Statistics 222, Spatial Statistics.

<u>Outline for the day:</u>

1. Purely spatial processes, Papangelou intensity and Georgii Zessin Nguyen.
2. Kernel smoothing.
3. F, G, J, K, and L functions.
4. Exercises.

1. Purely spatial processes, Papangelou intensity and the Georgii-Zessin Nguyen formula.

For point processes in R^2, there is no natural ordering as there is in time. One could just use the x-coordinate in place of time and define a conditional intensity, but most models for spatial processes would be very awkward to define this way.

Instead, a more natural and useful tool is the Papangelou intensity, $\lambda(x,y)$, which is the conditional rate of points around location (x,y), given information on everywhere else. Letting

$l(\theta) = \sum \log(\lambda(\tau_i)) - \int \lambda(x,y)\, dx\, dy$,

where $\lambda(x,y)$ is the Papangelou intensity,

$l(\theta)$ is called the *pseudo-loglikelihood*.

A key formula for space-time point processes is called the *martingale formula*:

for any predictable function $f(t,x,y)$,

$E \int f(t,x,y)\, dN = E \int f(t,x,y)\, \lambda(t,x,y)\, d\mu$.

$= E \sum_i f(t_i, x_i, y_i) = E \int f(t,x,y)\, \lambda(t,x,y)\, dt\, dx\, dy$

For spatial point processes the corresponding formula,

$E \int f(x,y)\, dN = E \int f(x,y)\, \lambda(x,y)\, dx\, dy$

is called the Georgii-Zessin-Nguyen formula.

When $f = 1$, this means $EN(B) = E \int \lambda\, d\mu$.

2. Kernel smoothing.

A simple way to start summarizing a spatial point process is by kernel smoothing.

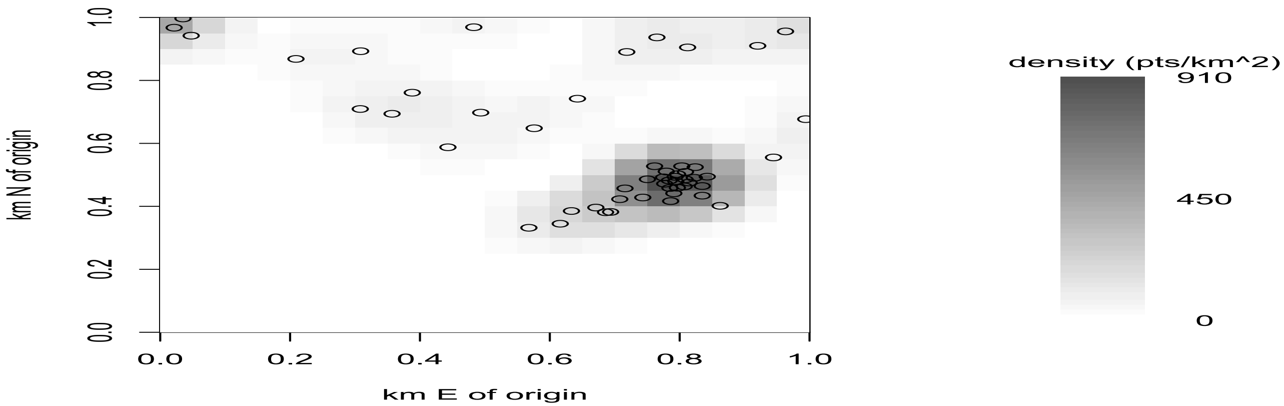
Suppose your observation region is B.

Let k(x,y) be a spatial density function, called a kernel, and construct, for each location (x,y),

$\hat{\lambda}(x,y) = \int_B k((x',y') - (x,y))\, dN(x',y') / \rho(x,y)$,

where $\rho(x,y) = \int_B k((x',y') - (x,y))\, dx'\, dy'$ is an edge correction term.

The resulting function $\hat{\lambda}(x,y)$ is a natural estimator of $\lambda(x,y)$ and, when N is a Poisson process, can be an asymptotically unbiased estimator of $\lambda(x,y)$.

