Statistics 222, Spatial Statistics.

Outline for the day:

1. Info about the exam.
2. Stoyan-Grabarnik statistic.
3. fitpoisstoyan.r
4. fitpoiscubicstoyan.r
5. fitpoisstoyancovariates.r
6. fithawkesstoyan.r

1. Info about the exam.

The exam on Nov20 will be from 12:30pm to 1:45pm Pacific Time, but it will be ONLINE.

I will post the exam on the course website,

http://www.stat.ucla.edu/~frederic/222/F23

at 12:30pm.

It will have 13 questions, all multiple choice.

Email me your answers to frederic@stat.ucla.edu by 1:45pm. Include your name. You do not need to write anything else aside from your name and your answers. You can just say, for instance

Rick Schoenberg

AFE CCB DEA HBG E.

Make sure your answers are in the correct order!

2. Stoyan-Grabarnik statistic. Baddeley et al., 2005.

$E \sum 1/\lambda_i = E \int 1/\lambda \; \lambda \; d\mu = E \int d\mu = |B|$.

So, $\sum 1/\lambda_i - |B|$ should be close to zero.

Stoyan-Grabarnik statistic (SG).

-- Stoyan and Grabarnik (1991) introduced the statistic $\sum 1/\lambda_i \div n$ in the context of marked Gibbs processes.

-- Baddeley et al. (2005) proposed using the SG statistic $\sum 1/\lambda_i$ as a goodness-of-fit diagnostic for a point process on observation region *B,* since $E[\sum 1/\lambda_i] = E \int 1/\lambda \, dN = E \int d\mu = |B|$.

-- Cronie and van Lieshout (2018) use the SG statistic as a way to choose the bandwidth when kernel smoothing an inhomogeneous Poisson process.

-- Kresin et al. (2022) proposed, for a general spatial-temporal point process, estimating the parameters θ by minimizing the squared difference between $\sum 1/\lambda_i$ and $|B|$.

Stoyan-Grabarnik statistic. Baddeley et al., 2005.

$E \sum 1/\lambda_i = E \int 1/\lambda \; \lambda \; d\mu = E \int d\mu = |B|$.

So, $\sum 1/\lambda_i - |B|$ should be close to zero.

What if we fit parameters by minimizing $(\sum 1/\lambda_i - |B|)^2$?

More specifically, imagine dividing up B into little grid cells, and within each cell, calculate this difference, $(\sum 1/\lambda_i - |B|)$,

and find the parameters minimizing the sum of squares?

Stoyan-Grabarnik statistic (SG).

Estimating the parameters θ by minimizing

$$\sum_j \left(\sum 1/\lambda_i - |B_j|\right)^2$$

Advantages:

-- No integral term.

-- Extremely easy to program.

-- Very fast to compute, even for relatively large datasets.

-- Only the conditional intensities at the observed points need to be specified.

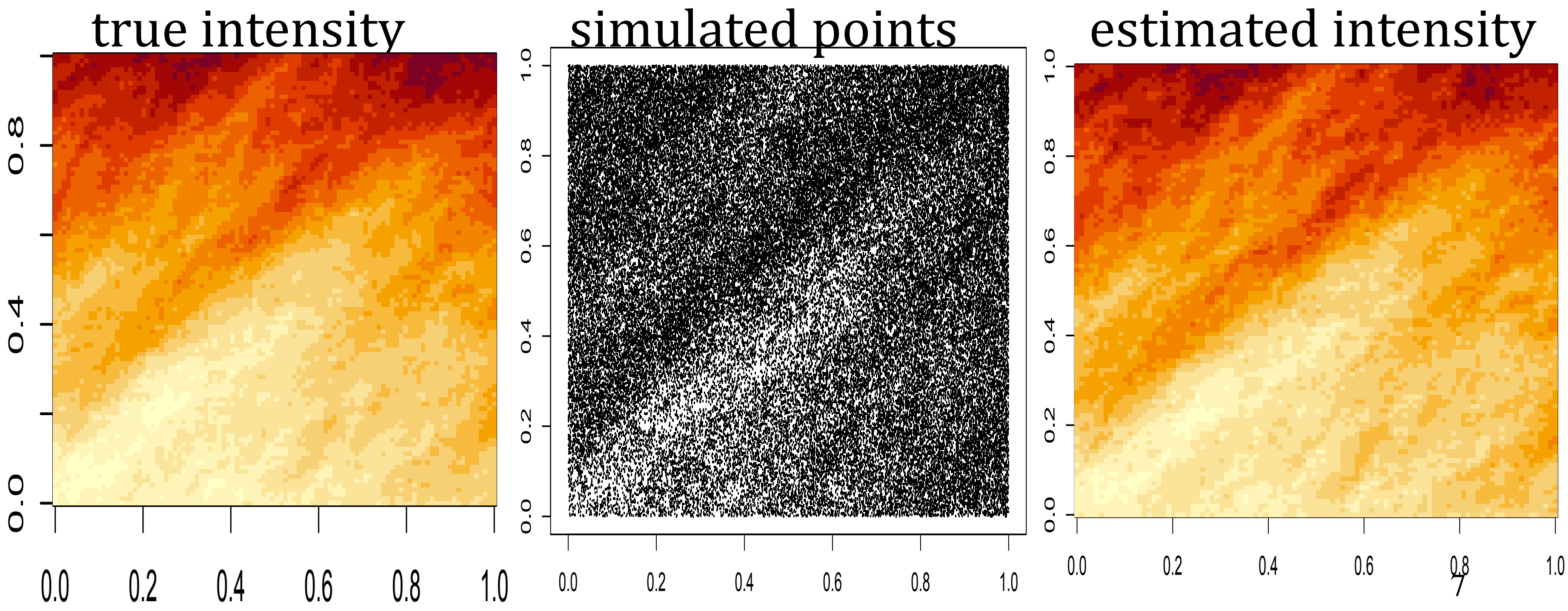
-- Consistent estimates.

