Stat 222 exam.

Rick Paik Schoenberg, 5/22/17, 9:30-10:45am.

PRINT YOUR NAME:

SIGN YOUR NAME:

Do not turn the page and start the exam until you are told to do so.

You may use a pen or pencil, and any notes.

There are 10 multiple choice questions each worth the same amount.

For each question, mark one answer only. No need to show work on these, and no partial credit will be given.

For the entire exam, let B mean the spatial-temporal region $[0, 10] \times [0, 1] \times [0, 1]$, i.e. the unit square observed from time 0 to time 10 days.

t x y.

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For the next 2 problems, suppose a point process N is constructed on B as follows. First, one point τ_1 is placed uniformly on $[0,5]\times[.25,.75]\times[.25,.75]$. Next, a Poisson random variable K with mean 3.5 is generated. Then K points, $\tau_2, \tau_3, ..., \tau_{K+1}$, are placed uniformly within a circle of radius 0.25 around τ_1 and within 0 to 2 days after τ_1. No further points are generated.

1. What is $E(\int_B dN)$?

a) 0. b) 1.0. c) 2.0. d) 3.0.
e) 4.5. f) 5.5. g) 7.0. h) 9.1.

2. What is Var(N(B))?

a) 1.0. b) 2.5. c) 3.5. d) 4.0.
e) 5.0. f) 6.0. g) 7.0. h) 12.0

For the next 2 problems, suppose a simple point process N has conditional intensity λ where $\lambda(t,x,y) = 9x^2$ on B, and $\lambda = 0$ outside of B.
Consider the statistic $D = \int_B \frac{1}{\sqrt{\lambda(t,x,y)}} dN$.

3. What is $E(D)$?

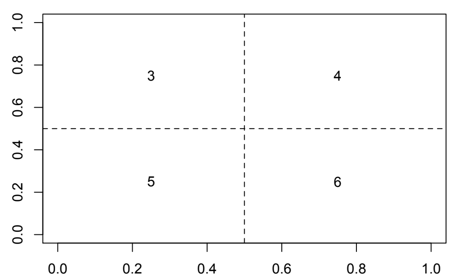
a) 15. b) 22. c) 27. d) log(18).
e) 900. f) log(900). g) 1/900. h) Cannot be determined.

4. Suppose N is a Hawkes process with background rate $\mu = 3.0$, productivity $K = 0.5$, and triggering density $g(t', x', y') = \frac{100}{\pi}$ for $0 < t' < 1$ and $||(x', y')|| < .1$, and $g = 0$ otherwise. In a given realization, N happens to have points only at (2.5, 0.5, 0.5) and (3.2,0.5,0.52). Calculate D for this point pattern.

a) 2/3. b) $\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3+\frac{50}{\pi}}}$. c) $log(3 - 100/\pi) + log(3)$. d) $log(6)$.
e) $\frac{2}{3} + \frac{1}{\pi+100}$. f) $\frac{2}{3} + \frac{\pi}{6}$. g) $\frac{2}{3} + 0.5log(\frac{100}{\pi})$. h) $\frac{4}{3} + \frac{log(\pi)}{1+\frac{1}{6\pi}}$.

5. Suppose your points N come from a Hawkes process but you model it instead as a Strauss process. Which of the following will be true of your superthinned residuals based on your model?
a) They will be highly clustered.
b) They will be more regularly spaced than one would expect from a stationary Poisson process.
c) They will resemble a stationary Poisson process.

6. Suppose N is a point process on B whose estimated conditional intensity $\hat{\lambda}$ is as follows:
$\hat{\lambda}(t, x, y) = 3$ for $x < .5$ and $y > .5$.
$\hat{\lambda}(t, x, y) = 4$ for $x > .5$ and $y > .5$.
$\hat{\lambda}(t, x, y) = 5$ for $x < .5$ and $y < .5$.
$\hat{\lambda}(t, x, y) = 6$ for $x > .5$ and $y < .5$.



Suppose you do superthinning with $c = 5$. Let M denote the superposed points in the superthinning. What is $var(M(B))$?
a) 1.5. b) 2.5. c) 3.5. d) 4.5.
e) 5.5. f) 6.5. g) 7.5. h) 8.5.

For the next 2 problems, let N be a purely spatial compound Poisson process on the unit square $Q = [0, 1] \times [0, 1]$, generated by first generating a stationary Poisson process on Q with rate 2.5, calling those points $\tau_1, \tau_2, ...$, generating iid Poisson random variables Z_1, Z_2, ... with mean 3, and then letting N have Z_i overlapping points at exactly location τ_i, for each i.

7. What is $EN(B)$?
a) 7.5. b) 10.5 c) 12.5. d) 15.5. e) 17.5.

8. What is the K-function, $K(r)$, for this process?
Ignore boundary effects, or assume the process is generated this way on the whole plane.
a) $3\pi r^2$. b) $2/7.5 + \pi r^2$. c) $3 + 4/3\pi r^2$. d) $\pi r^2/7.5$.
e) $19\pi r^2$. f) $22.5\pi r^2$. g) $3/7.5 + 2.5\pi r^2$. h) $1 + \pi r^2 + 10\log(2)$.

For the next 2 problems, let N be a point process on B with conditional intensity
$\lambda(t,x,y) = \mu + K \int_{t'<t} \int_0^1 \int_0^1 \frac{1}{\lambda(t',x',y')} f(t-t') g(x-x', y-y') dN(t',x',y')$.
This process is called a *recursive* process.
Suppose f is the exponential density $f(u) = \alpha e^{-\alpha u}$,
and g is the uniform density, $g(u,v) = \frac{100}{\pi}$ for $||(u,v)|| < 0.1$. Let $\mu = 5, K = 0.4$, and $\alpha = 4$.

9. N happens to have two points, one at (2.5,.5,.5) and the other at (3.5,0.5,0.52). What is the loglikelihood?
a) $\log(5) + \log(5 + \exp(-4)/\pi)$.
b) $2\log(5) - 50$.
c) $2\log(5) + \log(160\exp(-4)/\pi) - 50 - 0.4\pi/(5 + 160\exp(-4)) + 10\pi/(42\exp(7.2))$.
d) $2\log(5) + \log(5 + 160\exp(-4)/\pi) - 50 - 0.4/(5\pi)$.
e) $\log(5) + \log(5 + 32\exp(-4)/\pi) - 50 - 0.4/5 - 0.4/(5 + 32\exp(-4)/\pi)$.
f) $2\log(5) - 50 - .8/5$.
g) $\log(5) + \log(5 + 75\exp(-4)/\pi) - 50 - 0.4/5 - 0.4/(5 + 75\exp(-4)/\pi)$.

10. What is EN(B)? (Ignore boundary issues, or equivalently, assume the process is generated from time $t = -\infty$ to ∞ and over the entire plane, but we are only interested in the number occurring in B.)
a) 54. b) $5.4\exp(20 + 10/\pi)$. c) $50.4\exp(20 + 5/\pi)$.
d) $50.4(1 + \log(2))$. e) $50 + .4\exp(10)$. f) $50 + .4\log(2)$.
g) $50+.4(1+\exp(10))$. h) $50+.4(1+\log(10)+\exp(.2))$. i) $50+.4(1+\exp(10)+\log(.2))$.