

# Stat 35, Introduction to Probability. Rick Paik Schoenberg

## Outline for the day:

1. Discuss addiction.
2. Basic principle of counting.
3. Permutations and combinations.
4.  $R$ .
5. Conditional probability, independence, and multiplication rule.
6. Independence and dependence examples.
7. Negreanu and Elezra.
8. Odds ratios.
9.  $P(\text{have AA and flop a full house})?$
10.  $P(A\spadesuit \text{ after } 1^{\text{st}} \text{ ace})?$



1. Addiction handout.

2. **Basic Principle of Counting.**

If there are  $a_1$  distinct possible outcomes on trial #1, and for each of them, there are  $a_2$  distinct possible outcomes on trial #2, then there are  $a_1 \times a_2$  distinct possible *ordered* outcomes on both.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?  
 $52 \times 51$ . [ordered:  $(A\clubsuit, K\heartsuit) \neq (K\heartsuit, A\clubsuit)$  .]

In general, with  $j$  experiments, each with  $a_i$  possibilities, the # of distinct outcomes *where order matters* is  $a_1 \times a_2 \times \dots \times a_j$ .

### 3. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

# of distinct possibilities?

52 x 51. [ordered: (A♣, K♥) ≠ (K♥, A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck?  $52 \times 51 \times \dots \times 1 = 52!$

$$\sim 8.1 \times 10^{67}$$

A combination is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[ since now (A♣, K♥) = (K♥, A♣) .]

So, the number of *distinct* hands where *order doesn't matter* is

52 x 51 / 2.

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

“n choose k” = choose(n,k) =  $\frac{n!}{k! (n-k)!}$  .

4. *R*. To download and install *R*, go directly to [cran.stat.ucla.edu](http://cran.stat.ucla.edu), or as it says in the book at the bottom of p157, you can start at [www.r-project.org](http://www.r-project.org), in which case you click on “download *R*”, scroll down to UCLA, and click on [cran.stat.ucla.edu](http://cran.stat.ucla.edu). From there, click on “download *R* for ...”, and then get the latest version.

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**The R Project for Statistical Computing**

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PCA 5 vars  
princomp(x = data, cor = cor)

Fertility  
Catholic  
Agriculture  
Examination  
Education  
(1-3) 60%

Clustering 4 groups

Groups  
28  
16  
1  
2

Factor 1 [41%]  
Factor 3 [19%]

