Comparison of ETAS parameter estimates across different global tectonic zones

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Abstract

Branching point process models such as the ETAS (Epidemic-Type Aftershock Sequence) models introduced by Ogata (1988, 1998) are often used in the description, characterization, simulation, and declustering of modern earthquake catalogs. The present work investigates how the parameters in these models vary across different tectonic zones. After considering divisions of the surface of the Earth into several zones based on the plate boundary model of Bird (2003), ETAS models are fit to the occurrence times and locations of shallow earthquakes within each zone. Computationally, the EM-type algorithm of Veen and Schoenberg (2008) is employed for the purpose of model fitting. The fits and variations in parameter estimates for distinct zones are compared. Seismological explanations for the differences between the parameter estimates for the various zones are considered, and implications for seismic hazard estimation and earthquake forecasting are discussed.

Keywords: branching processes, earthquakes, epidemic-type aftershock sequence model, space-time point process models, maximum likelihood, global earthquake prediction, plate tectonics.

1 Introduction

Researchers have recently defined global tectonic zones according to geophysical characteristics such as crust types and relative plate velocities using plate boundary models (Bird, 2003; Bird and Kagan, 2004; Kagan et al., 2010). The primary purpose of this study is
to investigate the different properties and patterns of seismicity in these different zones by 
exploring the fit of space-time branching point process models to the data within each zone. 

Similarities and differences between the background rates and clustering behavior of earth-
quake occurrences in different tectonic zones may shed light on the interplay between tectonic 
forces and observed seismicity rates.

Similar investigations were performed by Kagan et al. (2010), using the branching point 
process model developed by Kagan (1991). Here, we explore the fit of Epidemic-Type Af-
tershock Sequence (ETAS) models of Ogata (1998) within each tectonic zone. Such models, 
which have become widely used in the study of the properties of earthquake clustering and 
in the forecasting of seismicity, are based on the notion that earthquakes occur at some 
background rate, those background events trigger aftershocks, and those aftershocks trigger 
subsequent aftershocks, etc. ETAS models have been used extensively in the analysis of local 
catalogs (e.g. Ogata, 1998; Schoenberg, 2003; Zhuang et al., 2005; Ogata and Zhuang, 2006; 
Helmstetter et al., 2007), and of global catalogs (Lombardi and Marzocchi, 2007; Marzocchi 
and Lombardi, 2008), but we are unaware of their prior use in the analysis of global tectonic 
zones such as those delineated by Bird (2003).

This article is organized as follows. We first present a brief description of the delineation 
of tectonic zones and global earthquake data in Section 2. Section 3 describes the ETAS 
model of Ogata (1998), as well as model diagnostics and the computational implementation 
used. Section 4 discusses the results of estimating ETAS models within each tectonic zone, 
including the maximum likelihood estimates (MLEs) of the parameters in the model, their 
standard errors, fitted triggering functions, estimates of the Gutenberg-Richter constant and 
branching ratio. Section 5 presents statistical diagnostics based on the estimated conditional
intensity, and diagnostics using comparisons between the empirical aftershock rates and fitted triggering functions. Concluding remarks and discussion of future work follow in Section 6.

2 Tectonic zones and earthquake catalog

2.1 Tectonic zones

Tectonic zones are related to the 7 plate boundary classes defined in global plate model PB2002 (Bird, 2003), but zones are defined differently so as to allow for earthquake classification even if location, depth, and focal mechanism are poorly known. As displayed in Figures 1, there are four major zones (Kagan et al., 2010):

1. Active continent (including continental parts of all orogens of PB2002, plus continental plate boundaries of PB2002).

2. Slow-spreading ridges (oceanic crust, spreading rate < 40 mm/yr, including transforms).

3. Fast-spreading ridges (oceanic crust, spreading rate ≥ 40 mm/yr, including transforms).

4. Trenches (including incipient subduction, and earthquakes in outer rise or upper plate).

In what follows, we refer to the zone consisting of the plate interiors, i.e. the rest of the Earth’s surface occupied by the non-colored areas in Figure 1, as zone 0.

It is reasonable to anticipate different branching behaviors between the zones, and our present definition of zones is designed with consideration for practical matters that will
permit such testing to be conducted relatively easily. Zones are defined here as collections
of polygons delineating the surface areas into which epicenters of shallow earthquakes may
fall. The zones are not divided based on the depths of the earthquakes because these depths
are usually poorly estimated unless there is local station control, so we follow Bird et al.
(2002) in limiting our analysis simply to shallow earthquakes of depth less than 70 km. The
resulting partition produces five major zones, and each zone contains a sufficient number of
earthquakes within the time period considered (1973-2006) to enable branching models to
be accurately and reliably estimated.

Our tectonic zones are defined by objective rules implemented in a program because this
is reproducible, easy to explain, easy to revise, and not very subject to procedural error.
Our program assigns tectonic zones to grid points rather than attempting to draw boundary
curves about each zone. It loops through latitudes (in 0.1° steps) and also loops through
longitudes (in 0.1° steps) to create a grid of zone index integers which identify the zone at
each grid point. The decision algorithm for each point is detailed in the Appendix of Kagan
et al. (2010). The necessary data are available in digital form: elevation from ETOPO5
(Anonymous, 1988), age of seafloor from Mueller et al. (1997), plate boundaries and Euler
poles from Bird (2003).

The output is a relatively compact (6 to 12 MB) representation, which is shown in Figure
1. The gridded zone integers may be downloaded from http://bemlar.ism.ac.jp/wiki/
index.php/Bird’s_Zones.

Note that some tectonic zones are the unions of non-continuous patches and may include
quite different types of earthquakes. For instance, strike-slip and normal-faulting earth-
quakes are contained in the same mid-ocean spreading ridges and continental rift zones,
and similarly, along trenches, subduction-related earthquakes are analyzed collectively along
with back-arc-spreading earthquakes.

