Assessing the accuracy of strike angle estimates

Assessing the predictive accuracy of earthquake strike angle estimates using non-parametric Hawkes processes

Research Article

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Abstract

Earthquake focal mechanism estimates have been posited to have predictive value for forecasting future seismicity. In particular, for strike-slip earthquakes, aftershocks should occur roughly along the estimated mainshock strike. However, the errors in such estimated strike angles is considerable. We compare the degree to which estimated strike angles forecast the direction of future seismicity around a given earthquake to that of uniformly distributed angles and to strike angles estimated based on previous seismicity. The fit of non-parametrically estimated Hawkes models using the estimated strike angle that best fits the post-mainshock set of events for each main-shock is compared to that of corresponding models that exclude these estimates. Strike angle estimates are shown to have marginal predictive value for forecasting the direction of future seismicity, but no more than the better-fitting of a uniformly distributed angle and its complement.

Keywords.
Focal mechanism, Hawkes process, point process, seismology, Voronoi deviance residuals.
1 Introduction

When an earthquake occurs in an area with sufficient coverage of seismometers, the waveform data produced are used to estimate a focal mechanism which describes the deformation that occurred at the earthquake source. Such earthquake focal mechanism estimates have become increasingly common recently with the deployment of broadband seismometers and more powerful computers (Clinton et al. 2006). One component of such a focal mechanism, a double-couple, represents the deformation as two nodal planes, one of which represents the fault along which the rupture occurred while the other is known as the auxiliary plane and is orthogonal to the fault plane (Aki and Richards 2002). The hope among seismologists is that such information will have high predictive value for earthquake forecasting, particularly with respect to information about the orientation of the fault plane on which triggered events primarily occur (Kagan 2014).

Aftershocks have indeed been observed to occur more densely at the ends and along fault ruptures and less densely in areas orthogonal to the rupture, in agreement with theoretical studies of Coulomb stress (Das and Henry 2003). Henry and Das (2001) report in particular a tendency for aftershocks to occur along the estimated strike of strike-slip mainshocks, again in agreement with geophysical theory of Coulomb stress. Strader and Jackson (2014) found that resolving Coulomb stress onto the more favorable nodal plane of a receiver earthquake resulted in an increased number of earthquakes occurring in areas of high-stress. Thus, estimated focal mechanisms have been used to project the changes in stress in order to highlight areas of increased anticipated aftershock activity following earthquakes. For additional information about Coulomb stress, its derivation and its properties, see Scholz (2002).
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Unfortunately, focal mechanism estimates are known to suffer from large uncertainties (Kagan 2003). In particular, large errors are associated with estimates of the orientation of nodal planes which describe the deformation that occurs at an earthquake’s source (Kagan 2014). In addition, determining which nodal plane is the fault plane and which is the auxiliary plane is not possible from the estimation procedure and relies on the use of other geological data (Aki and Richards 2002).

Figure 1 illustrates how large the errors can be in the strike angle estimates obtained from estimated nodal planes. As shown in Figure 1, there can be considerable discrepancy between the two nodal plane estimates and the direction of subsequent seismicity. This disagreement cannot reasonably be explained as resulting from location errors, since the local seismicity itself is nearly linear in this example and in many others. Figure 1 also highlights the ambiguity that can exist in determining which nodal plane is the fault plane and which is auxiliary.

The objective of this paper is to describe and quantify the degree to which such estimated strike angles increase our ability to forecast future seismicity, from a statistical perspective. One way to assess the benefit of these measurements would be to compare the performance of earthquake forecasts made with focal mechanism estimates to those made without the use of focal mechanism estimates. Unfortunately, however, such estimates remain presently unused in almost all models for forecasting seismicity, such as ETAS and other models used in forecasting studies such as the Regional Earthquake Likelihood Model (RELM) working group and the Collaboratory for the Study of Earthquake Predictability (CSEP) (Schorlemmer et al. 2010; Zechar et al. 2013). An exception is Kagan and Jackson (2014), which used previous focal mechanisms to forecast future focal mechanisms as well as future seismicity, and Kagan et al.
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(2007) used focal mechanisms in their five year forecast to orient anisotropic smoothing, though such inclusion resulted in negligible improvement in forecasting (Wang et al. 2011). Ogata (1998) found that using anisotropic smoothing, based on the general direction of faulting in Japan, led to improved forecasts for Epidemic Type Aftershock Sequence (ETAS) models, and Ogata and Zhuang (2006) and Ogata (2011) report improved fit from ETAS models with spatially varying parameters, but such parameters are typically not estimated presently using focal mechanism estimates. Because many competing models have been offered, with widely varying implications in terms of seismic hazard (Schoenberg and Patel 2012) and there remains considerable disagreement about which model is optimal, we instead examine the impact of estimated strike angles non-parametrically, using data-driven methods and keeping modeling assumptions about particular functional forms to a minimum.