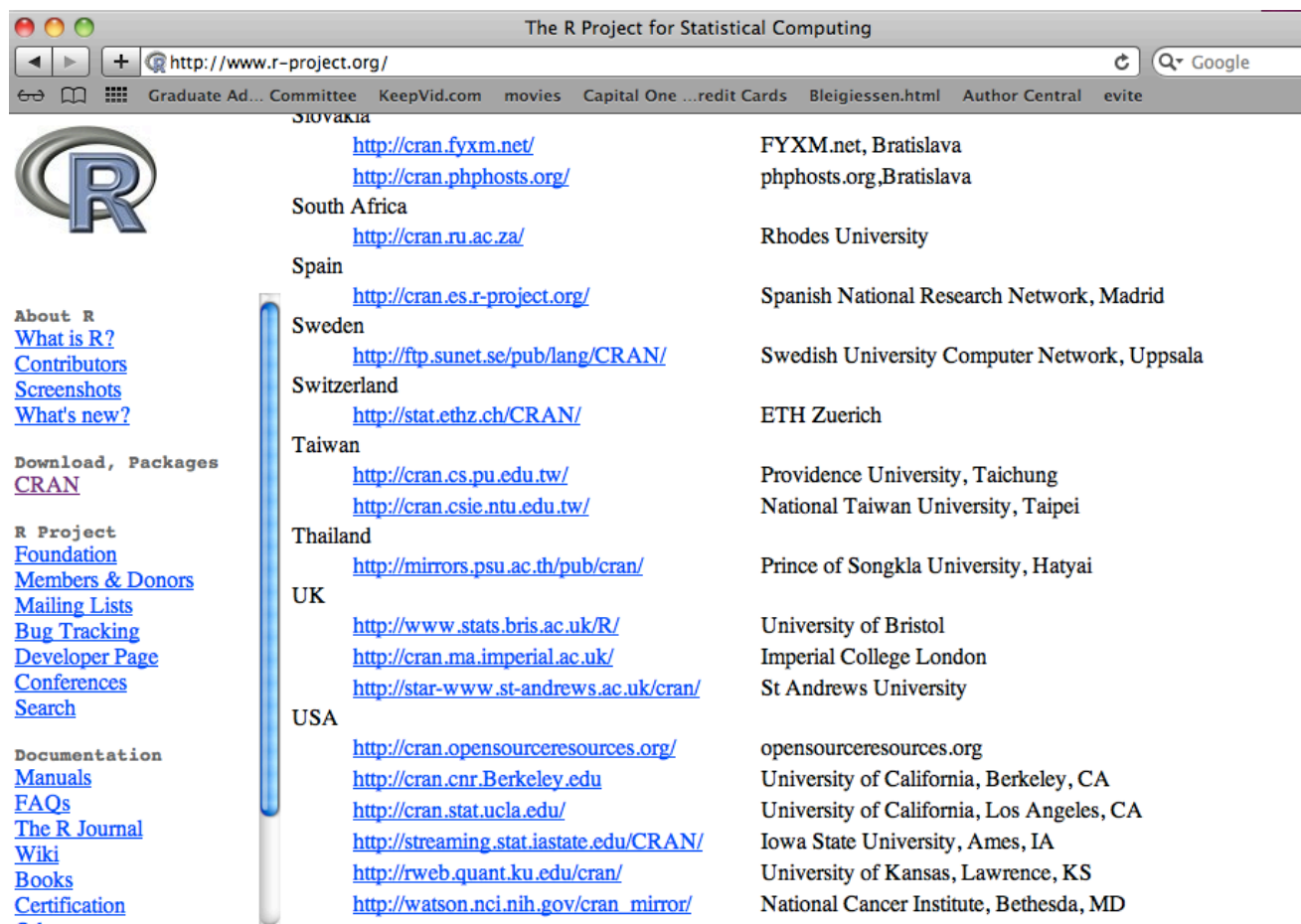
V. De Geneve

**Getting Started:**

- *R* is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To **download *R***, please choose your preferred [CRAN mirror](#).
- If you have questions about *R* like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.

To download and install *R*, go directly to [cran.stat.ucla.edu](http://cran.stat.ucla.edu), or as it says in the book at the bottom of p157, you can start at [www.r-project.org](http://www.r-project.org), in which case you click on “download *R*”, scroll down to UCLA, and click on [cran.stat.ucla.edu](http://cran.stat.ucla.edu).

From there, click on “download *R* for ...”, and then get the latest version.