2.2 Earthquake catalog

The data used for our analysis were retrieved from the PDE (Preliminary Determination
of Epicenters) database (USGS, 2008). The time window we study is January 1, 1973 to
December 31, 2006. Only shallow earthquakes (0 to 70 kilometers in depth) are considered.
The shallow events constitute approximately 80% of all events in the catalog. Summaries of
catalog size and area are shown in Table 1. For each zone’s analysis, a lower magnitude cutoff
threshold of $m_t$ 5.0 is used, following the example of Kagan et al. (2010). The PDE dataset
with lower magnitude threshold 5.0 consists of mostly (97%) body wave (55%), surface wave
(8%), moment magnitude (31%), and small percentage of local magnitude (3%).

Table 1 displays each zone’s surface area, in proportion to that of the globe, and the pro-
portion of all observed events occurring in each zone. Among these zones, zone 4 (trenches)
has the largest number of events (64.5% out of total 40504 events) while only occupying
2.98% of the global surface area. Zone 1 (active continent) also contains a disproporti-
ate number of events, with 16.6% of the events in the global catalog and only 6.58% of the
global surface area. Zones 1 and 4 are unions of irregularly shaped polygons, with one or two
larger polygons corresponding to the orogens, and several small and strip-shaped polygons
corresponding to the plate boundaries. Zones 2 (slow-spreading ridges) and 3 (fast-spreading
ridges) are similar to one another in number and shape. They each have about 7% of the
total events (7.47% for zone 2 and 6.35% for zone 3), and occupy a small portion of the
global surface area (3.09% for zone 2 and 1.65% for zone 3). Zones 2 and 3 have very long, narrow shapes completely constituted by plate boundaries. Lastly, zone 0 (plate-interior), the regime excluding zones 1, 2, 3 and 4, occupies 43.8% of the whole global surface and contains 5.06% of the global events.

3 Model and methods

3.1 Model

The ETAS model is a type of Hawkes point process model; such models are also sometimes called branching or self-exciting point processes. For a temporal Hawkes process, the conditional rate of events at time $t$, given information $H_t$ on all events prior to time $t$, has the form

$$\lambda(t|H_t) = \mu + \sum_{i: t_i < t} g(t - t_i),$$

where $\mu > 0$ is the background rate, $g(u) \geq 0$ is the triggering function dictating the rate of aftershock activity associated with a prior event, and $\int_0^\infty g(u)du < 1$ in order to ensure stationarity (Hawkes, 1971).

These models were called epidemic by Ogata (1988) because of their natural branching structure: an earthquake can produce aftershocks, and these aftershocks produce their own aftershocks, etc. An example is the time-magnitude ETAS model of Ogata (1988), which has magnitude-dependent triggering function

$$g(u_i; m_i) = \frac{K_0}{(u_i + c)^p} e^{a(m_i - m_t)},$$
where $u_i = t - t_i$ is the time elapsed since earthquake $i$, $K_0 > 0$ is a normalizing constant governing the expected number of direct aftershocks triggered by earthquake $i$, $m_i$ denotes the recorded magnitude of earthquake $i$, and $m_t$ is the lower cutoff magnitude for the earthquake catalog. The term $K_0/(u_i + c)^p$ describing the temporal distribution of aftershocks is known as the modified Omori-Utsu law (Utsu et al., 1995).

Ogata (1998) extended the ETAS model to describe the space-time-magnitude distribution of earthquake occurrences, introducing circular or elliptical spatial functions into the triggering function so that the squared distance between an aftershock and its triggering event follows a Pareto distribution. For instance, a form for the conditional rate proposed in Ogata (1998) is

$$\lambda(t, x, y | \mathcal{H}_t) = \mu + \sum_{i: t_i < t} g(t - t_i, x - x_i, y - y_i, m_i),$$

where

$$g(t - t_i, x - x_i, y - y_i, m_i) = K_0 (t - t_i + c)^{-p} \left( \frac{(x - x_i)^2 + (y - y_i)^2}{e^{a(m_i - m_t)}} + d \right)^{-q}, \quad (3)$$

and where $(x_i, y_i)$ represent the Cartesian coordinates of earthquake $i$.

With the triggering function (3), the spatial distribution of aftershocks interacts dramatically with mainshock magnitude. This interaction is sometimes referred to as magnitude scaling, since a typical feature of the model and of earthquake catalogs is a gradual widening of the spatial-temporal aftershock distribution as the magnitude of the mainshock increases (Utsu et al., 1995).

Typically, in local catalogs, one calculates the squared epicentral distance in the Cartesian plane, via $D_i^2 = (x - x_i)^2 + (y - y_i)^2$ as in (3), but for global seismicity it is important to
account for the sphericity of the Earth. Hence in this analysis we replace this term in (3) with the squared great circle distance between \((x, y)\) and \((x_i, y_i)\).

Ogata (1998) and others have extended the model (1) to include a spatially inhomogeneous background rate \(\mu(x, y)\). Note that in our analysis, we fit a homogeneous background rate in each zone, since our primary goal is to examine the differences in clustering behavior and overall rates between the various zones, rather than the spatial heterogeneity within each zone. Further, ETAS models are fit to the data within each zone individually; no triggering between zones is assumed. Implications and extensions to more complex models are discussed in Section 6.

