Stat 222 exam.

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Email your answers to me at frederic@stat.ucla.edu as soon as the exam is over. Indicate your name in your email.

There are 10 multiple choice questions each worth the same amount.

For each question, choose one answer only. No need to show work, and no partial credit will be given.

For the entire exam, let B mean the spatial-temporal region $[0, 10] \times [0, 1] \times [0, 1]$, i.e. the unit square observed from time 0 to time 10 days. t x y.

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For the next 2 problems, suppose a point process N is constructed on B as follows. First, 2 points are placed independently in B according to a uniform distribution on B. If both of those points occur at times later than 5.0, then one more point is placed in B uniformly. Otherwise, no further points are generated.

1. What is $E(\int_B dN)$?

a) 0. b) 1.5. c) 2.0. d) 2.25.
e) 2.5. f) 3.0. g) 4.5. h) None of the above.

2. What is $P\{N(C) = 3\}$ where $C = [0, 10] \times [0, .5] \times [0, 1]$?
a) 1/32. b) 1/24. c) 1/12. d) 1/8.
e) 1/4. f) 1/2. g) 1. h) None of the above.

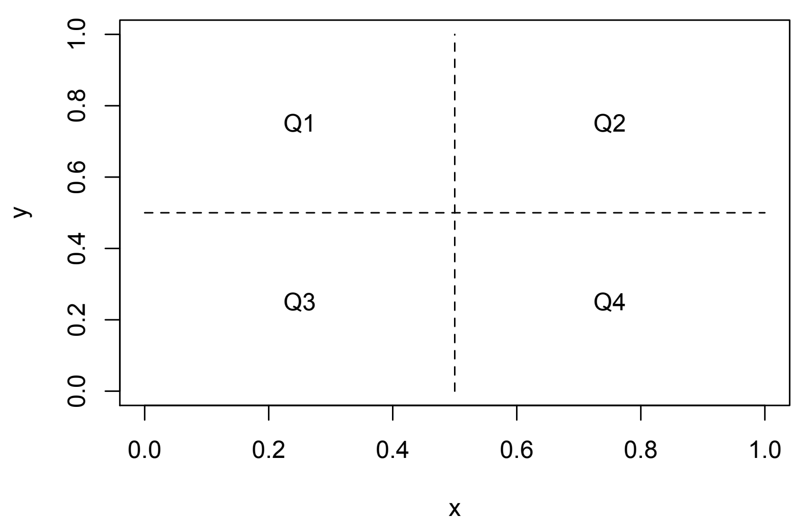
3. Suppose a purely spatial random field X_s is 2nd order stationary with variance $V(X_s) = 11$ and linear semivariogram $\gamma(h) = 1 + 2|h|$ for $0 \leq |h| \leq 5$, and $\gamma(h) = 11$ for $|h| > 5$.
What is the partial sill for X?
a) 1. b) 2. c) 4. d) 5.
e) 8. f) 10. g) 12. h) None of the above.

4. Several point processes are described below. Which one is a Poisson process?
a) a Poisson cluster process with mean cluster size of 3.
b) a simple point process with conditional intensity $\lambda(t, x, y) = 3t + 4x^2 + exp(y)$.
c) a compound Poisson process with $E(Z_i) = 3$.
d) a mixed Poisson process where c is a Poisson random variable with mean 3.
e) a simple Hawkes process with productivity $K = 0.5$.
f) None of the above.

5. Suppose that N is a Hawkes process on B with background rate $\mu = 3.0$, productivity $K = 0.5$, and triggering density $g(t', x', y') = \frac{100}{\pi}$ for $0 < t' < 1$ and $||(x', y')|| < .1$, and $g = 0$ otherwise. In a given realization, N happens to have points only at (2.5, 0.5, 0.5) and (3.2,0.5,0.52). Calculate L for this point pattern. Round your answer to the nearest integer if necessary.
a) -450. b) -292. c) -101. d) -79.
e) -27. f) -17. g) -3. h) None of the above.

6. Suppose N is a mixed Poisson process with $\lambda(s) = c$, where $c = 10$ with probability 0.5 and $c = 20$ with probability 0.5. What is $E(L)$ over the observation region B? Round the answer to the nearest integer if necessary.
a) 265. b) 274. c) 288. d) 302.
e) 307. f) 313. g) 322. h) None of the above.

7. Suppose N is a simple point process on B where with probability 0.5, $\lambda(t, x, y) = e^1$, e^2, e^3, or e^4 depending on what quadrant (x, y) is in, where the 4 quadrants are shown in the figure below, and with probability 0.5, $\lambda(t, x, y) = e^2$, e^3, e^4, or e^5 in these 4 quadrants respectively. What is $E(L)$ over the observation region B? Remember that t goes from 0 to 10. Round the answer to the nearest integer if necessary.
a) 377. b) 522. c) 609. d) 741.
e) 883. f) 1021. g) 1291. h) None of the above.



8. Suppose N is a point process on B whose estimated conditional intensity $\hat{\lambda}$ is as follows:
$\hat{\lambda}(t, x, y) = 3$ in quadrant 1, i.e. for $x < .5$ and $y > .5$.
$\hat{\lambda}(t, x, y) = 4$ in quadrant 2, for $x > .5$ and $y > .5$.
$\hat{\lambda}(t, x, y) = 5$ in quadrant 3, for $x < .5$ and $y < .5$.
$\hat{\lambda}(t, x, y) = 6$ in quadrant 4, for $x > .5$ and $y < .5$.

Suppose you do superthinning with $c = 5$, resulting in residual point process M. Let Q denote the number of points common to both M and N. In other words, Q is the number of points of N that are not deleted in the thinning to form M. What is $var(Q)$?
a) 12.5. b) 14.5. c) 17.5. d) 25.5.
e) 30.5. f) 42.5. g) 50.5. h) None of the above.

9. A 2nd-order stationary process with covariance function $\rho(h) = \frac{exp(-\beta|h|)}{2\beta}$, for all h, where $\beta > 0$, is called an Ornstein-Uhlenbeck process. What is the corresponding semivariogram, $\gamma(h)$?

a) $\frac{1}{|h|} - \frac{exp(-2\beta|h|)}{\beta}$. b) $\frac{|h|}{\beta} - \frac{exp(-\beta|h|)}{|h|+\beta}$. c) $\frac{|h|}{2\beta} - \frac{exp(-2\beta|h|)}{|h|+\beta}$.
d) $\frac{1}{2\beta} - \frac{exp(-2\beta|h|)}{|h|+\beta}$. e) $\frac{1}{2\beta} - \frac{exp(-\beta|h|)}{2\beta}$. f) None of the above.

10. Consider the point process on B with conditional intensity
$\lambda(t,x,y) = \mu + K \int_{t'<t} \int_0^1 \int_0^1 \frac{1}{\lambda(t',x',y')} f(t-t') g(x-x', y-y') dN(t', x', y')$.
This process is called a *recursive* process.
Suppose f is the exponential density $f(u) = \alpha e^{-\alpha u}$,
and g is the uniform density, $g(u,v) = \frac{100}{\pi}$ for $||(u,v)|| < 0.1$. Let $\mu = 5, K = 0.4$, and $\alpha = 4$.
Suppose N is a recursive process with n points in B and let D denote the total productivity, $\sum_{i=1}^{n} \frac{K}{\lambda(t_i,x_i,y_i)}$, of these n points. What is $E(D)$? Round the answer to the nearest integer if necessary.

a) 1. b) 2. c) 4. d) 7.
e) 9. f) 11. g) 12. h) None of the above.