Statistics 222, Spatial Statistics.

Outline for the day:

1. Marked G and J functions.
2. Weighted K function.
3. Project order.
4. Kernel smoothing, summary functions, model fitting, and weighted K function for spatial point processes, unmarked and marked, in R.

1. Marked G and J functions.

$G(r) = P_0$(point within r), where P_0 means given a pt. at 0.
It is estimated with $G\hat{}(r) = 1/n \sum_i 1(\text{there is } j: |\tau_i - \tau_j| \le r)$.
$= 1/n \sum_i 1(\min_{i \neq j} |\tau_i - \tau_j| \le r)$

One could alternatively compute a *marked* G-function
$1/n_1 \sum_i 1(\min_j |\tau_i - \tau_j| \le r)$
where the sum is over the n_1 points τ_i with mark in some range M_1, and the minimum is over the points τ_j with mark in some range M_2.
This is the *marked* or *cross* G-function.
One can similarly define a marked or cross J-function
as $J(r) = (1-G(r)) / (1-F(r))$ accordingly, plugging in the corresponding G function.

Marie-Collette van Lieshout

van Lieshout, M.N.M. (2006). A J-function for marked point patterns. *AISM* 58, 235-259.

2. Weighted K function.

For a stationary Poisson process with rate μ,
$K(r) = 1/\mu\ E$(# of other points within distance *r* of a randomly chosen point).
Estimated via $K4(r) = 1/(\lambda\hat{}\ n)\ \sum_{i \neq j} (|\tau_i - \tau_j| \leq r)\ w(\tau_i, \tau_j)$,
where $\lambda\hat{} = n/|S|$, and $w(\tau_i, \tau_j) = 1/$proportion of circle centered at i going through j that is in S = border correction term.
If N is inhomogeneous, can instead weight each point by $1/\lambda$, obtaining $K_w(r) = 1/n\ \sum_{i \neq j} (|\tau_i - \tau_j| \leq r)\ w(\tau_i, \tau_j) / \lambda(\tau_i) / \lambda(\tau_j)$.

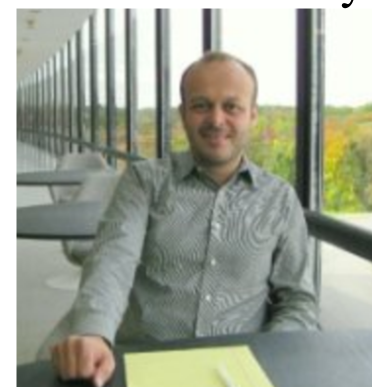
$K_w(r) \sim N(\pi r^2, 2\pi r^2 |S| / E(n)^2)$, if $\inf \lambda = 1$.

Baddeley, A., Møller, J., Waagepetersen, R. (2000). Non and semi-parametric estimation of interaction in inhomogeneous point patterns. *Statistica Neerlandica*, 54(3), 329-350.

Veen, A. and Schoenberg, F.P. (2006). Assessing spatial point process models for California earthquakes using weighted K-functions: analysis of California earthquakes, in *Case Studies in Spatial Point Process Models*, Baddeley, A., Gregori, P., Mateu, J., Stoica, R., and Stoyan, D. (eds.), Springer, NY, pp. 293-306.

Adelfio, G. and Schoenberg, F.P. (2009). Point process diagnostics based on weighted second-order statistics and their asymptotic properties. *Annals of the Institute of Statistical Mathematics*, 61(4), 929-948.


Adran Baddeley


Alejandro Veen


Giada Adelfio

3. Presentation times.

I will now randomly assign people to presentation times. If you want to change oral presentation dates and times with another person, feel free but let me know.