### 3.2 Methods

The parameters in the ETAS model may be estimated by maximizing the log-likelihood:

\[
\log L(\theta) = \sum_i \log \lambda(t_i, x_i, y_i | H_{t_i}) - \int_0^T \int_{y_0}^{y_1} \int_{x_0}^{x_1} \lambda(t, x, y) \, dx \, dy \, dt
\]

where \(\theta = (\mu, a, K_0, c, p, d, q)\) is the vector of parameters to be estimated and \([x_0, x_1] \times [y_0, y_1] \times [0, T]\) is the space-time window where points in the marked point process dataset \((x_i, y_i, t_i, m_i)\) are observed (Ogata, 1998; Daley and Vere-Jones, 2003, ch. 7). Maximum likelihood estimates of the parameters in the ETAS model are obtained based on the EM (Expectation-Maximization type algorithm) proposed in Veen and Schoenberg (2008), with modification to accommodate the magnitude scaling in (1) and the use of great circle distance mentioned in Section 3.1. The algorithm of Veen and Schoenberg (2008) is similar to the original, EM algorithm of Dempster, Laird and Rubin (1977), which is now widely used to
obtain estimates of parameters in the presence of unobserved random variables influencing
the likelihood. The EM algorithm is based on an expectation (E) step where the expectation
of the likelihood given the unobserved variables is computed and a maximization (M) step
where the parameters maximizing the expected likelihood computed in the E step are found.
The above steps are typically iterated until a certain tolerance is reached. In the EM-type
estimation procedure proposed by Veen and Schoenberg (2008), the branching structure
dictating which earthquake triggered which aftershocks comprises the unobserved random
variables and is estimated according to the ETAS model in each E step. This assignment
changes in each iteration as the parameters in the ETAS model change following each M
step. The idea is similar to the stochastic reconstruction idea of Zhuang, Ogata and Vera-
Jones (2002, 2004), in which the ETAS model is similarly used to assign a random branching
structure to an observed earthquake catalog.

Note that the ETAS model allows assignments where a triggered event has larger magni-
tude than its parent event and that despite the fact that the branching structure estimated
in each iteration of the EM algorithm will often be quite erroneous, the resulting estimation
procedure nevertheless produces quite stable and accurate parameter estimates, as shown by
Veen and Schoenberg (2008).

For zones 1, 2, 3, and 0, the time window 1973 to 2001 was used for computation of
MLEs (maximum likelihood estimates). Due to concerns associated with computer memory
limitations, the time window 1999 to 2006 was used to obtain the MLEs of zone 4, the
trench zone; we have ensured that in all of these cases, the MLEs appear already to have
converged and stabilized within these shortened time windows. The standard error for each
parameter estimate may be estimated using the square root of the diagonal elements of the
inverse of the Hessian of the log-likelihood (Ogata, 1978). As noted in Wang et al. (2010) and Kagan et al. (2010), such asymptotic standard error estimates may be unreliable in the presence of strong correlations between parameter estimates, making the statistical significance of differences between parameter estimates for different tectonic zones difficult to assess. One method for obtaining more accurate standard errors would be to simulate the ETAS model repeatedly, after fitting its parameters by MLE (maximum likelihood estimation), and subsequently re-estimating the model’s parameters for each simulation. One may then use the standard deviation of these estimates as an approximation of the standard error. Such repeated simulation and re-estimation of the models is extraordinarily computationally intensive, however, and approximate standard errors for each parameter estimate obtained individually by investigating the square root of the inverse Hessian when optimizing each parameter individually are used as an approximation.

In addition to studying the behavior of ETAS parameter estimates in the different tectonic zones, one may also investigate other properties of the models, such as their information gain relative to the stationary Poisson model. The information gain is defined as (Kagan and Knopoff, 1977; Kagan, 1991):

\[
I = \frac{\ell - \ell_0}{N}
\]

(5)

where \( \ell \) is the logarithm of the likelihood for the ETAS model, \( \ell_0 \) is the logarithm of the likelihood for the stationary Poisson process, and \( N \) is the catalog size.

Since the ETAS model contains the Poisson process as a special case when the triggering function is 0, the information gain must necessarily be non-negative, and since the difference
between ETAS and the stationary Poisson model is that the former is capable of modeling intense clustering behavior, higher values of the information score in this case generally signify increased clustering in the earthquake catalog.

Another comparison of the triggering properties in different zones can be made by investigating the branching ratio. Letting \( m = m_i - m_t \), then \( G \), the total expected number of directly triggered aftershocks per earthquake of magnitude \( m_i \) within each zone, can be defined as the integral of (3) over time and space:

\[
G(m) = \int_0^\infty \int_{-\infty}^{-\infty} g(t, x, y|H_i) \, dx \, dy \, dt = \pi K_0 (p - 1)^{-1} c^{1-p} (q - 1)^{-1} d^{1-q} e^{am}. \tag{6}
\]

In the subcritical case, the expected value \( E(G) = \int G(m) f(m) \, dm \) coincides with the branching ratio (Veen, 2006), and estimates of this branching ratio for each zone are discussed in Section 4.2.

In addition to the aforementioned assessment methods, we compare the estimated conditional intensity at a grid of locations, integrated over time, with the corresponding observed rate of earthquake occurrences within a small rectangle around each location in the grid. Letting \((0, T)\) denote the observed time window,

\[
\int_0^T \lambda(t, x, y) \, dt = \mu T + K_0 \sum_i \int_{0}^{T-t_i} (t + c)^{-p} (R_i^2/e^{a(m_i-m_t)} + d)^{-q} \, dt \tag{7}
\]

\[
= \mu T + K_0 \sum_i (R_i^2/e^{a(m_i-m_t)} + d)^{-q} (c^{p-1} - (T - t_i + c)^{p-1})/(p - 1) \tag{8}
\]

where \( R_i \) is the distance between the location \((x, y)\) and a previous earthquake’s location, \((x_i, y_i)\); particularly, the great circle distance between \((x, y)\) and \((x_i, y_i)\) is used in the global
case. Comparison of the integrals in (8) with observed seismicity rates for various locations $(x, y)$ may be assessed directly, or via the various standardizations of these differences, as discussed in Baddeley et al. (2005). As a further diagnostic, we compare the estimated triggering function $g$ in (3), with the empirical, observed aftershock rate. The procedure to obtain the empirical aftershock rate is the following: we select earthquakes in a given fixed magnitude range and consider their productivity of direct aftershocks by inspecting subsequent earthquakes within some small time-space window within the catalog. Note that since the background rate is much smaller than the triggering of the conditional intensity in (1) at such locations and times, the empirical triggering function and $g$ should be similar in such regions if the model is appropriate.