Recent articles by Guo et al. (2015) and Guo et al. (2017) investigated how fault geometry estimated using previous seismicity for large ($M \geq 7.5$) earthquakes in China and Japan impacted parameter estimates of Epidemic Type Aftershock Sequence (ETAS) models (Ogata 1998). However, these studies did not quantify the extent to which including fault geometry in the models improved earthquake forecasts. In this article, we compare the degree to which estimated strike angles forecast the direction of future seismicity around a given earthquake to that of uniformly distributed angles or angles estimated based on seismicity occurring before the mainshock. Also, using the estimation method of Marsan and Lengliné (2008b), we consider purely non-parametrically estimated Hawkes models using the estimated strike angle that best fits the post-mainshock set of events for each mainshock and compare their predictive performance to corresponding models that exclude these estimates.
2 Data

Data on estimated origin times, locations, and magnitudes of 17,734 shallow (depth \( \leq 75 \text{ km} \)) earthquakes of magnitude \( M \geq 2.8 \) in Southern California from 1980 to 2016 were recorded by the Southern California Seismic Network (SCSN) and obtained from the Southern California Earthquake Data Center (SCEDC) \cite{caltech1926,scedc2013}. Most of these events contain no focal mechanism estimates. Each earthquake in the catalog was assigned a location quality grade and we restrict our attention exclusively to those earthquakes considered to have a location quality of \( C \) or better. SCEDC also compiles the focal mechanism estimates for a subset of 899 of these events dating from 1999 to 2016, with variance reduction \( \geq 40 \), variance reduction being a measurement of focal mechanism estimation quality.

In what follows, to distinguish these 899 events with focal mechanism estimates we refer to them as mainshocks, though the branching structure of the earthquakes is unknown, and many of these 899 events may actually be aftershocks of other events. Each nodal plane is described by a set of three angles. The azimuthal strike angle is the angle created by intersecting the nodal plane with a horizontal surface, such as the surface of the Earth, and are measured as counter-clockwise degrees from North. Dip angles describe the downward angle of the plane from the horizontal surface of the Earth. A dip angle of 0° would lie parallel to the surface of the Earth whereas a dip angle of 90° would be perpendicular to the surface. The rake angle describes the direction of planar movement relative to the horizontal surface, i.e. the movement of the fault in the direction of the strike angle. For each mainshock, we examine the dip angle of the nodal plane whose strike angle best fits the post mainshock set of events (using fitting criteria described in Section 3.1) and exclude events with dip...
angles \leq 75^\circ$. This leaves us with nodal planes that are near-vertical as in the case of strike-slip faulting. Additionally, we exclude mainshocks that had fewer than three earthquakes occur prior to the mainshock and within a distance of $10^{-0.5(M_i-M_c)}$, which is of standard use in seismology as a maximum distance of aftershocks around small-to-medium sized mainshocks for catalogs with magnitude cutoff $M_c$ (Scholz 2002). For this subset of 333 strike-slip mainshocks, the strike angle of the best fitting nodal plane should closely resemble the fault plane and have enough previous seismicity to estimate additional strike angles statistically.