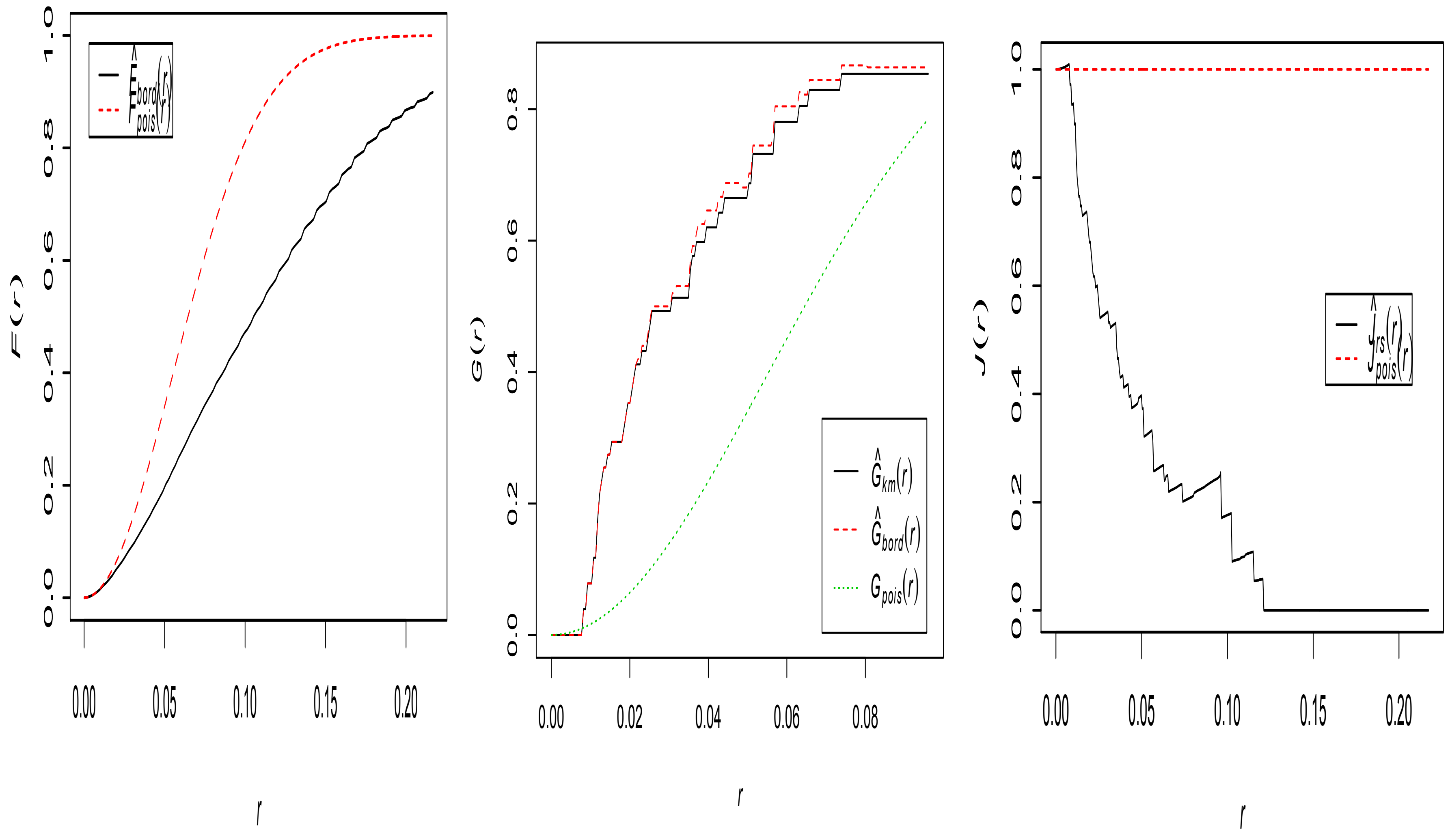


3. F, G, J, K, and L functions.
Let F(r) be the probability that the distance from a randomly chosen location to its nearest *point* of the process is $\leq$ r.
Let G(r) be the probability that the distance from a randomly chosen *point* to its nearest neighbor is $\leq$ r.
F is the empty space function and G is the nearest neighbor distribution function.
Matern (1971) showed that for a homogeneous Poisson process, $F(r) = G(r) = 1 - \exp(-\lambda \pi r^2)$.

Marie-Collette van Lieshout

Let $J(r) = (1-G(r)) / (1-F(r))$, for any r such that $F(r)<1$.
$J > 1$ indicates inhibition, and $J < 1$ indicates clustering.
For a stationary Poisson process with rate μ, let
$K(r) = 1/\mu$ E(# of other points within distance *r* of a randomly chosen point).
K is the reduced 2nd moment measure or *Ripley*'s K-function (Ripley, 1976).
van Lieshout, M.C. (2010). A *J*–function for inhomogeneous point processes. *Statistica Neerlandica*, 65(2), 183-201.
and references therein gives extensions to the inhomogeneous Poisson process and to marked point processes.

F, G, J, K, and L functions, continued.