4 Results

4.1 Maximum likelihood estimates of ETAS parameters

The MLEs (maximum likelihood estimates), their standard errors, and the log-likelihoods of the ETAS model and the reference model (stationary Poisson) are shown in Table 2. The information gains are all larger than 1, showing apparent improvement in fit due to the ETAS model compared to the stationary Poisson model, and indicating significant clustering in each zone. Zone 0 (plate-interior) has the largest information gain, 4.20, due to the more pronounced spatial and temporal heterogeneity of seismicity within this zone. The information gains in zones 1 (active continent) and 4 (trenches) are also somewhat large (2.62 for zone 1 and 2.24 for zone 4), due to the high level of clustering in these two zones.
Zones 2 (slow-spreading ridges) and 3 (fast-spreading ridges) have the smallest information gains (1.79 for zone 2 and 1.20 for zone 3), as the seismicity within these zones is relatively homogeneous and less clustered than the other zones, so the gain due to the ETAS model, relative to a stationary Poisson model, is comparatively small. Note that the analyses in Kagan et al. (2010) use the same tectonic zone partitions and calculate the information gain relative to an inhomogeneous Poisson model. Kagan et al. (2010) also use different parametrization of their critical branching model. Here we apply homogeneous models and compute the information gain relative to a homogeneous Poisson model.

The estimated background rate, $\hat{\mu}$, differs substantially from one zone to another. Not surprisingly, the estimate is largest in zone 4, which has by far the highest seismicity rate, as seen in Table 1. Zone 3 has the second largest estimated rate of background seismicity, but the estimate of this rate in zone 3 is only one fourth of the corresponding estimated rate in zone 4. The estimated background rates in zones 1 and 2 are somewhat similar to one another. The fact that the estimated background seismicity rate in zone 4 is approximately 500 times that in zone 0 demonstrates the tremendous variability in overall seismicity from one tectonic zone to another, with the vast majority of background events concentrating in the trench zone.

The MLEs of other parameters can similarly be used to summarize patterns of seismicity, including levels of clustering, swarms, and spatial triggering behavior, within the different tectonic zones. The exponential term in (2) is known as the productivity law in seismology, in which the parameter $a$ governs the relationship between the magnitude of an earthquake and its expected number of generated offspring and is also useful in characterizing earthquake sequences quantitatively in relation to their classification into seismic types (Mogi,
1963; Utsu, 1970; Ogata, 1988). Smaller estimates of $a$ in zones 2 and 3 are characteristic of earthquake swarms, so the relatively low estimates of $a$ in zones 2 and 3 appear to indicate a correspondingly high incidence of swarming in these tectonic zones. Conversely, the estimates of $a$ are largest in zones 1 and 4, indicating a large disparity in aftershock productivity between smaller and larger events. Lastly, $\hat{a}$ is moderately sized in zone 0, indicating relatively moderate distinction between large and small events in terms of the productivity of subsequent aftershocks in this zone.

The parameters $c$ and $p$ govern the temporal decay of aftershock activity. The modified Omori formula describes the frequency of aftershocks per unit time interval since a preceding event as $n(t) = K_0/(t + c)^p$ (Utsu, 1961). As seen in Table 2, the values of $\hat{p}$ are similar across the zones; this stability indicates the seismic activity in the different tectonic zones is somewhat similar in terms of the rate at which the aftershock behavior after an earthquake decreases over time. Note that while $\hat{p}$ seems similar across the zones, the estimate of $c$ in zone 0 is somewhat higher than that in the other zones. Larger values of $\hat{c}$ indicate a more gradual decaying rate of seismicity following an event, according to the modified Omori formula. The estimates of $c$ are much smaller in Zones 2, 3, and 4 than the other zones, indicating that these ridges, oceanic transform zones and trenches appear to have sharp temporal decays in aftershock activity following earthquakes. Conversely, the apparent aftershock activity decays much more slowly and gradually in active continent zones and especially in interplate regions, where little aftershock activity is observed even in the close temporal proximity of an event.

The parameter $q$ governs the spatial distribution of aftershocks. In general, smaller $q$ corresponds to a more gradual spatial decay of aftershock productivity (long range decay)
while larger $q$ indicates that the triggering effect is mainly confined to a relatively small area around an earthquake (short range decay). Our results indicate that, among the major zones, zones 1 and 4 have relatively far-reaching triggering effects, and conversely, zones 2 and 3 have triggering effects that reach a smaller area. Additionally, the value of $\hat{q}$ for zone 0 is much smaller than the corresponding estimates in the major zones, indicating that aftershock activity is much more gradually spatially decaying in interplate regions, according to the fitted ETAS model. This is likely due in large part to faulting being attributed to clustering in the ETAS model, as explained further below.

Along with $q$, the square root of $d$ indicates the scale, in km, of the spatial decay in the generation of aftershocks. That is, if at a certain time after a particular event of magnitude $m_i$, the rate of aftershocks at distance $R = 0$ from the event is some value $s$, then at distance $\sqrt{d}e^{a(m_i-m_t)/2}$ the rate decreases to $2^{-q}s$, i.e. the rate decreases by a factor of $2^{-q}$. As seen from Table 2, among the major zones, the estimate of $d$ is smallest in the active continental earthquake zone and highest in the ridges and oceanic transform zones. Note that, for two models with similar values of $q$, a higher value of $d$ indicates more gradual spatial decay in clustering as one moves further from the mainshock. However, such interpretations can be misleading without simultaneously regarding the corresponding estimates of $q$; zone 3, for instance, has the largest estimate of $d$, indicating more gradual spatial decay with distance, but also the largest estimate of $q$, which indicates sharper spatial decay. The cumulative effects of these two parameters on the spatial decay function are explored below.