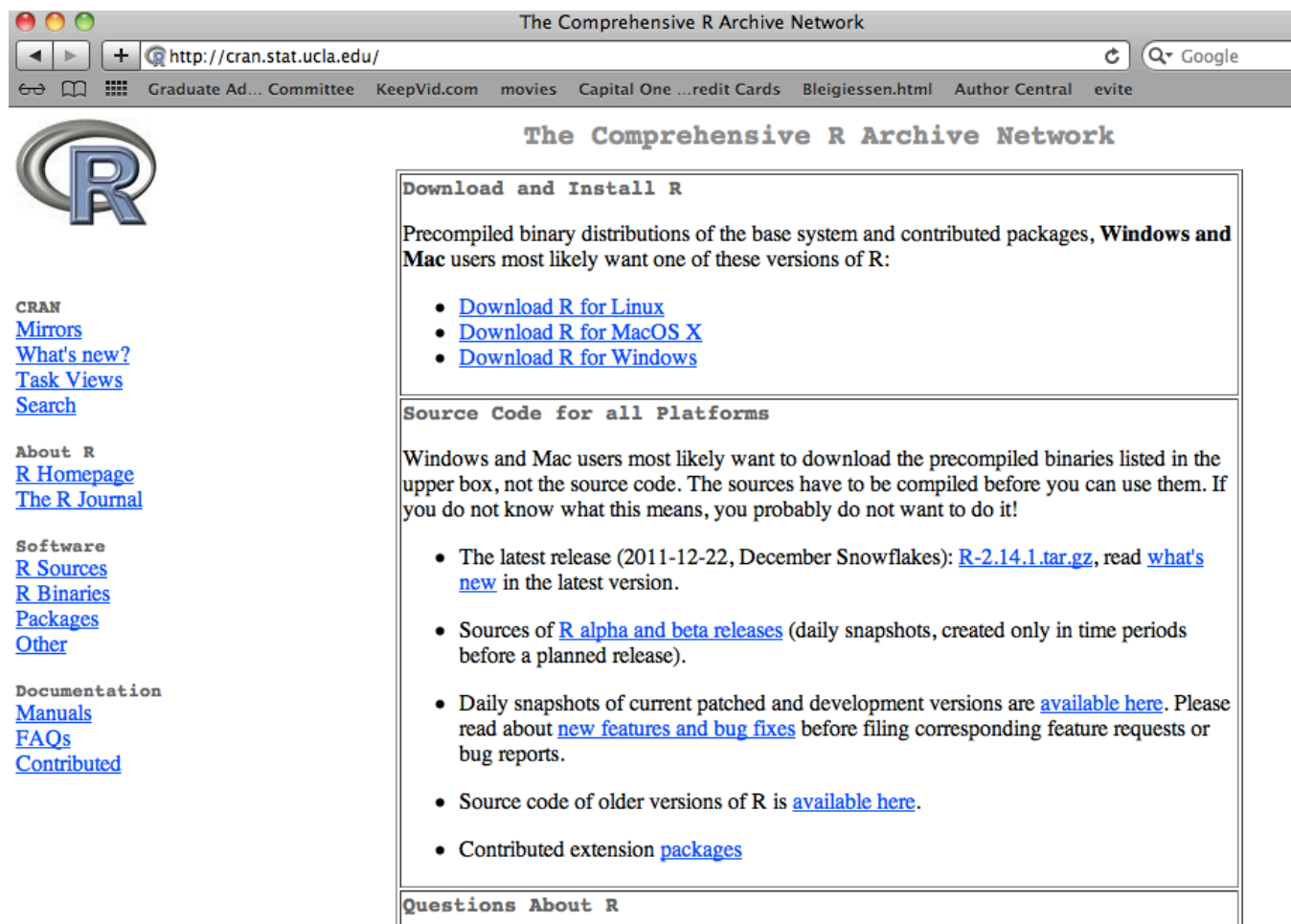


The screenshot shows a web browser window titled "The R Project for Statistical Computing" with the address bar displaying "http://www.r-project.org/". The page content includes a navigation menu on the left and a list of mirrors on the right. The mirrors are organized by country, with each country listing one or more mirror URLs and their corresponding institutions.

Country	Mirror URL	Institution
Slovakia	<a href="http://cran.fyxm.net/">http://cran.fyxm.net/</a>	FYXM.net, Bratislava
	<a href="http://cran.phphosts.org/">http://cran.phphosts.org/</a>	phphosts.org, Bratislava
South Africa	<a href="http://cran.ru.ac.za/">http://cran.ru.ac.za/</a>	Rhodes University
Spain	<a href="http://cran.es.r-project.org/">http://cran.es.r-project.org/</a>	Spanish National Research Network, Madrid
Sweden	<a href="http://ftp.sunet.se/pub/lang/CRAN/">http://ftp.sunet.se/pub/lang/CRAN/</a>	Swedish University Computer Network, Uppsala
Switzerland	<a href="http://stat.ethz.ch/CRAN/">http://stat.ethz.ch/CRAN/</a>	ETH Zuerich
Taiwan	<a href="http://cran.cs.pu.edu.tw/">http://cran.cs.pu.edu.tw/</a>	Providence University, Taichung
	<a href="http://cran.csie.ntu.edu.tw/">http://cran.csie.ntu.edu.tw/</a>	National Taiwan University, Taipei
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	<a href="http://cran.ma.imperial.ac.uk/">http://cran.ma.imperial.ac.uk/</a>	Imperial College London
	<a href="http://star-www.st-andrews.ac.uk/cran/">http://star-www.st-andrews.ac.uk/cran/</a>	St Andrews University
USA	<a href="http://cran.opensourceresources.org/">http://cran.opensourceresources.org/</a>	opensourceresources.org
	<a href="http://cran.cnr.Berkeley.edu">http://cran.cnr.Berkeley.edu</a>	University of California, Berkeley, CA
	<a href="http://cran.stat.ucla.edu/">http://cran.stat.ucla.edu/</a>	University of California, Los Angeles, CA
	<a href="http://streaming.stat.iastate.edu/CRAN/">http://streaming.stat.iastate.edu/CRAN/</a>	Iowa State University, Ames, IA
	<a href="http://rweb.quant.ku.edu/cran/">http://rweb.quant.ku.edu/cran/</a>	University of Kansas, Lawrence, KS
	<a href="http://watson.nci.nih.gov/cran_mirror/">http://watson.nci.nih.gov/cran_mirror/</a>	National Cancer Institute, Bethesda, MD

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From there, click on “download *R* for ...”, and then get the latest version.