The catalog of 17,734 earthquakes without estimated focal mechanisms is used in our analysis for strike angle estimation and evaluation. That is, we compare the estimated strike angles for the 333 strike-slip mainshocks to strike angle estimates obtained using the other observed events that occurred before each mainshock, and evaluate the overall fit of our different strike angle estimates on events that occurred after each mainshock. To ease the computational burden of the estimation procedure for the non-parametric Hawkes model described in Section 3.2, we raise the lower magnitude cutoff for both catalogs used in the estimation to $M > 3.25$ leaving us with 5,649 earthquakes without focal mechanisms and 330 mainshocks.

3 Methods

3.1 Fault angle estimation and evaluation

To help assess how well the recorded strike angle estimates forecast the direction of future seismicity, we consider three alternative estimates of strike angle, based on prior seismicity rather than recorded double-couple estimates. First, let $\hat{\theta_i}, i = 1, \ldots, 330,$
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denote the strike angle estimates based on the estimated nodal planes obtained from SCEDC. Given mainshock \( i \) at location \((x_i, y_i)\) with magnitude \( M_i \), consider the set \( S_i \) of events that occurred prior to the mainshock within the mainshock’s aftershock radius, which is taken as
\[
d(M_i) = 10^{-0.5(M_i - M_c)},
\]
following Scholz (2002).

Consider an alternative strike angle estimate, \( \theta^D_i \), computed using Deming regression (Deming 1943), so that \( \theta^D_i \) is the slope of the line minimizing the sum of squared orthogonal distances to the events in \( S_i \). That is, assuming an equal amount of error in the \( x \) and \( y \) directions, we compute the slope of the line as
\[
\hat{\beta}_1^* = \frac{s_y^2 - s_x^2 + \sqrt{(s_y^2 - s_x^2)^2 + 4 \cdot \text{cov}^2(x, y)}}{2 \cdot \text{cov}(x, y)}
\]
with intercept
\[
\hat{\beta}_0^* = \bar{y} - \hat{\beta}_1^* \bar{x}
\]
where \( x, y \) are sets of points in \( S_i \), \( \bar{x}, \bar{y} \) are the mean values of \( x \) and \( y \), and \( s_x^2, s_y^2 \) are the sample variances for \( x \) and \( y \), respectively. Another possible strike angle estimate, \( \theta^a_i \), may be defined as the slope of the line through \((x_i, y_i)\) minimizing the mean absolute error (MAE) where errors are defined as the angles formed by the \( \theta^a_i \) and the rays from \((x_i, y_i)\) to each preceding earthquake in \( S_i \). In what follows we also consider the estimate \( \theta^u_i \) obtained simply by choosing an angle between \( 0 - 360^\circ \) uniformly at random.

To evaluate the strike angle estimates \( \hat{\theta}_i, \theta^D_i, \theta^a_i, \) and \( \theta^u_i \), we consider two different fitting criteria. One is the root mean squared error (RMSE), where errors are defined as the orthogonal distance between the line whose slope is derived from the strike
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angle estimates and the events occurring after mainshock $i$ within a distance of $d(M_i)$. That is, for points, $(x_j, y_j)$, $j = 1, \ldots, n$, occurring after a mainshock and within the aftershock radius, let $(x_j^*, y_j^*)$ denote the points on the line which form orthogonal angles between the line and points $(x_j, y_j)$. Then

$$\text{RMSE} = \sum_j \sqrt{(x_j - x_j^*)^2 + (y_j - y_j^*)^2}.$$ 

We also consider the mean absolute value of the angles formed by the line whose slope is derived from the corresponding strike angle estimate and the same set of events occurring after each mainshock. This allows us to compare how well each strike angle estimate forecast the direction of seismicity that occurred after each mainshock. Estimates with smaller RMSE or smaller angular MAE would have better forecast the direction of future seismicity.

Since the two estimated nodal planes of a double-couple are indistinguishable, we calculate the RMSE and minimum angular MAE for each nodal plane strike angle, and for each fitting criteria, let the best fitting nodal plane strike angle $\tilde{\theta}_i$ denote the nodal plane strike angle that minimizes the RMSE or angular MAE. Similarly, since the focal mechanisms are essentially given two chances to fit a strike angle to the events occurring after a mainshock, one may also consider a second, orthogonal angle for each of our strike angle estimates based on previous seismicity, as well as the estimate chosen uniformly at random, and label the better fitting of the two angles $\tilde{\theta}_i^D$, $\tilde{\theta}_i^a$, or $\tilde{\theta}_i^u$, correspondingly. This allows us to measure how the selection of the better fitting angle among a strike angle estimate and its complement affects the overall performance in forecasting the direction of future seismicity.

