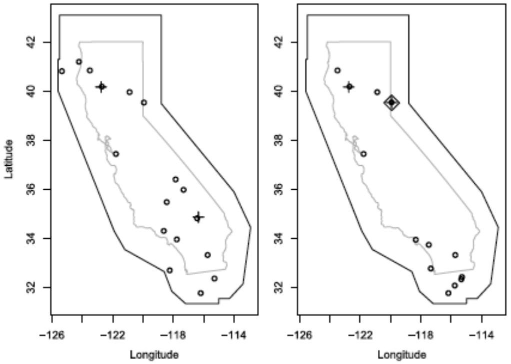
FIG. 3. (a) *Estimated rates under the Shen, Jackson and Kagan* (2007) *model, with epicentral locations of observed earthquakes with* $M \geq 4.0$ *in Southern California between January* 1, 2006 *and January* 1, 2011 *overlaid.* (b) *Pixel deviance plot with blue favoring model A, Helmstetter, Kagan and Jackson* (2007), *versus model B, Shen, Jackson and Kagan* (2007). *Coloration is on a linear scale.* (c) *Voronoi deviance plot with blue favoring model A, Helmstetter, Kagan and Jackson* (2007), *versus model B, Shen, Jackson and Kagan* (2007). *Coloration is on a linear scale.*

1. Superthinning. (Clements et al., 2012)

Choose some number $c \sim \text{mean}(\hat{\lambda})$.

Superpose: where $\hat{\lambda}(t,x,y) < c$, add in points of a simulated Poisson process of rate $c - \hat{\lambda}(t, x, y)$.

Thin: where $\hat{\lambda}(t_i,x_i,y_i) > c$, keep each point (t_i,x_i,y_i) with prob. $c / \hat{\lambda}(t_i,x_i,y_i)$.



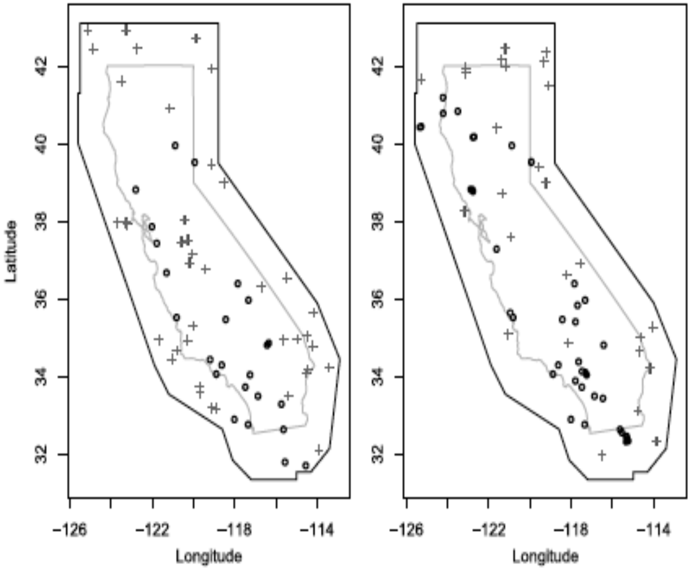


FIG. 11. *One realization of super-thinned residuals for the five models considered (circles = observed earthquakes; plus signs = simulated points). Top-left panel* (a): *model A* (*k* = 2.76). *Top-center panel* (b): *model B* (*k* = 2.95). *Top-right panel* (c): *model C* (*k* = 2.73). *Bottom-left panel* (d): *ETAS* (*k* = 1.35). *Bottom-right panel* (e): *STEP* (*k* = 0.75).