The screenshot shows a web browser window titled "The Comprehensive R Archive Network" with the address bar displaying "http://cran.stat.ucla.edu/". The browser's address bar also shows a search engine (Google) and a list of bookmarks including "Graduate Ad... Committee", "KeepVid.com", "movies", "Capital One ...redit Cards", "Bleigiessen.html", "Author Central", and "evite".

The main content of the page is titled "The Comprehensive R Archive Network" and is divided into several sections:

- Download and Install R**

Precompiled binary distributions of the base system and contributed packages, **Windows and Mac** users most likely want one of these versions of R:

  - [Download R for Linux](#)
  - [Download R for MacOS X](#)
  - [Download R for Windows](#)
- Source Code for all Platforms**

Windows and Mac users most likely want to download the precompiled binaries listed in the upper box, not the source code. The sources have to be compiled before you can use them. If you do not know what this means, you probably do not want to do it!

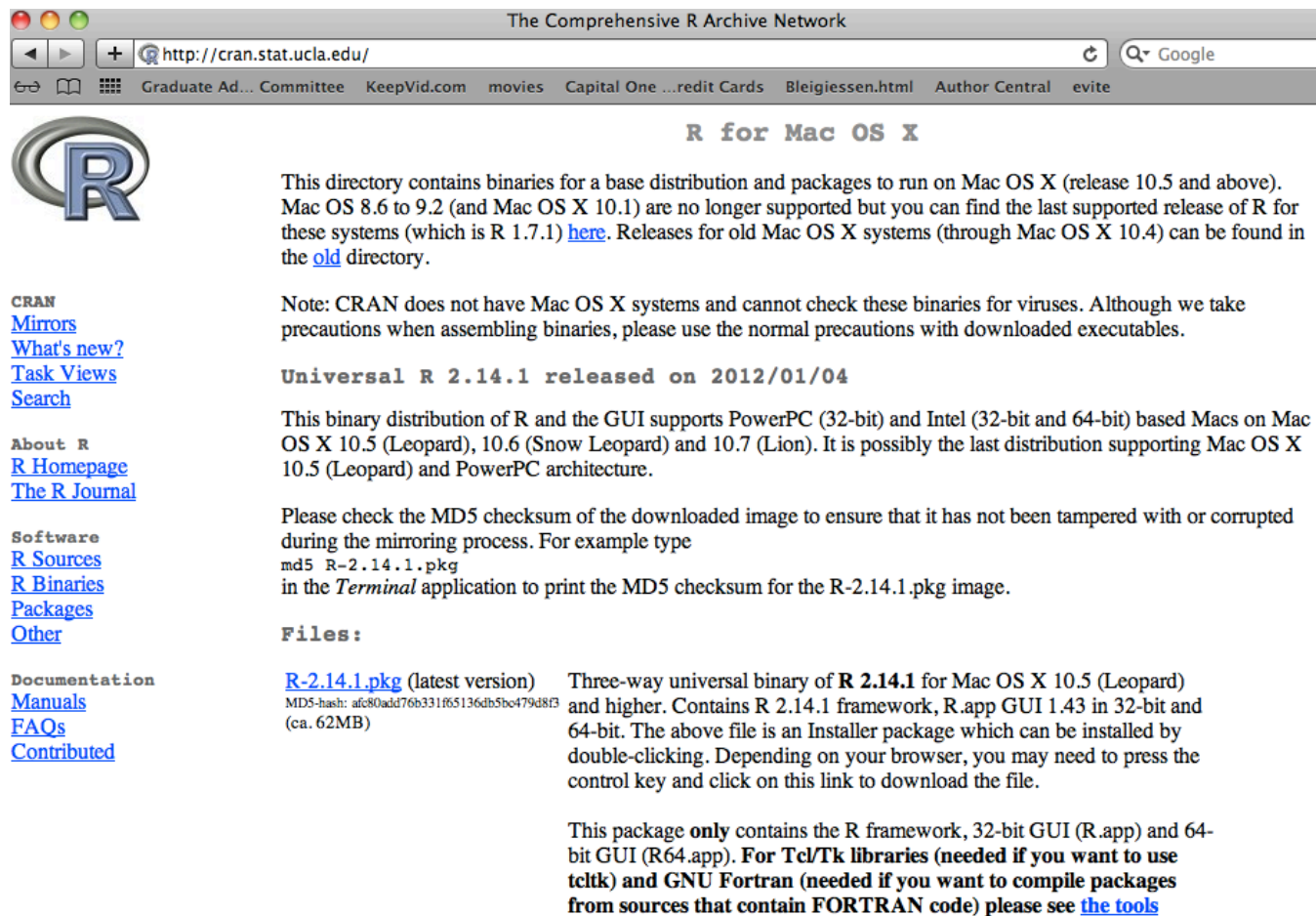
  - The latest release (2011-12-22, December Snowflakes): [R-2.14.1.tar.gz](#), read [what's new](#) in the latest version.
  - Sources of [R alpha and beta releases](#) (daily snapshots, created only in time periods before a planned release).
  - Daily snapshots of current patched and development versions are [available here](#). Please read about [new features and bug fixes](#) before filing corresponding feature requests or bug reports.
  - Source code of older versions of R is [available here](#).
  - Contributed extension [packages](#)
- Questions About R**