For each of the $n = 330$ mainshocks and for each strike angle estimation method described above, we compute the mean and standard error (SE) of the RMSE and
angular MAE for the post mainshock set of events, where the SE is calculated as the standard deviation of the RMSEs divided by $\sqrt{n}$. In order to compare the estimates to the angle best fitting the seismicity following each mainshock, we also compute retrospective or *gold* angle estimates, $\theta^g_i$, defined for each mainshock and each fitting criterion as the angle with minimal RMSE or minimal mean absolute angle from the post mainshock set of events.

### 3.2 Non-parametric Hawkes model

In order to compare the predictive performance of point process models with strike angle estimates to those without strike angle estimates, we fit Hawkes point process models to the data non-parametrically, using the method of Marsan and Lengliné (2008). We briefly review some details of this procedure and some point process preliminaries here.

A point process is a collection of points $\{\tau_1, \tau_2, \ldots\}$ occurring in some metric space (Daley and Vere-Jones 2007; Daley and Vere-Jones 2008). A point process in space-time is typically modeled via its conditional intensity function, $\lambda(s,t)$, which is a stochastic process representing the infinitesimal rate at which points are expected to accumulate around location $s$ and time $t$, given all points occurring prior to time $t$.

The Hawkes, or self-exciting, point process model (Hawkes 1971) is a type of branching point process which models the conditional intensity of a process given its history, $\mathcal{H}_t$, up to time $t$ as

$$\lambda(s,t|\mathcal{H}_t) = \mu(s,t) + \sum_{i:t_i<t} g(s-s_i,t-t_i)$$  \hspace{1cm} (1)

where $\mu(s,t)$ is the background rate of events occurring and the triggering function

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\( g \) describes the spatial-temporal rate at which points induce subsequent points. The model extends easily to the marked case where the triggering function \( g \) depends on earthquake magnitude as well, and such marked versions are commonly used in earthquake forecasting in both parametric (Ogata 1988, Ogata 1998) and non-parametric (Marsan and Lengliné 2008b, Fox et al. 2016) forms.

In order to adapt the model to account for earthquake strike angles, we follow Gordon and Schoenberg (2017) and adopt the following product form for the conditional intensity function:

\[
\lambda(x, y, t|H_t) = \mu(x, y) + \sum_{i: t_i < t} g_t(t - t_i) g_{xy}(x - x_i, y - y_i; m_i, \tilde{\theta}_i) g_{m}(m_i)
\] (2)

where \( \mu > 0 \) is the background rate, \( g_t \) and \( g_{xy} \) are densities governing the temporal and spatial triggering, respectively, and \( g_m \) dictates how the productivity of aftershocks depends on the mainshock magnitude. The functions \( g_t(t - t_i), g_{xy}(x - x_i, y - y_i; m_i, \tilde{\theta}_i), \) and \( g_{m}(m_i) \) are each computed using the method of Marsan and Lengliné (2008a) whereby the estimation of a Hawkes model is computed by maximizing the expectation of the complete data log-likelihood and assigning the probability of each earthquake being a background event or an aftershock of a previous event. Space-time-magnitudes were shown by Zhang (2017) to be approximately separable hence we assume separability in our model. Here \( t_i \) is the time of the \( i \)th event, \((x_i, y_i)\) is the two-dimension epicentral location of the \( i \)th event, \( m_i \) is the magnitude of the \( i \)th event and \( \tilde{\theta}_i \) is the best fitting strike angle from the \( i \)th double-couple.