$K_0$ is a normalizing constant governing the total expected number of aftershocks per earthquake (Ogata 1988, 1998). Estimates of $K_0$ vary dramatically between the various tectonic zones, with the highest observed triggering rates in zones 2 and 3. This is quite
counterintuitive, since one would expect higher productivity of aftershocks in the more active zones. Additionally, the estimate $\hat{K}_0$ in zone 0 is approximately the same as that in zone 1 and zone 4. As noted in Veen (2006), the variance of $\hat{K}_0$ may be very large due to the flatness of (1) and thus large changes in $\hat{K}_0$ can correspond to very small improvements in the log-likelihood (4). In addition, the temporal and spatial components of the triggering function $g$ are densities and integrate to unity when integrated over infinite time and space, but when integrated over relatively small, thin zones, such as zones 2 and 3 in particular, the integral of the spatial component will often deviate substantially from one. This complicates the interpretation of $\hat{K}_0$, since the expected number of aftershocks observed within the given zone will deviate markedly from $\hat{K}_0$ in such cases. Hence it may be preferable to interpret the model’s triggering density in total rather than interpret each parameter estimate individually. This is explained further below.

Figure 2 shows both the relationship between the estimated triggering function $g$ and $\Delta R$ or $(\Delta R)^2$, where $\Delta R$ denotes the distance between a mainshock and its aftershock. The triggering component is much more pronounced in the active zone (zone 1), as expected, compared to the other zones, but this triggering decays vary rapidly as the distance from the mainshock increases. Zone 0’s estimated triggering function is noticeably larger than that of the other zones. A plausible explanation for the strangely high degree of triggering in zone 0 lies in the extreme inhomogeneity of the Earth’s seismicity. Recall that the fitted ETAS model has a homogeneous background rate, $\mu$. Within zones 1-4, the seismicity is relatively homogeneous, but in zone 0, the ETAS model with homogeneous background rate tends to assign a low background rate in order to accommodate areas in zone 0 with no seismicity at all, and as a result, the clusters of seismicity that one observes along faults gets
assigned to the triggering portion of the model, although many of the earthquakes along these faults might more reasonably be thought of as background events.

We now return to our concern of the relationship between parameter estimates for a given fitted model in a given tectonic zone. Since $c$ and $p$ are both time-related, and $d$ and $q$ are both space-related, one might expect strong correlations for these pairs of parameters. As noted from simulation studies in Schoenberg et al. (2010), such estimates can be strongly correlated. In Figure 3, we observe significant correlations among the pairs $(\hat{d}, \hat{q})$, $(\hat{d}, \hat{K}_0)$, $(\hat{K}_0, \hat{q})$, and $(\hat{K}_0$, information gain), while significant correlation is not observed for some other pairs (e.g. for $\hat{a}$, Figure 4).

4.2 Gutenberg-Richter constant, branching ratio

Maximum likelihood estimates of the Gutenberg-Richter constant $\beta$ and its corresponding $b$-value differ between tectonic zones, as shown in Table 3. The probability density function of $m = m_i - m_t$, derived from the Gutenberg-Richter law is (Gutenberg and Richter 1944):

$$f(m) = \frac{\beta e^{-\beta m}}{1 - e^{-\beta (M_{GR}^{max} - m_t)}}$$  \hspace{1cm} (9)$$

where $M_{GR}^{max}$ is defined as the pre-specified maximum magnitude, and the exponent $\beta$ relates to the $b$-value as $\beta = bln(10) \approx 2.3b$ (Veen, 2006). Larger values of $\beta$ in the above truncated exponential density of $m$ indicate more rapid decay in the density function as magnitude increases, implying a larger proportion of smaller events. This is the case in zone 2. Zone 3, by contrast, has a smaller estimate of $\beta$, indicating a smaller proportion of small events.

Note that $\hat{a} < \hat{\beta}$ for all the tectonic zones, and the difference $\hat{\beta} - \hat{a}$ is largest in zones 2,
3 and 0, implying that the smallest earthquakes contribute most to the overall seismic rate (Helmstetter 2003, Sornette and Werner 2005a, 2005b), especially in zones 2, 3 and 0. This agrees with the observation in Section 4.1 that zones 2 and 3 tend to contain more swarms rather than clusters of clearly distinct mainshocks and aftershocks.

Table 3 displays the estimated branching ratio (expected number of triggered events per mainshock) for the fitted models. Zone 0 has the largest estimated number of aftershocks per mainshock. Zone 3 appears to have the fewest aftershocks per mainshock, in agreement with the results in Kagan et al. (2010). Among the major zones of primary interest, zone 4 has the highest estimated branching ratio. This is not surprising because zone 4 is the most seismically active regime.

4.3 Triggering of aftershocks

Figure 5 shows how the estimated branching ratio, $G$, based on equation (6), varies as a function of mainshock magnitude for different tectonic zones. The curves for zones 2 and 3 are very similar; the curves corresponding to zones 1 and 4 are also similar. Zone 0 exhibits a phenomenon between these two types. For zones 1 and 4, a typical M6 event would have about 2 expected aftershocks of magnitude $\geq 5.0$; a typical M7 event would have about 3 expected aftershocks of this size; an M8 event would have about 6 or 7 expected aftershocks with $M \geq 5.0$; and an M9 event has about 18 or 19 expected aftershocks with magnitude $\geq 5.0$ within these zones. The number of expected aftershocks increases much more dramatically with mainshock magnitude in zones 1 and 4, compared to zones 2, 3 and 0. For zones 2 and 3, one would expect a typical M6 or M7 event to have only about
one aftershock greater than magnitude 5.0; a M8-9 event would have about two expected aftershocks of magnitude $\geq 5.0$ in this zone. There is no dramatic change in the number of expected aftershocks as the magnitude increases. For zone 0, a typical M6 or M7 event would have about 1 or 2 expected aftershocks of magnitude $\geq 5.0$, which is somewhat similar to zones 2 and 3; a typical M8 or M9 event would have about 3 or 4 direct aftershocks of this size. In summary, the aftershock rates of zones 2, 3 and 0 are much more gradual than the other zones. This observation coincides with the implication mentioned in Sections 4.1 and 4.2 that among the major zones, zones 2 and 3 have primarily swarm-type seismicities, and zones 1 and 4 have relatively clear distinctions between mainshocks and aftershocks.