On the left side of the page, there is a navigation menu with the following links:

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The Comprehensive R Archive Network

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## R for Mac OS X

This directory contains binaries for a base distribution and packages to run on Mac OS X (release 10.5 and above). Mac OS 8.6 to 9.2 (and Mac OS X 10.1) are no longer supported but you can find the last supported release of R for these systems (which is R 1.7.1) [here](#). Releases for old Mac OS X systems (through Mac OS X 10.4) can be found in the [old](#) directory.

Note: CRAN does not have Mac OS X systems and cannot check these binaries for viruses. Although we take precautions when assembling binaries, please use the normal precautions with downloaded executables.

### Universal R 2.14.1 released on 2012/01/04

This binary distribution of R and the GUI supports PowerPC (32-bit) and Intel (32-bit and 64-bit) based Macs on Mac OS X 10.5 (Leopard), 10.6 (Snow Leopard) and 10.7 (Lion). It is possibly the last distribution supporting Mac OS X 10.5 (Leopard) and PowerPC architecture.

Please check the MD5 checksum of the downloaded image to ensure that it has not been tampered with or corrupted during the mirroring process. For example type

```
md5 R-2.14.1.pkg
```

in the *Terminal* application to print the MD5 checksum for the R-2.14.1.pkg image.

**Files:**

[R-2.14.1.pkg](#) (latest version) Three-way universal binary of **R 2.14.1** for Mac OS X 10.5 (Leopard) and higher. Contains R 2.14.1 framework, R.app GUI 1.43 in 32-bit and 64-bit. The above file is an Installer package which can be installed by double-clicking. Depending on your browser, you may need to press the control key and click on this link to download the file.

This package **only** contains the R framework, 32-bit GUI (R.app) and 64-bit GUI (R64.app). **For Tcl/Tk libraries (needed if you want to use tcltk) and GNU Fortran (needed if you want to compile packages from sources that contain FORTRAN code) please see [the tools](#)**

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## 5. Conditional probability, independence, & multiplication rule.

$P(A \& B)$  is often written “ $P(AB)$ ”.

“ $P(A \cup B)$ ” means  $P(A \text{ or } B \text{ [or both]})$ .

Conditional Probability:

$P(A \text{ given } B)$  [written “ $P(A|B)$ ”] =  $P(AB) / P(B)$ .

Independent: A and B are “independent” if  $P(A|B) = P(A)$ .

Fact (*multiplication rule for independent events*):

If A and B are independent, then  $P(AB) = P(A) \times P(B)$

Fact (*general multiplication rule*):

$$P(AB) = P(A) P(B|A)$$

$$P(ABC\dots) = P(A) \times P(B|A) \times P(C|A\&B) \dots$$

## 6. Independence and dependence examples.

Independence:  $P(A | B) = P(A)$  [and  $P(B|A) = P(B)$ ].

So, when independent,  $P(A\&B) = P(A)P(B|A) = P(A)P(B)$ .