For the spatial triggering, we take

\[
g_{xy}(x - x_i, y - y_i; m_i, \tilde{\theta}_i) = g_r(r; m_i) g_\phi(\phi; \tilde{\theta}_i)
\] (3)

where \( g_r \) accounts for triggering as a function of distance from earthquake \( i \) and \( g_\phi \)
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accounts for the angular locations of triggering relative to strike angle $\tilde{\theta}_i$. We then compare the fit of (2) to the corresponding model without strike angle estimates, i.e.

$$\lambda(x, y, t|H_t) = \mu(x, y) + \sum_{\tau_i: t_i < t} g_t(t - t_i)g_{xy}(x - x_i, y - y_i; m_i)g_{m}(m_i).$$ (4)

For both model (2) and (4), the functions $\mu$, $g_t$, $g_{xy}$, and $g_{m}$ are estimated entirely non-parametrically using the E-M type method as described in Marsan and Lengliné (2008b).

### 3.3 Voronoi deviance residuals

To compare the fit of models (2) and (4), one may inspect a residual process aggregated over a (typically rectangular) grid of pixels as in Baddeley et al. (2005). Specifically, we examine deviance residuals (Clements et al. 2011) over such a rectangular grid. We also compute Voronoi deviance residuals as in Bray et al. (2014). Such residuals are helpful in evaluating the goodness of fit of point process models with highly volatile conditional intensities, and Voronoi residual deviances have the advantage of being less skewed in their distribution when compared to grid based residuals (Bray et al. 2014).

To construct Voronoi residuals, we partition the space into Voronoi cells, where each cell $B_i$ is defined as the set of locations closer to the observed point $\tau_i$ than to any of the other observed points. Each Voronoi cell, $B_i$, then contains only the single point $\tau_i$ by construction.

For each bin $B_i$ in a rectangular grid, or for each cell $B_i$ in the Voronoi tessellation of the observed points, we compute the deviance, or ratio of two corresponding log-likelihoods, to evaluate the relative fit of two models. For two different intensity
estimates, $\hat{\lambda}_1$ and $\hat{\lambda}_2$, the deviance residual computed over cell $B_i$ is

$$R(B_i) = \sum_{i: (t_i, x_i, y_i) \in B_i} \log \left( \hat{\lambda}_1(t_i, x_i, y_i) \right) - \int_{B_i} \hat{\lambda}_1(t, x, y) \, dt \, dx \, dy$$

$$- \left( \sum_{i: (t_i, x_i, y_i) \in B_i} \log \left( \hat{\lambda}_2(t_i, x_i, y_i) \right) - \int_{B_i} \hat{\lambda}_2(t, x, y) \, dt \, dx \, dy \right).$$

A positive deviance residual implies that model $\hat{\lambda}_1$ provides a better fit to the data in the given region while negative deviance residuals imply the opposite. Voronoi deviance residuals are sometimes scaled by the dividing the log-likelihood ratio for each Voronoi cell by its area. We use unscaled residuals so the total deviance for the competing models is simply the sum of the binned deviances, $\sum_i R(B_i)$. Total deviance values close to zero indicate minimal difference in fit between the two competing models.

### 3.4 Stoyan-Grabarnik diagnostic

The temporal fit of (2) is evaluated using the Stoyan-Grabarnik diagnostic (Stoyan and Grabarnik 1991)

$$E \left( \sum_{x_i \in N \cap B} \frac{1}{\lambda(x_i)} \right) = |B|$$

where $N$ is the point process, $B$ is a spatio-temporal window and $\lambda(x_i)$ is the conditional intensity of point $x_i$. Dividing the entire temporal window into $n$ temporal bins, $B_j$ with $j = 1, \ldots, n$, if the estimated intensity $\hat{\lambda}$ is accurate then sum of the estimated inverse intensities for points in bin $B_i$ should approximate the volume of $B_j$ with variance

$$\text{Var} \left( \sum_{x_i \in N \cap B_j} \frac{1}{\hat{\lambda}(x_i)} \right) = \int_S \int_T \frac{1}{\lambda(t, x, y)} \, dt \, dx \, dy + \sum_{x_i \in N \cap B_j} \frac{1}{\hat{\lambda}(x_i)} - |B_j|^2,$$

where $S$ and $T$ are the spatial and temporal windows, respectively. Baddeley et al. (2008) notes that it is possible for $\hat{\lambda}(x_i)$ to be zero and for the diagnostic to have a
large variance. Following their suggestions, we set $1/\hat{\lambda}(x_i) = 0$ for values of $\hat{\lambda}(x_i)$ that are zero or close to zero.