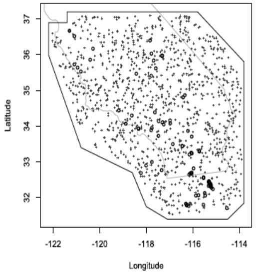


FIG. 9. *Superposed residuals for model C. Simulated points make up 90.7% of all points.*

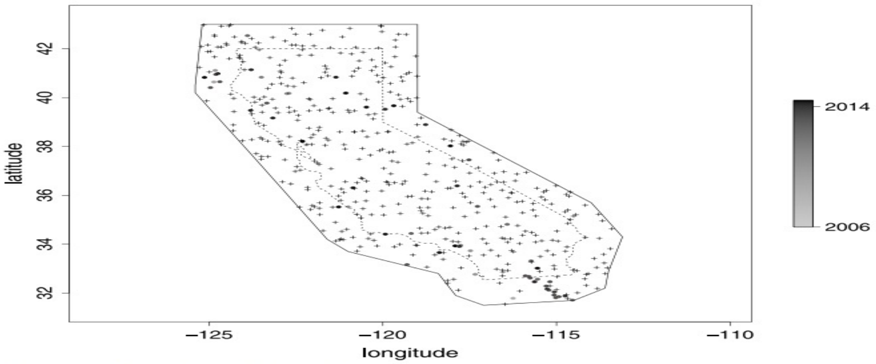
1. Superthinning. (Clements et al., 2013)

Choose some number c. mean($\hat{\lambda}$) at the points is a suggested default.

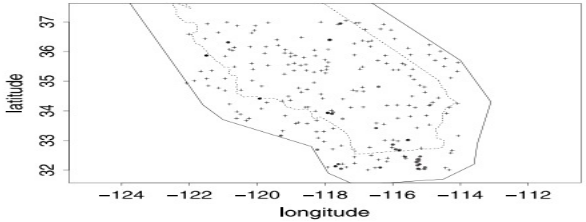
Superpose: where $\hat{\lambda}$ (t ,x,y) < c, add in points of a simulated Poisson process of rate c - $\hat{\lambda}(t, x, y)$.

Thin: where $\hat{\lambda}(t_i,x_i,y_i) > c$, keep each point (t_i,x_i,y_i) with prob. c / $\hat{\lambda}(t_i,x_i,y_i)$.

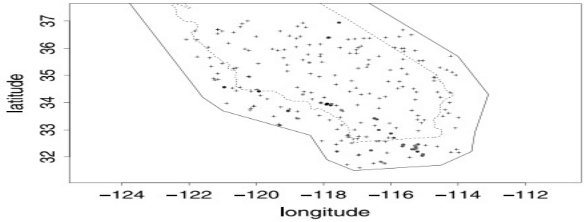
Result is Poisson with rate c, if the model for λ is correct.



(a) Super-thinned residuals Model A.



(b) Super-thinned residuals Model B.



(c) Super-thinned residuals Model C.

Exercises. Superposition.

Suppose N_1 is a Poisson process with rate 3,
and N_2 is a Poisson process with rate $2 + x + 4t$,
independent of N_1, and both are on $[0,10] \times [0,1] \times [0,1]$.
t x y.

Let $M = N_1 + N_2$. Is M a Poisson process? What is its intensity?

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Let $M = N_1 + N_2$. Is M a Poisson process? What is its intensity?

For any disjoint measurable sets $B_1, B_2, ...,$
$M(B_i) = N_1(B_i) + N_2(B_i)$ is independent of $\{N_1(B_j), j \neq i\}$ and $\{N_2(B_j), j \neq i\}$ and thus is independent of $\{N_1(B_j) + N_2(B_j), j \neq i\}$.

So yes, M is a Poisson process and since $EM(B) = EN_1(B) + EN_2(B)$, M has rate $5 + x + 4t$.

Exercises.

Suppose N is homogeneous Poisson process with rate 1,

and M is a clustered Hawkes process.

Both M and N have 40 points on B = [0,10] x [0,1] x [0,1]

t x y.

Let v1 = the average size of a Voronoi cell in a Voronoi tessellation of N,

and v2 = the average size of a Voronoi cell in a Voronoi tesselation of M.

Which is bigger, v1 or v2, or will they be the same?

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Which is bigger, v1 or v2, or will they be the same?

The same, since v1 = v2 = ¼. Each cell has one point, and the 40 cells occupy an area of size 10.