The spatial and temporal ranges of a typical event’s direct (first-generation) aftershocks are shown in Figure 5. The top right panel in Figure 5 shows the time (in years) to cover a certain proportion, $\rho$, of direct aftershocks. All the curves are quite similar for $\rho < 75\%$; in all zones, at least 75% of the expected direct aftershocks would be expected to occur within 1 year of the corresponding triggering earthquake. The zones differ, however, for larger values of $\rho$. For example, we expect earthquakes in zone 4 to have 85% of their direct aftershocks within 1 or 2 years; an event in zone 1 would have 85% of its expected aftershocks within 5 years; for zones 0 and 2, 85% of the direct aftershocks are expected to occur within 10 years of the mainshock; and for zone 3, one expects it to take nearly 10 years for merely 80% of the direct aftershocks to occur. Among the major zones, zones 2 and 3 have less strongly clustered seismicity compared to the other zones.

Figure 5 also shows the spatial range expected to cover a certain proportion of direct aftershocks for a typical M6 event. The curves of zones 1, 2, 3 and 4 show that up to 90% of the direct aftershocks are within 100 km of their triggering earthquakes, while almost all
aftershocks are within 200 km of their corresponding mainshocks. Zone 0 behaves similarly to the other zones except at the upper tail of the percentage of aftershocks to be contained. To contain more than 95% of the direct aftershocks, the radius around a mainshock would have to be larger than 200 km. Among the major zones, the aftershock radii in zone 2 are the smallest, and zone 4 has the largest spatial radii. This is an indication that zone 4 exhibits the strongest triggering, so that mainshocks in zone 4 can trigger events to somewhat further distances than earthquakes in the other zones. Note that the radii for zone 0 are even larger than those for zone 4, which may largely be attributable to the large area of zone 0.

The bottom right panel in Figure 5 shows the radius (in km) required to contain 95% of the expected direct aftershocks for a typical event. The curves of zones 2 and 3 are similar; an M7, M8 or M9 event would have 95% of its expected aftershocks within approximately 100 km. For zones 1 and 4, an M7 event would have 95% of its expected aftershocks within 150 km; an M8 event would have 95% of its expected aftershocks within 150 to 200 km; and in the case of zone 4, we see somewhat larger radii. For zone 1, an M9 event would have 95% of its expected aftershocks within 300 to 350 km; and an M9 event in zone 4 would have 95% of its expected aftershocks within 400 km. Zone 0 is somewhat distinct from the other zones for events of M5, M6 and M7 (95% radii are approximately 100 km, 150 km and 200 km, respectively), but similar to zones 1 and 4 for events of M8 and M9 (95% radii are approximately 250 km and 300 km, respectively). Our estimates of the sizes of aftershock zones are considerably wider than those estimated in previous research (Abercrombie 1995, Kagan et al. 2010). Below (Section 6) we try to explain this ETAS result and its difference from other estimates of a focal zone size.
5 Model diagnostics

5.1 Estimated conditional intensity in space

The fitted models in each zone may most directly be inspected by examining spatial plots of the integrated conditional intensity. In Figure 6, we present an example for each zone:

1. a sub-region of zone 1 (active continent): Southern California, (−122, −114) in longitude by (32, 37) in latitude.

2. a sub-region of zone 2 (slow-spreading ridges): oceanic ridge in the North Atlantic Ocean, (−47, −42) in longitude by (20, 30) in latitude.

3. a sub-region of zone 3 (fast-spreading ridges): oceanic ridge to the south of Australia, (145, 150) in longitude by (−60, −55) in latitude.

4. a sub-region of zone 4 (trenches): east of Honshu, Japan, (140, 145) in longitude by (30, 40) in latitude.

The approximation 1° ≈ 111 km, also used in Bird (2003), is employed here.

One sees in Figure 6 that areas of high estimated intensity correspond to areas of high seismicity, and the spatial distribution of the triggering functions appear to fit the data well.

5.2 Triggering function versus empirical aftershock rate

Another basic diagnostic tool is to compare the triggering function to the empirical aftershock rate. Figure 7 shows the spatial and temporal distribution of all events of magnitude at least 5.0 occurring within a given space-time window around any event with magnitude in a given
range. For example, for zone 1, we consider in Figure 7 all events occurring after an event of estimated magnitude approximately 6.0.

Note that the estimated background rates for all the zones depicted are small compared with the contributions from the triggering functions (see Table 2), so most of the events depicted in Figure 7 are attributed by the ETAS model to aftershocks. One can see the steep temporal decrease in seismicity following an earthquake, in agreement with the Omori law. Spatially, one sees that zone 1 and zone 2 have most of their aftershocks within a 50 to 100 km radius of the mainshock, but in zone 4, radii of up to 200 km are required to contain most of the aftershocks. The spatial plots in Figure 7 do not indicate any strong departures from isotropy, at least not in the aggregate over all earthquakes within a zone, although in zone 1, aftershocks in the North-South direction appear to be somewhat more prevalent. Note that locally, the spatial patterns of aftershocks may be highly anisotropic, but that in the aggregate over an entire global zone, when the aftershocks are plotted relative to North-South and West-East directions rather than relative to local fault directions, they appear nearly isotropic in Figure 7.