4 Results

4.1 RMSE of orthogonal distances

The RMSEs for each of the strike angle estimates described in Section 3.1 are summarized in Figure 2 panel (a). The average RMSE for the estimate $\theta^D$ obtained using Deming regression on prior seismicity was 2.84 km, just slightly above the mean RMSE of 2.81 km corresponding to $\tilde{\theta}$, the best fitting of the two possible strike angles obtained using the estimated nodal plane. Both $\theta^D$ and $\tilde{\theta}$ fit subsequent seismicity significantly better than the random uniformly distributed angle $\theta^u$, which had a mean RMSE of 3.74 km, and both also fit better than the angle $\theta^a$ minimizing the mean absolute angle to previous seismicity, which had a mean RMSE of 3.16 km. Each estimate fit the aftershock seismicity significantly worse than the retrospective best fitting angle $\theta^g$, as the mean RMSE for $\theta^g$ was just 1.72 km.

Much of the apparent success of the estimated strike angle $\tilde{\theta}$ in forecasting future seismicity is attributable to the fact that $\tilde{\theta}$ is defined as the better fitting of the estimated strike angle and its complement. Indeed, when each alternative estimator of the strike angle is replaced with the better fitting of the estimate and its complement, the resulting RMSEs decrease markedly, as shown in Figure 2 panel (b). The estimates $\tilde{\theta}^D$ and $\tilde{\theta}^a$ have mean RMSEs of 2.60 km and 2.64 km, respectively, from subsequent seismicity. Both of these strike angle estimates, which are obtained using only prior seismicity and not using focal mechanism estimates, fit subsequent seismicity better
than the nodal plane estimate $\tilde{\theta}$, which had a mean RMSE of 2.81 km. The RMSEs for $\tilde{\theta}$ are not significantly lower than those of the uniformly distributed angles $\tilde{\theta}^u$. Even after accounting for the the orthogonal complements, each strike angle estimate in Figure 2 panel (b) had a significantly larger RMSE than the retrospective best-fitting angle $\theta^g$.

4.2 Mean absolute angles

The results are similar when considering angular MAE as the fitting criterion. Figure 2 panel (c) summarizes the angular MAE for each strike angle estimate, with an error defined as the angular distance between an aftershock and the estimated strike angle. The average angular MAE for the best fitting nodal plane estimate $\tilde{\theta}$ ($35.55^\circ$) is significantly lower than the average MAEs of $\theta^D$ ($39.24^\circ$), $\theta^a$ ($39.60^\circ$), and $\theta^u$ ($46.15^\circ$). However, the advantage of the nodal plane estimate appears to be entirely due to the selection between the two orthogonal nodal plane estimates, as shown in Figure 2 panel (d). Comparing the average MAEs of the best fitting strike angles for each method, the estimates based on prior seismicity ($34.56^\circ$ for $\tilde{\theta}^D$ and $34.37^\circ$ for $\tilde{\theta}^a$) fit aftershock activity slightly better than $\tilde{\theta}$ ($35.55^\circ$). In fact, using MAE as a metric, the fit of the nodal plane estimate $\tilde{\theta}$ was indistinguishable from that of the uniformly distributed angle $\tilde{\theta}^u$ ($35.43^\circ$). All four of the prospective strike angle estimates fit significantly worse than the retrospective optimum $\theta^g$ ($29.49^\circ$).

4.3 Non-parametric Hawkes Process

Figure 3 shows the estimated contribution of the strike angles to the triggering function of a non-parametric Hawkes model. The $y$ axis shows the estimated triggering density,
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\( \hat{g}_\phi \), as a function of the angle the location in question makes with a mainshock, relative to the estimated nodal plane strike angle \( \hat{\theta} \) of that mainshock. The LOESS smoothing of the non-parametric triggering density estimates in Figure 3 shows a downward trend indicating fewer events were generally triggered at larger angles relative to the mainshock strike angle.