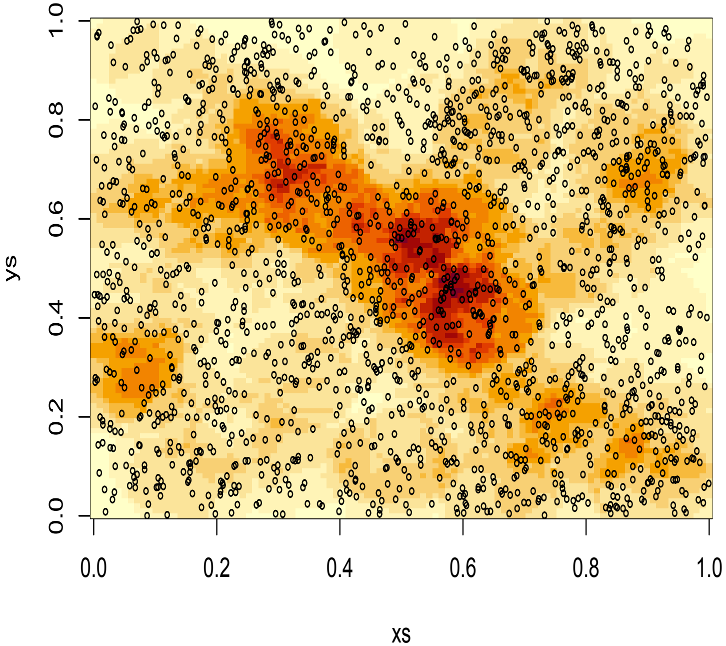
Simulated examples.

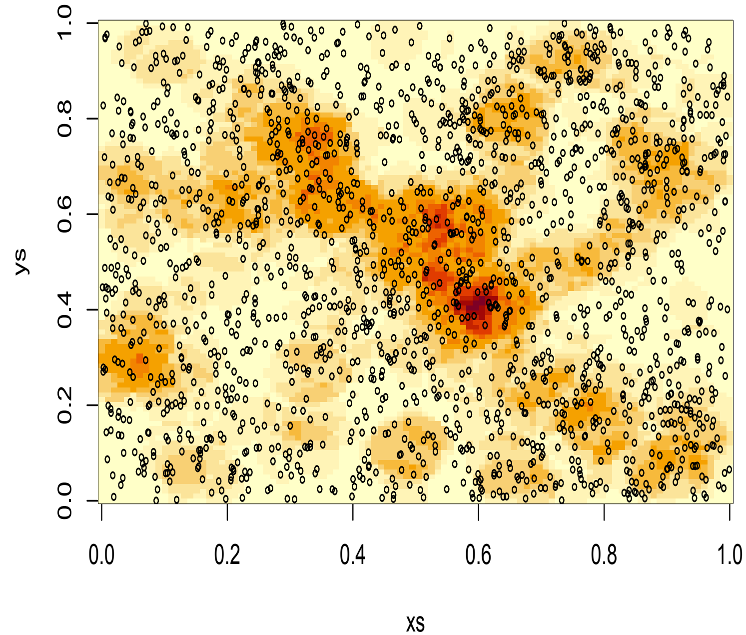
$$\lambda(x,y,t) = e^{\alpha x} + \beta e^{y} + \gamma xy + \delta x^2 + \varepsilon y^2 + W(x,y),$$

where $W(x,y)$ = Brownian bridge.

