Statistics 222, Spatial Statistics.

Outline for the day:

1. Integrals for the exam.
2. Purely spatial processes, Papangelou intensity, and the Georgii-Zessin-Nguyen formula.
3. Exercises and code.
4. Discuss Van Lieshout pp 11-15, 23-26.

1. Integrals for the exam.

For the exam, you need to know the very basics of integrals,

like $\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$,

and be able to compute the integral of f(x) dx, where f(x) is

$f(x) = c$,

$f(x) = \log(x)$,

$f(x) = x^a$ where a is any real number,

$f(x) = e^{ax}$.

What is $\int_1^3 \int_1^3 (4+3/x)\, dx\, dy$?

$2(4x + 3\log(x)]_1^3) = 2(12 + 3\log(3) - 4 - 3\log(1)) = 2(8 + 3\log(3))$.

2. Purely spatial processes, Papangelou intensity and the Georgii-Zessin Nguyen formula.

For point processes in R^2, there is no natural ordering as there is in time. One could just use the x-coordinate in place of time and define a conditional intensity, but most models for spatial processes would be very awkward to define this way.

Instead, a more natural and useful tool is the Papangelou intensity, $\lambda(x,y)$, which is the conditional rate of points around location (x,y), given information on everywhere else. Letting

$L(\theta) = \sum \log(\lambda(\tau_i)) - \int \lambda(x,y)\, dx\, dy$,

where $\lambda(x,y)$ is the Papangelou intensity,

$L(\theta)$ is called the *pseudo-loglikelihood*.

A key formula for space-time point processes is called the *martingale formula*:

for any predictable function f(t,x,y),

$E \int f(t,x,y)\, dN = E \int f(t,x,y)\, \lambda(t,x,y)\, d\mu.$

$= E \sum_i f(t_i,x_i,y_i) = E \int f(t,x,y)\, \lambda(t,x,y)\, dt\, dx\, dy$

For spatial point processes the corresponding formula,

$E \int f(x,y)\, dN = E \int f(x,y)\, \lambda(x,y)\, dx\, dy$

is called the Georgii-Zessin-Nguyen formula.

When f = 1, this means $EN(B) = E \int \lambda\, d\mu$.

3. exercises.

a. Suppose N is a Poisson process with intensity $\lambda(t,x,y) = \exp(-3t)$ over t in [0,10], x in [0,1], y in [0,5].

N happens to have points at (1.5, .4, 2.7)
(2, .52, 4.1)
(4, .1, 2.9)
(5, .71, 0.5).

What is the log-likelihood of this realization?

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What is the log-likelihood of this realization?

$-4.5-6-12-15 - \iiint \exp(-3t)\, dt\, dx\, dy$

$= -37.5 - 5\int_0^{10} \exp(-3t)\, dt$, because x goes from 0 to 1 and y goes from 0 to 5,

$= -37.5 - 5\exp(-3t) / (-3)]_0^{10}$

$= -37.5 + 5\exp(-30)/3 - 5\exp(0)/3$

$= -37.5 + 5\exp(-30)/3 - 5/3$

$\sim -39.2.$

exercises.

Which of the following is not typically true of the MLE of a spatial-temporal point process?

a. It is unbiased.
b. It is consistent.
c. It is asymptotically normal.
d. It is asymptotically efficient.

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