The empirical densities shown in Figure 7 may be compared to the expected densities according to the fitted ETAS model, and this comparison is shown in Figure 8. We observe that the empirical and expected aftershock density curves coincide very well in each of the zones.
6 Conclusions

We have found that the ETAS models appear to fit rather well to each of the major tectonic zones, despite the fact that the estimates of parameters in the ETAS models differ across the zones. Most of these differences in parameter estimates can be attributed to the differences in overall seismic rates for different zones. Zones 1 (active continent) and 4 (trenches) for instance, have the highest overall rates of seismicity and also appear to have the most intense estimated triggering functions. Zones 2 (slow-spreading ridges) and 3 (fast-spreading ridges), by contrast, appear to have lower background rates, have estimated triggering functions that decay more sharply spatially, and generally have parameter estimates consistent with swarms rather than traditional mainshock-aftershock clustering.

As pointed out in Schoenberg et al. (2010), maximum likelihood estimates of ETAS parameters may be significantly biased due to the lower cutoff $m_t$. Although the same cutoff magnitude of 5.0 is applied to each zone, the bias resulting from this choice of cutoff may affect different zones differently due to the varying sparsity of the catalog within each zone. A similar sort of truncation bias may be introduced by the irregularly shaped spatial windows delineating the different tectonic zones. This may have contributed to biases in the parameter estimates, especially for the spatial parameters $d$ and $q$. Further study is needed in order to determine the biases introduced by spatial windows for parameters in ETAS models. The time windows selected here, by contrast, appear not to be a major source of bias in our parameter estimates; we have tested different time windows of various lengths and obtained very similar results for all of the parameters, including the temporal aftershock decay parameters $\hat{c}$ and $\hat{p}$. However, the correlations between parameter estimates appear
to be significant, and these correlations can also bias parameter estimates and estimates of
standard errors.

Another major source of error in the present analysis is due to the use of homogeneous
background rates. We have elected to use a homogeneous background rate model because
of our focus on the triggering behavior in the different tectonic zones. Furthermore, vari-
ous different methods for estimating inhomogeneous background rates have been proposed,
including kernel methods and Bayesian smoothing methods, and each may provide substan-
tially different estimates; we desired our conclusions not to depend critically on our decision
to use a particular choice of background rate estimation technique. However, our results es-
pecially for zone 0 appear to depend critically on our choice of homogeneous background rate,
and a similar study using inhomogeneous ETAS models may be an important direction for
future work. In addition, one may consider larger models allowing triggering between zones.
The study of such models with interactions is an important concern for future research. As
noted in Bird (2003), very small plates within the orogens are likely to be identified in the
near future, so further modeling of seismicity within single orogens may yield increasingly
accurate estimates and forecasts of seismicity in the future.
Data and Resources

All data used in this paper came from published sources listed in the references.


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earthquake from the ETAS Model, Baath’s Law, and observed aftershock sequences.
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Utsu, T. (1970). Aftershocks and earthquake statistics (II) - Further investigation of af-
tershocks and other earthquake sequences based on a new classification of earthquake


Veen, A., and F. Schoenberg (2008). Estimation of space-time branching process models in
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tion*, **103**(482), 614-624.

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mates in the ETAS Model, ms, accepted by *Bull. Seismol. Soc. Amer.*


Tables
<table>
<thead>
<tr>
<th>Zone</th>
<th>Area (x $10^7$ km$^2$)</th>
<th>Proportion of area</th>
<th>No. events in zone</th>
<th>No. events in zone / total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Active continent</td>
<td>3.36</td>
<td>6.58%</td>
<td>6728</td>
<td>16.6%</td>
</tr>
<tr>
<td>2. Slow-spreading ridges</td>
<td>1.58</td>
<td>3.09%</td>
<td>3028</td>
<td>7.47%</td>
</tr>
<tr>
<td>3. Fast-spreading ridges</td>
<td>0.844</td>
<td>1.65%</td>
<td>2574</td>
<td>6.35%</td>
</tr>
<tr>
<td>4. Trenches</td>
<td>1.52</td>
<td>2.98%</td>
<td>26125</td>
<td>64.5%</td>
</tr>
<tr>
<td>0. Plate-interior</td>
<td>43.8</td>
<td>85.8%</td>
<td>2049</td>
<td>5.06%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>51.0</strong></td>
<td><strong>100%</strong></td>
<td><strong>40504</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

Table 1: Areas and PDE catalog sizes for each tectonic zone. Time window is January 1, 1973 to December 31, 2006. Only shallow events ($\leq 70$ km in depth) with reported magnitudes of at least 5.0 are considered.
### Table 2: Maximum likelihood estimates of ETAS parameters.

Standard errors (diagonal terms in the Hessian matrix) are reported in parentheses. 'Log-lik (ETAS)' and 'Log-like (Poisson)' refer to the log-likelihood for the ETAS and homogeneous Poisson models, respectively.