Figure 4 compares the conditional intensity over \( 0.05^\circ \times 0.05^\circ \) grid cells across the southern California region for the null model (4) assuming isotropic triggering to that of model (2) incorporating the strike angle estimate \( \hat{\theta} \). Both fitted models agree closely with one another, though model (2) tends to be slightly more diffuse in low seismicity regions.

To compare the areas where model (2) improves the fit of the data over the null model (4), the deviance residuals are shown in Figure 5 and the unscaled Voronoi deviance residuals, where each Voronoi cell encloses each of the 330 mainshocks, in Figure 6. Red areas indicate cells where the inclusion of \( \hat{\theta} \) in model (2) led to an improvement in model fit when compared to the null model (4), with the total deviance of Figure 5 being 615.86 in favor of model (2). This corresponds to an overall information gain (Daley and Vere-Jones (2007) and Daley and Vere-Jones (2008)) of 0.103 per earthquake. Figures 5 and 6 indicate that including \( \hat{\theta} \) in the triggering function tends to improve the model’s fit especially in the Northwest and Southeast portions of the San Andreas Fault, with only a few small isolated areas where model (4) happened to outperform model (2).

The temporal fit of model (2) is shown in Figure 7. The model appears to fit adequately, given the close agreement of the Stoyan-Grabarnik statistic to its expected value under the assumption that the fitted model is correct.
5 Discussion

Nodal plane strike angles, $\tilde{\theta}$, fit post-mainshock events slightly better in terms of RMSE and substantially better in terms of mean absolute angle compared to strike angles estimated based on previous seismicity, $\theta^D$ and $\theta^a$. However, including a second, orthogonal estimate for $\theta^D$ and $\theta^a$ and using the angle that fit the data best for each mainshock, as was done for $\tilde{\theta}$, led to such a substantial improvement in both $\tilde{\theta}^D$ and $\tilde{\theta}^a$, in terms of RMSE and mean absolute angle, that both $\tilde{\theta}^D$ and $\tilde{\theta}^a$ actually fit better than $\tilde{\theta}$. In fact, $\tilde{\theta}$ fit aftershock seismicity no better than $\tilde{\theta}^a$, the latter of which was simply the better-fitting of a random, uniformly distributed angle and its complement. The forecasting ability of $\tilde{\theta}$ thus seems to be entirely explained by the fact that the better-fitting of the nodal plane and its complement is chosen as the estimated nodal plane.

Comparing the results of each evaluation method to $\theta^g$ provides a sense of the amount of error inherent in the strike angle estimates. For instance, comparing $\tilde{\theta}$ to $\theta^g$ using the average mean absolute angle, the mean absolute angle to aftershocks for $\theta^g$ was 29.49° and that of $\tilde{\theta}$ was 35.55°, corresponding to an increase of 6.06°. Thus, the nodal plane estimate includes about 6.06° more uncertainty, on average, beyond what can be explained by the variability in the aftershock events alone.

Figure 3 shows the additional triggering of seismicity relative to $\tilde{\theta}$ for the non-parametric Hawkes model which includes $\tilde{\theta}$ in the conditional intensity function. While there is a large amount of variation in the additional triggering estimates, we do note that the LOESS curve has a general downward trend as the angle bins become more orthogonal in agreement with previous results indicating preferential occurrence of aftershocks along the strike angle. The large amount of variation corresponding to each
angle bin may be due to errors in the strike angle estimates. Whether the improve-
ment in model fit seem in the deviance and Voronoi deviance residuals comes from
the accuracy of the estimated nodal planes or from choosing the better fitting strike
angle of the two nodal planes is unclear and demonstrates some of the challenges of
incorporating fault geometry to forecast future events.

The analysis here used only strike-slip mainshocks so that benefits of using nodal
plane strike angles would be most easily discerned. Given that each focal mechanism
includes two potential estimates for the true strike angle and each estimate contains a
considerable degree of error, we attempt to determine, statistically, which strike angle
estimate is most plausibly estimating the true strike angle and include this estimate
in the model. Even for such strike-slip events and choosing the most plausible strike
angle estimate from the focal mechanisms, we find no significant increase in accuracy
of the nodal plane strike angles compared to strike angles based on previous seismicity
or those chosen uniformly at random.

