**Stat 222 exam.**

Rick Paik Schoenberg, 9:30-10:45am.

**PRINT YOUR NAME:**

**SIGN YOUR NAME:**

**Do not turn the page and start the exam until you are told to do so.**

You may use a pen or pencil, and any notes.

There are 10 multiple choice questions each worth the same amount.

For each question, mark one answer only. No need to show work on these, and no partial credit will be given.

For the entire exam, let $B$ mean the spatial-temporal region $[0, 10] \times [0, 1] \times [0, 1]$, i.e. the unit square observed from time 0 to time 10 days.

t x y.

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For the next 3 problems, a point process $N$ is constructed on $B$ as follows. First, one point $\tau_1$ is placed uniformly on $[0, 5] \times [.25, .75] \times [.25, .75]$. Next, a second point $\tau_2$ is placed uniformly within a circle of radius 0.25 around $\tau_1$ and within 0 to 2 days after $\tau_1$. No further points are generated.

**1.** What is $E(\int_B dN)$?

a) 0. b) 1.0. c) 2.0. d) 3.0.
e) 4.0. f) 5.0. g) 10.0. h) 20.0.

**2.** What is Var(N(B))?

a) 0. b) 1.0. c) 2.0. d) 3.0.
e) 4.0. f) 5.0. g) 10.0. h) 20.0.

**3.** For one realization, the points happen to occur at $(2.5, 0.5, 0.5)$ and $(3.2, 0.5, 0.52)$. Let $\lambda$ denote the conditional intensity for this point pattern. What is $\lambda(3, .45, .5)$ ?

a) 0. b) $\pi/16$. c) $\pi/4$. d) $2\pi/5$.
e) $4\pi$. f) $2\pi/7$. g) $8/\pi$. h) 7.

For the next 2 problems, suppose a simple point process $N$ has conditional intensity $\lambda$ on $B$. Consider the statistic $V = \int_B \frac{1}{\lambda(t,x,y)} dN$. This is called the *Stoyan-Grabarnik* statistic.

**4.** What is $E(V)$?

a) 1. b) $\pi r^2$. c) $log(t, x, y)$. d) $log(20)$.
e) 10. f) 20. g) 1/20. h) Cannot be determined without knowing what $\lambda$ is.

**5.** Suppose $N$ is a Hawkes process with background rate $\mu = 3.0$, productivity $K = 0.5$, and triggering density $g(t', x', y') = \frac{100}{\pi}$ for $0 < t' < 1$ and $||(x', y')|| < .1$, and $g = 0$ otherwise. In a given realization, $N$ happens to have points only at (2.5, .5, .5) and (3.2,.5,.52). Calculate $V$ for this point pattern.

a) 2/3. b) $\frac{1}{3} + \frac{1}{3+\frac{50}{\pi}}$. c) $log(3 - 100/\pi) + log(3)$. d) $log(6)$.
e) $\frac{2}{3} + \frac{1}{\pi+100}$. f) $\frac{2}{3} + \frac{\pi}{6}$. g) $\frac{2}{3} + 0.5log(\frac{100}{\pi})$. h) $\frac{4}{3} + \frac{log(\pi)}{1+\frac{1}{6\pi}}$.

For the next two problems, suppose
$N_1$ is a Poisson process on $B$ with $\lambda(t,x,y) = 1 + 2t + 3x + 4y$,
$N_2$ is a Poisson process on $B$, independent of $N_1$, with $\lambda(t,x,y) = 2 + 3t$,
and $N = N_1 + N_2$.

**6.** What is $E(N_1(B))$?
a) 10. b) $10\pi/4$. c) 25. d) 100.
e) $100\pi/4$. f) 145. g) $exp(28)$. h) 28.

**7.** What is $Var(N(B))$?
a) 22. b) 40. c) 45. d) 125.
e) 315. f) 445. g) 512. h) 810.

For the next two problems, suppose $N$ is a Poisson cluster process on $B$ with background rate 2 and, for each background point, exactly 3 offspring points are placed uniformly in a circle of radius 0.1 around the background point, and uniformly within time $[0,1]$ after the background point. If any of the offspring points is outside of $B$, then it is discarded and a new offspring point is generated. Thus each background point is guaranteed to have exactly 3 offspring points within $B$. $N$ contains just the offspring points, not the background points.

**8.** What is $Var(N(B))$?
a) 14. b) 20. c) 180. d) 200.
e) exp(30). f) 20 + log(40). g) 2 log(30). h) $30e^2$.

**9.** Consider now the same process but extended to the entire plane, and ignoring time. In other words, $N$ is a Poisson cluster process on $\mathbf{R}^2$ with background rate 2 and, for each background point, exactly 3 offspring points placed uniformly in a circle of radius 0.1 around the background point. Again, $N$ contains just the offspring points, not the background points. Let $K(r)$ be the $K$-function for this process. What is $K(.2)$?
a) $\frac{1}{2}$. b) $\frac{1}{3} + \frac{\pi}{6}$. c) $\frac{1}{2} + \frac{3\pi}{7}$ d) $\frac{1}{\pi} + \frac{2}{7}$.
e) $\frac{\pi}{3} + e^2$. f) $\frac{1}{2} + \frac{\pi}{3}$. g) $\frac{1}{2} + \frac{3\pi}{16}$ h) $\frac{1}{3} + \frac{\pi}{25}$.
i) $\frac{2}{3} + \frac{\pi}{2}$. j) $2 + \pi/4$. k) $\frac{2}{3} + \frac{\pi}{16}$. l) $\pi/4$.

**10.** Suppose $N$ is a purely spatial Matern (I) hard-core process,
on the unit square $[0,1] \times [0,1]$, with Papangelou intensity
$\lambda(x,y) = 2$ if there is no other point within distance $r = 0.25$ of $(x,y)$,
and $\lambda(x,y) = 0$ otherwise.
Suppose the process happens to have just two points, at (0.5, 0.25) and (0.5, 0.75).
Calculate the pseudo-loglikelihood for this point pattern.
a) $-2$. b) $2log(2) - \pi/4$. c) $2log(2) - \pi/16$. d) $2log(2) - 2 + \pi/4$.
e) 0. f) $-\pi/16$. g) $2log(2) - \pi/4$. h) $2log(2) - 2$.
i) $2log(2) - 4$. j) $4 - \pi/4$. k) $4 - \pi/16$. l) $\pi/4$.
m) $e^2 + 2log(2) - 4 + \pi/16$. n) $e^2 + 4 - \pi/4$. o) $e^2 + 4 - \pi/16$. p) $e^2 - \pi/4$.