<table>
<thead>
<tr>
<th>Estimates</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
<th>Zone 0</th>
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<tbody>
<tr>
<td></td>
<td>Act-Con.</td>
<td>Slow</td>
<td>Fast</td>
<td>Trench</td>
<td>Inter.</td>
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<tr>
<td>$a$</td>
<td>0.980</td>
<td>0.180</td>
<td>0.324</td>
<td>0.942</td>
<td>0.439</td>
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<tr>
<td>(1/magnitude)</td>
<td>(0.0179)</td>
<td>(0.0543)</td>
<td>(0.0530)</td>
<td>(0.0181)</td>
<td>(0.0466)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.0781</td>
<td>0.0319</td>
<td>0.0307</td>
<td>0.0408</td>
<td>0.497</td>
</tr>
<tr>
<td>(days)</td>
<td>(0.00341)</td>
<td>(0.00282)</td>
<td>(0.00320)</td>
<td>(0.00195)</td>
<td>(0.0396)</td>
</tr>
<tr>
<td>$d$</td>
<td>69.3</td>
<td>227</td>
<td>327</td>
<td>141</td>
<td>69.0</td>
</tr>
<tr>
<td>(km$^2$)</td>
<td>(0.958)</td>
<td>(4.35)</td>
<td>(6.91)</td>
<td>(2.06)</td>
<td>(2.60)</td>
</tr>
<tr>
<td>$K_0$</td>
<td>0.539</td>
<td>23.3</td>
<td>77.6</td>
<td>1.33</td>
<td>0.169</td>
</tr>
<tr>
<td>(events/day/km$^2$)</td>
<td>(0.00963)</td>
<td>(0.664)</td>
<td>(2.78)</td>
<td>(0.0233)</td>
<td>(0.00469)</td>
</tr>
<tr>
<td>$p$</td>
<td>1.19</td>
<td>1.16</td>
<td>1.15</td>
<td>1.25</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>(0.00309)</td>
<td>(0.00414)</td>
<td>(0.00488)</td>
<td>(0.00430)</td>
<td>(0.00440)</td>
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<tr>
<td>$q$</td>
<td>1.91</td>
<td>2.33</td>
<td>2.49</td>
<td>1.95</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>(0.00322)</td>
<td>(0.00462)</td>
<td>(0.00542)</td>
<td>(0.00296)</td>
<td>(0.00449)</td>
</tr>
<tr>
<td>$\mu \times 10^{-9}$</td>
<td>7.63</td>
<td>8.43</td>
<td>15.6</td>
<td>61.7</td>
<td>0.131</td>
</tr>
<tr>
<td>(events/day/km$^2$)</td>
<td>(0.132)</td>
<td>(0.239)</td>
<td>(0.398)</td>
<td>(0.775)</td>
<td>(0.00542)</td>
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<td>Log-lik.(ETAS)</td>
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<td>-52071</td>
<td>-44562</td>
<td>-380005</td>
<td>-37876</td>
</tr>
<tr>
<td>Log-lik.(Poisson)</td>
<td>-127451</td>
<td>-57488</td>
<td>-47674</td>
<td>-438746</td>
<td>-46508</td>
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<tr>
<td>Info. gain</td>
<td>2.62</td>
<td>1.79</td>
<td>1.20</td>
<td>2.24</td>
<td>4.20</td>
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<tr>
<td>Zone</td>
<td>( a )</td>
<td>( \beta )</td>
<td>( b )</td>
<td>Branching ratio</td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------</td>
<td>------------</td>
<td>--------</td>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td>1. Active continent</td>
<td>0.980</td>
<td>2.73</td>
<td>1.18</td>
<td>0.521</td>
<td></td>
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<tr>
<td>2. Slow-spreading ridges</td>
<td>0.180</td>
<td>3.15</td>
<td>1.37</td>
<td>0.481</td>
<td></td>
</tr>
<tr>
<td>3. Fast-spreading ridges</td>
<td>0.324</td>
<td>2.42</td>
<td>1.05</td>
<td>0.386</td>
<td></td>
</tr>
<tr>
<td>4. Trenches</td>
<td>0.942</td>
<td>2.69</td>
<td>1.17</td>
<td>0.559</td>
<td></td>
</tr>
<tr>
<td>0. Plate-interior</td>
<td>0.439</td>
<td>3.07</td>
<td>1.33</td>
<td>0.690</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Estimates of parameters \( a \) and \( \beta \) governing the Gutenberg-Richter distribution of magnitudes, as well as the corresponding \( b \)-value \( (b = \beta/\ln(10)) \) and expected branching ratio.
Figure Captions
Figure 1: Tectonic zones plotted with world map (Mercator projection). Blue: zone 1 (active continent), green: zone 2 (slow-spreading ridges), yellow: zone 3 (fast-spreading ridges), red: zone 4 (trenches), white: zone 0 (plate-interior).

Figure 2: Estimated triggering function $g$ (triggered events / day / km$^2$), fit by maximum likelihood, as a function of (a) spatial radius or of (b) squared radius. Zone 1: active continent, zone 2: slow-spreading ridges, zone 3: fast-spreading ridges, zone 4: trenches, zone 0: plate-interior.

Figure 3: Scatterplots of estimated ETAS parameters as a function of estimates of $K_0$, for each zone. The numbers in the plots indicate the corresponding zone numbers. The estimates for zones 0 and 1 nearly overlap in the plot of the estimate of $d$ versus the estimate of $K_0$. Zone 1: active continent, zone 2: slow-spreading ridges, zone 3: fast-spreading ridges, zone 4: trenches, zone 0: plate-interior.
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Figure 6: Estimated conditional intensities according to ETAS (in color, on logarithmic scale) and recorded earthquakes (black dots) for particular regions. The events are shallow events ($\leq 70$ km deep) of magnitude $\geq 5.0$. (a) A sub-region of zone 1 (active continent) in California with 122 events in 1973-2006. (b) A sub-region of zone 2 (slow-spreading ridges), an oceanic ridge in the North Atlantic Ocean, with 113 events in 1973-2006. (c) A sub-region of zone 3 (fast-spreading ridges), an oceanic ridge South of Australia, containing 86 events in 1973-2006. (d) A sub-region of zone 4 (trenches), East of Honshu, Japan, containing 185 events in 1999-2006.
Figure 7: Histograms (left) and scatterplots (right) of events within 30 days and occurring within the given great circle distance (GCD) of a given event of recorded magnitude. (a) and (b): Magnitude 6.0 for zone 1, the active continent. (c) and (d): Magnitude 5.3 for zone 2, the slow-spreading ridge zone. (e) and (f): Magnitude 6.0 for zone 4, the trench zone.

Figure 8: Estimated triggering density $g$ (solid curves) and empirical rate (dotted curves) of events occurring within 30 days and within a given spatial radius, following an earthquake of a given magnitude. (a) mainshocks of M6.0 in zone 1 (active continent). (b) mainshocks of M5.3 in zone 2 (slow-spreading ridges). (c) mainshocks of M5.3 in zone 3 (fast-spreading ridges). (d) mainshocks of M6.0 in zone 4 (trenches). (e) Mainshocks of M5.3 in zone 0 (plate-interior).
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