$(\alpha, \beta, \gamma, \delta, \varepsilon) = (-2,3,4,5,-6)$.







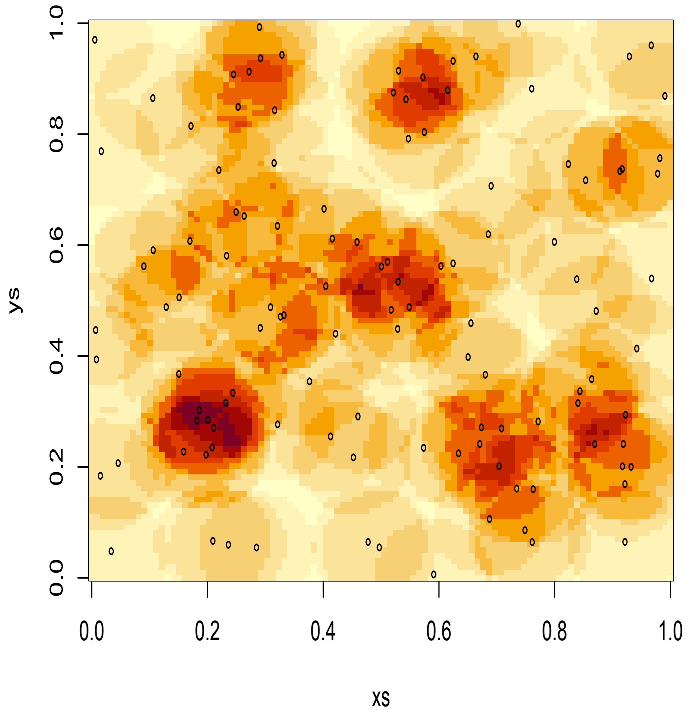
True λ Estimated λ

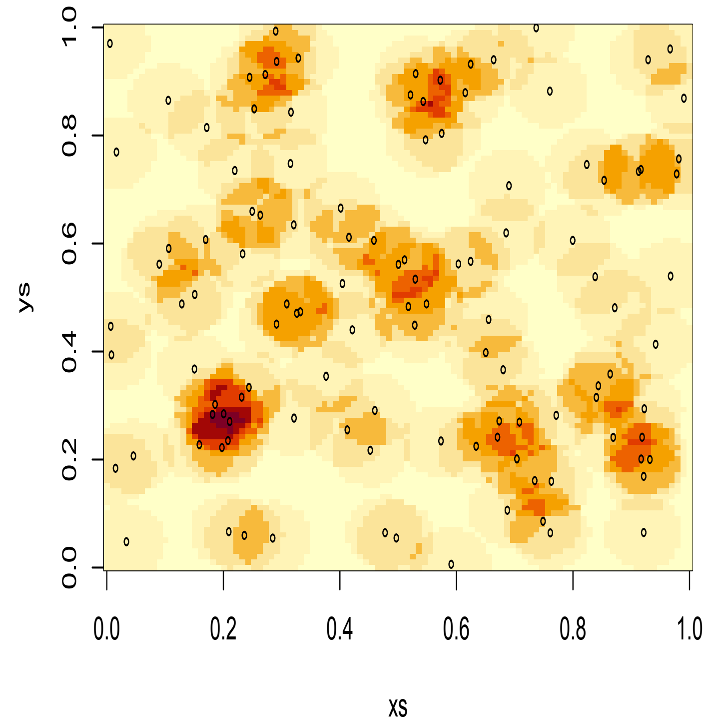
Hawkes model,

$\lambda(t,x,y) = \mu + \kappa \sum g(t-t_i)h(x-x_i,y-y_i)$,

$g(t) = 1/\alpha$ on $[0, \alpha]$, $h(x,y) = 1/(\pi r^2)$ for r in $[0, \beta]$,

$t = 1000$.





True λ Estimated λ

Hawkes model,

$\lambda(t,x,y) = \mu + \kappa \sum g(t-t_i)h(x-x_i,y-y_i)$,

$g(t) = 1/\alpha$ on $[0, \alpha]$, $h(x,y) = 1/(\pi r^2)$ for r in $[0, \beta]$,

$t = 100$.