Although we find that strike-angle estimates from focal mechanisms tend not to
outperform strike-angle estimates based on prior seismicity, we do find that including
the most plausible strike-angle in model (2) was better than nothing in the sense of
improving the fit of a non-parametric Hawkes model compared to model (4), as shown
in Figures 5 and 6. The models are largely in agreement in most areas, but there
are more places where including strike-angle estimates improved the model fit. Those
pixels where model (2) underperformed tended to do so more drastically than areas
where model (2) tended to perform better.

Similarly, Figure 6 shows that for the areas surrounding a mainshock with a strike
angle estimate, including said strike angle estimate overwhelmingly tended to improve
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the fit of the model in all but a few Voronoi cells. That is, even though focal mecha-
nism based strike angle estimates tended to underperform other strike angle estimates,
including them in the non-parametric Hawkes model tended to improve the overall
spatial fit of the model.

The values of the Stoyan-Grabarnik diagnostic for model (2) are shown in Figure 7 and tend to lie inside the error bounds, despite considerable variation. In few in-
stances did the fitted model forecast significantly too many events, corresponding to
Stoyan-Grabarnik values below the lower error bound, or significantly too few events,
corresponding to Stoyan-Grabarnik values above the upper error bound. The large
variability is not unexpected as the variance of the Stoyan-Grabarnik diagnostic tends
to rise in the presence of very small conditional intensities.

For this article, we chose to use a non-parametric form for the conditional intensity
to keep the number of modeling assumptions to a minimum. This allows us to examine
the effect of including fault plane geometry in the model in the absence of additional
modeling assumptions. Investigating how the inclusion of fault geometry in other
parametric model forms and how the effect of this inclusion compares to our work
performed with the non-parametric model are important areas for future research. It
is also important to note that this work was focused solely on earthquakes occurring
in Southern California and further research should be undertaken to determine the
generalizability of our results to seismicity in other areas of the world.
6 Conclusion

In this article, we examine different methods of forecasting the direction of aftershock seismicity using strike angles from nodal plane estimates, strike angle estimates based on previous seismicity, or random uniformly distributed angles. We evaluate the ability of these estimates to forecast the direction of future seismicity based on minimizing orthogonal distances and the mean absolute angle between the estimated strike angle and the locations of future events. We also include the estimated strike angles for each mainshock in a non-parametric Hawkes model and compare the deviance to a null Hawkes model that does not include the strike angle.

The best fitting nodal plane strike angle estimates are shown to be marginally better at forecasting the direction of future seismicity compared to estimates based on previous seismicity and angles chosen uniformly at random. These best fitting angles also seem to improve the estimates of the conditional intensity function in our non-parametric Hawkes model. However, by including orthogonal complements to the estimates based on previous seismicity and estimates chosen uniformly at random and then choosing the better fitting angle estimate for each method (as the better fitting nodal plane strike angle estimates were selected), the small advantage in forecasting future events shown by using the better fitting nodal plane strike angle estimates completely disappears. This indicates that the errors in nodal plane estimates are so large that their improvement in the conditional intensity estimate is entirely attributable to the fact that the better fitting of the two orthogonal nodal planes is selected. Our results thus confirm previous findings that the errors in estimated strike angles remain very large (Kagan 2003).

Important subjects for future work are to determine whether using strike angle
estimates based solely on previous seismicity in conjunction with estimates based on nodal planes would lead to better fitting models, whether incorporating the dip and rake angles of the estimated nodal planes might prove beneficial in forecasting future seismicity, and how increased accuracy in future estimates of nodal planes would translate into increased accuracy in projections of seismicity.

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References


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Figure 1: Strike angle estimates (dashed lines) based on estimated nodal planes for a 5.12 Ml mainshock (red dot) in Southern California, with an estimated depth of 10.1 km, occurring on May 5, 2010. Black points indicate earthquake with epicenters in the range \([-116.066, -115.627]\) in longitude, \([32.481, 32.833]\) in latitude, with magnitude at least 2.8, and occurring between 0.3 and 2137.7 days after the mainshock.
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