Statistics 222, Spatial Statistics.

Outline for the day:

1. Marked G and J functions.
2. Weighted K function.
3. van Lieshout p26, 37-38.
4. Project order.
5. Kernel smoothing, summary functions, model fitting, and weighted K function for spatial point processes, unmarked and marked, in R. day07.r.

1. Marked G and J functions.

Marie-Collette van Lieshout

$G(r) = P_0$(point within r), where P_0 means given a pt. at 0.
It is estimated with $G\hat{}(r) = 1/n \sum_i 1(\text{there is } j: |\tau_i - \tau_j| \le r)$.
$= 1/n \sum_i 1(\min_{i \neq j} |\tau_i - \tau_j| \le r)$

One could alternatively compute a *marked* G-function
$1/n_1 \sum_i 1(\min_j |\tau_i - \tau_j| \le r)$
where the sum is over the n_1 points τ_i with mark in some range M_1, and the minimum is over the points τ_j with mark in some range M_2.
This is the *marked* or *cross* G-function.
One can similarly define a marked or cross J-function as $J(r) = (1-G(r)) / (1-F(r))$ accordingly, plugging in the corresponding G function.

van Lieshout, M.N.M. (2006). A J-function for marked point patterns. *AISM* 58, 235-259.

2. Weighted K function.

For a stationary Poisson process with rate μ,
$K(r) = 1/\mu$ E(# of other points within distance *r* of a randomly chosen point).
Estimated via $K_4(r) = 1/(\lambda\hat{}\ n)\ \sum_{i\neq j} (|\tau_i - \tau_j| \le r)\ w(\tau_i, \tau_j)$,
where $\lambda\hat{} = n/|S|$, and $w(\tau_i, \tau_j)$ = 1/proportion of circle centered at i going through j that is in S = border correction term.
If N is inhomogeneous, can instead weight each point by $1/\lambda$,
obtaining $K_w(r) = 1/n\ \sum_{i\neq j} (|\tau_i - \tau_j| \le r)\ w(\tau_i, \tau_j) / \lambda(\tau_i) / \lambda(\tau_j)$.

$K_w(r) \sim N(\pi r^2, 2\pi r^2|S| / E(n)^2)$, if inf $\lambda = 1$.

Baddeley, A., Møller, J., Waagepetersen, R. (2000). Non and semi-parametric estimation of interaction in inhomogeneous point patterns. *Statistica Neerlandica*, 54(3), 329-350.

Veen, A. and Schoenberg, F.P. (2006). Assessing spatial point process models for California earthquakes using weighted K-functions: analysis of California earthquakes, in *Case Studies in Spatial Point Process Models*, Baddeley, A., Gregori, P., Mateu, J., Stoica, R., and Stoyan, D. (eds.), Springer, NY, pp. 293-306.

Adelfio, G. and Schoenberg, F.P. (2009). Point process diagnostics based on weighted second-order statistics and their asymptotic properties. *Annals of the Institute of Statistical Mathematics*, 61(4), 929-948.

Adrian Baddeley

Alejandro Veen

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3. $\gamma(t) = \rho(0) - \rho(t)$, for 2nd order stationary processes, from van Lieshout p23. Why?
2nd order stationary = weakly stationary and means
$E(X_t^2) < \infty$, $E(X_0) = E(X_1) = \ldots = E(X_t)$ for all t,
and $Cov(X_0, X_t) = Cov(X_1, X_{t+1}) = Cov(X_2, X_{t+2})$, etc., for any t.
If 2nd order stat., then letting $t = 0$, $Var(X_0) = Var(X_1) = \ldots = Var(X_t)$ for all t.

The semivariogram $\gamma(t) = Var(X_t - X_0)/2$.
The covariogram $\rho(t) = Cov(X_0, X_t)$.

So $\rho(0) - \rho(t) = Cov(X_0, X_0) - Cov(X_0, X_t) = Var(X_0) - Cov(X_0, X_t)$.

$$\begin{aligned}
\gamma(t) &= Var(X_t - X_0)/2 \\
&= Cov(X_t - X_0, X_t - X_0)/2 \\
&= \{Cov(X_t, X_t) + Cov(X_0, X_0) - 2Cov(X_0, X_t)\}/2 \\
&= Var(X_t)/2 + Var(X_0)/2 - Cov(X_0, X_t) \\
&= Var(X_0) - Cov(X_0, X_t) \\
&= \rho(0) - \rho(t).
\end{aligned}$$

4. Presentation times.
I will now randomly assign people to presentation times. If you want to change oral presentation dates and times with another person, feel free but let me know.

5. Kernel smoothing, summary functions, model fitting, and weighted K function for spatial point processes, unmarked and marked, in R. day07.r.