Reasonable to assume the following are independent:

- a) Outcomes on different rolls of a die.
- b) Outcomes on different flips of a coin.
- c) Outcomes on different spins of a spinner.
- d) Outcomes on different poker hands.
- e) Outcomes when sampling from a large population.

Ex:  $P(\text{you get AA on 1st hand and I get AA on 2nd hand})$

$$= P(\text{you get AA on 1st}) \times P(\text{I get AA on 2nd})$$

$$= 1/221 \times 1/221 = 1/48841.$$

$P(\text{you get AA on 1st hand and I get AA on 1st hand})$

$$= P(\text{you get AA}) \times P(\text{I get AA} | \text{you have AA})$$

$$= 1/221 \times 1/(50 \text{ choose } 2) = 1/221 \times 1/1225 = 1/270725.$$

## 7. Negreanu and Elezra example: High Stakes Poker, 1/8/07.

Greenstein folds, Todd Brunson folds, Harman folds. Elezra calls \$600, Farha (K♠ J♥) raises to \$2600, Sheikhan folds. Negreanu calls, Elezra calls. Pot is \$8,800.

Flop: 6♠ 10♠ 8♥ .

Negreanu bets \$5000. Elezra raises to \$15000. Farha folds.

Negreanu thinks for 2 minutes..... then goes all-in for another \$96,000.

Elezra: 8♣ 6♣. (Elezra calls. Pot is \$214,800.)

Negreanu: A♦ 10♥ .

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At this point, the odds on tv show 73% for Elezra and 25% for Negreanu.

They “run it twice”. First: 2♠ 4♥. Second time? A♥ 8♦!

P(Negreanu hits an A or 10 on turn & still loses)?

Given both their hands, and the flop, and the first “run”, what is P(Negreanu hits an A or 10 on the turn & loses)?

Since he can't lose if he hits a 10 on the turn, it's:

P(A on turn & Negreanu loses)

$$= P(\text{A on turn}) \times P(\text{Negreanu loses} \mid \text{A on the turn})$$

$$= \frac{3}{43} \times \frac{4}{42}$$

$$= 0.66\% \text{ (1 in 150.5)}$$

Note: this is very different from:

P(A or 10 on turn) x P(Negreanu loses),

which would be about  $\frac{5}{43} \times 73\% = 8.49\%$  (1 in 12)

## 8. Odds ratios.

Odds ratio of A =  $P(A)/P(A^c)$

Odds *against* A = Odds ratio of  $A^c = P(A^c)/P(A)$ .

Ex: (from Phil Gordon's *Little Blue Book*, p189)

Day 3 of the 2001 WSOP, \$10,000 No-limit holdem championship.

613 players entered. Now 13 players left, at 2 tables.

Phil Gordon's table has 5 other players. Blinds are 3,000/6,000 + 1,000 antes.

Matusow has 400,000; Helmuth has 600,000; Gordon 620,000.

(the 3 other players have 100,000; 305,000; 193,000).

Matusow raises to 20,000. Next player folds.

Gordon's next, in the *cutoff seat* with  $K\clubsuit K\spadesuit$  and re-raises to 100,000.

Next player folds. Helmuth goes all-in. Big blind folds. Matusow folds.

Gordon's decision.... Fold!

Odds against Gordon winning, if he called and Helmuth had AA?

What were the odds against Gordon winning, if he called and Helmuth had AA?

$$P(\text{exactly one K, and no aces}) = 2 \times C(44,4) / C(48,5) \sim 15.9\%.$$

$$P(\text{two Kings on the board}) = C(46,3) / C(48,5) \sim 0.9\%.$$

[also some chance of a straight, or a flush...]

Using [www.cardplayer.com](http://www.cardplayer.com)'s poker odds calculator,

$P(\text{Gordon wins})$  is about 18%, so the odds against this are:

$$P(A^c)/P(A) = 82\% / 18\% = 4.6 \text{ (or “4.6 to 1” or “4.6:1”).}$$

9. P(you get dealt AA and flop a full house)?

This = P(you get dealt AA) x P(you flop a full house | AA)

$$= C(4,2) / C(52,2) * P(\text{triplet or Axx} | AA)$$

$$= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,2)$$

$$= .0709\%.$$

10. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

11. Which is more likely, given no info about your cards:

\* flopping 3 of a kind,

or

\* eventually making 4 of a kind?