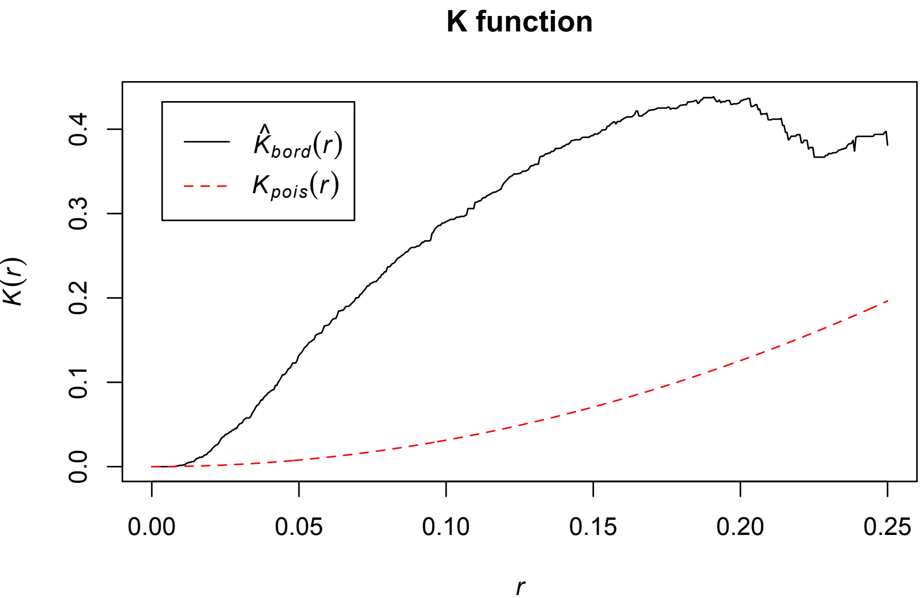


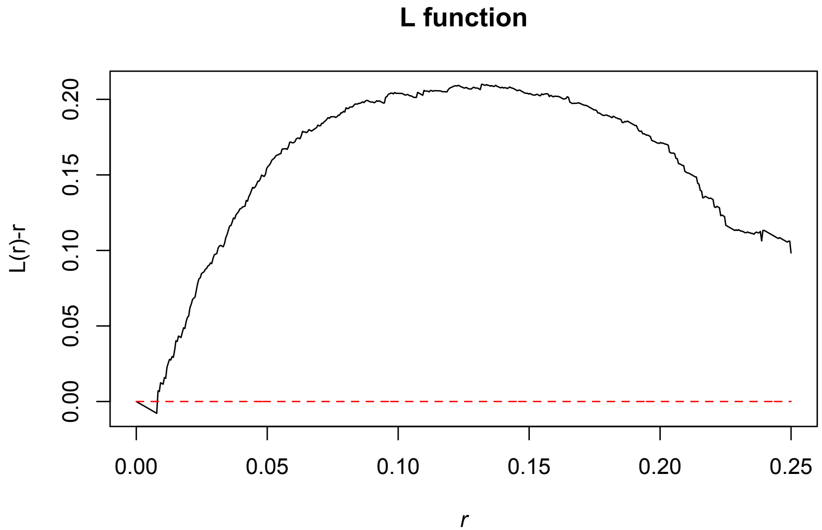
F, G, J, K, and L functions, continued.

$K(r) = 1/\mu$ E(# of other points within distance r of a randomly chosen point).

K is estimated in the obvious way using data, but various edge correction ideas are available.

For a stationary Poisson process in R^2, $K(r) = \pi r^2$, so one may consider
$L(r) = \sqrt{(K(r)/\pi)}$.
For a stationary Poisson process in R^2, $L(r) - r = 0$ and $\hat{L}(r)-r$ should be approx. 0.





4. Exercises.

Suppose N is a spatial point process with clustering for distances $\leq$ d.
Let F(r) be the empty space function and let G(r) be the nearest neighbor distribution function.
Which of the following is true.

a. $F(d) = G(d)$.
b. $F(d) < G(d)$.
c. $F(d) > G(d)$

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Entering data and kernel smoothing example.

```
## First, input 54 points using the mouse.
n = 54
plot(c(0,1),c(0,1),type="n",xlab="longitude",ylab="latitude",
                                main="locations")
  x1 = rep(0,n)
  y1 = rep(0,n)
  for(i in 1:n){
  z1 = locator(1)
  x1[i] = z1$x
  y1[i] = z1$y
  points(x1[i],y1[i])
  }
```

```
##### PLOT THE POINTS WITH A 2D KERNEL SMOOTHING IN
     GREYSCALE PLUS A LEGEND
library(splancs)
bdw = sqrt(bw.nrd0(x1)^2+bw.nrd0(y1)^2)  ## possible default bandwidth
b1 = as.points(x1,y1)
bdry = matrix(c(0,0,1,0,1,1,0,1,0,0),ncol=2,byrow=T)
z = kernel2d(b1,bdry,bdw)
attributes(z)
par(mfrow=c(1,2))
image(z,col=gray((64:20)/64),xlab="km E of origin",ylab="km N of
     origin")
points(b1)
x4 = seq(min(z$z),max(z$z),length=100)
```

```
plot(c(0,10),c(.8*min(x4),1.2*max(x4)),type="n",
        axes=F,xlab="",ylab="")
image(c(-1:1),x4,matrix(rep(x4,2),ncol=100,byrow=T),
add=T,col=gray((64:20)/64))
text(2,min(x4),as.character(signif(min(x4),2)),cex=1)
text(2,(max(x4)+min(x4))/2,
        as.character(signif((max(x4)+min(x4))/2,2)),cex=1)
text(2,max(x4),as.character(signif(max(x4),2)),cex=1)
mtext(s=3,l=-3,at=1,"density (pts/km^2)")

library(spatstat)
b2 = as.ppp(b1,c(0,1,0,1))
k = Kest(b2,correction="border")
plot(k, main="K function")
plot(k, sqrt(./pi)-r ~ r, ylab="L(r)-r", main="L function",legend=F)
```

```
par(mfrow=c(1,3))
plot(Fest(b2,correction="border"),main="")
plot(Gest(b2,correction="border"),main="")
plot(Jest(b2,correction="border"),main="")
```