Short-term Exciting, Long-term Correcting Models for Earthquake Catalogs

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Abstract

A class of probability models for earthquake occurrences, called Short-term Exciting Long-term Correcting (SELC) models, is presented. This class encompasses features of different models presently used in hazard analysis to characterize earthquake catalogs, such as that of F. Omori. It offers the potential for a unified approach to the analysis and description of different types of earthquake catalogs. Maximum likelihood estimation methods for the seismicity model parameters and standard errors are presented. Sample SELC models are shown to provide satisfactory fit to a seven-year catalog of microearthquakes occurring in Parkfield, California and a longer seismicity sequence from the San Andreas fault zone in Central California. Inferences on seismicity patterns and mechanisms are discussed. Both significant clustering and strain release are detected.

Introduction

Two widely noted features of earthquake catalogs are the following:

1) Earthquakes tend to occur in clusters. This clustering is both spatial and temporal, and is sometimes referred to with terms such as "swarms", "foreshock activity", and "aftershock activity" (Hawkes and Adamopoulos, 1973; Kagan and Knopoff, 1984; Ogata and Tanemura, 1984; Bullen and Bolt, 1985; Ogata and Katsura, 1988).

2) The fault ruptures that generate earthquakes decrease the amount of strain present at the locations along the fault where rupture occurs. This tectonic strain is thought to rebuild gradually over time, eventually achieving a critical level at which time another earthquake, or sequence of earthquakes, is generated (Reid, 1911; Ogata and Vere-Jones, 1984; Wang et al., 1991; Ogata, 1994).

Several probability models reflect efforts at modeling this first type of behavior. Earthquake catalogs are modeled as realizations of triggering, branching or epidemic-type point processes, and all the referenced models have the feature that the instance of an earthquake at point (x, t_1) in space and time increases the likelihood of an earthquake at point (y, t_2) in space and time, where $t_1 < t_2$. The likelihood of a particular realization may be given by the conditional rate of the point process; these models prescribe that the conditional rate of the earthquake process *increases* if more earthquakes have occurred. The amount which the conditional rate increases as a result of one previous earthquake is generally assumed to taper off as both time and distance from the previous earthquake increase.

Numerous researchers have instead focused on the second facet of earthquake behavior listed above. In some cases small events and/or aftershocks are removed from earthquake catalogs under consideration (e.g.

Gonzalez-Ruiz and McNally, 1988; Nishenko and Jacob, 1990). The remaining earthquakes may be modeled as part of a Markov process or strain-release mechanism, in which strain gradually increases until it reaches a critical point, at which time an earthquake occurs and relieves some of the strain, and the process continues (e.g. Zheng and Vere-Jones, 1994). According to such models the conditional rate at a point (y, t_2) in space and time depends on the strain present at point y and time t_2. As the occurrance of an earthquake at a nearby point x at a previous time t_1 decreases the strain at point y, such an event will generally *decrease* the conditional rate at (y, t_2).

Gaussian and log-normal renewal models, which are also often used to model earthquake occurrences (see e.g. Bakun and Lindh, 1985; McEvilly and Lindh, 1986; Nadeau et al., 1995), fit loosely into the second category described above. Such models are grounded on the theory that an earthquake at point (x, t_1) in space and time releases a constant (or approximately constant) amount of the strain near x, which gradually rebuilds until achieving a critical level and causing an event at (y, t_2), where y is near x and $t_2 > t_1$. For such models, the conditional rate at (y, t_2) is assumed to depend *only* on the time t_1 of the most recent event near y, and earthquakes occurring prior to t_1 only contribute indirectly to the conditional rate at time t_2. Such models describe catalogs of well-spaced, quasi-periodic events which exhibit little short-term clustering behavior, and generally have the property that an earthquake at (x, t_1), by releasing the strain near point x in space, *decreases* the conditional rate at (y, t_2), at least for t_2 near t_1.

It would appear that the two classes of models above are diametrically opposed: the first prescribes that earthquakes make future nearby earthquakes more likely; the other predicts that earthquakes make future nearby events less likely. Published examples indicate that the first type of model tends to provide close fit to earthquake catalogs, especially catalogs containing many events, while the second type generally fits poorly unless aftershocks are screened out of the catalog (Kagan and Jackson, 1991; Kagan and Jackson, 1999). Although the second class still generally provides less than spectacular fit to the data, such models are commonly employed (Kagan, 1997) largely because of their agreement with basic seismological strain-release theory.

The dilemma suggests that an alternative type of model, which incorporates both aspects of earthquake behavior, may be an improvement. In order to account for any short-term clustering behavior of an earthquake sequence, the alternative model proposed here displays *self-exciting* behavior in the short run; that is, an event at (x, t_1) increases the conditional rate at (y, t_2), for t_1 slightly less than t_2 and x near y. In addition, in order to agree with strain-release theory, the model exhibits *self-correcting* behavior over the longer term; i.e. an event at (x, t_1) decreases the conditional rate at (y, t_2), for t_1 much less than t_2 and x near y. Such a model may be called a Short-term Exciting, Long-term Correcting (SELC) model.

The purpose of the present paper is to introduce various examples of SELC models and to illustrate their properties and potential uses. Following a review of some existing models which are useful for comparison, several examples of SELC models are presented, and some of their features examined below.

Trigger, strain-release and renewal models

Point processes are useful mathematical constructs for modeling earthquake catalogs (Brillinger, 1982). A point process N is a random collection of points falling in some space S. For instance, the origin times of earthquakes may be viewed as a point process on the real line, with each point corresponding to an earthquake. Similarly, an earthquake catalog containing a list of earthquake origin times as well as their locations and magnitudes may be viewed as a point process in a higher-dimensional space S. For simplicity we consider first the one-dimensional case, where N consists of the origin times of earthquakes.

For any subset B of the real line, let $N(B)$ denote the number of points (earthquakes) in the set B. When the limit exists, the conditional rate process λ associated with N may be defined by:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{E\{N[t, t+\Delta t)|H_t\}}{\Delta t}, \tag{1}$$

where H_t denotes the history of the point process up to time t. Here $N[t, t+\Delta t)$ may be interpreted as the number of earthquakes between times t and $t+\Delta t$, and λ may be regarded as the probability density of an earthquake occurring at time t, given the entire history of the process up to time t. Because all finite-dimensional distributions of N may be derived from the conditional rate (see e.g. Daley and Vere-Jones, 1988), in modeling N it suffices to set down a model for λ.

Consider the first or "self-exciting" type of seismicity behavior described in the previous Section. For the trigger model of Hawkes and Adamopoulos (1973), λ is given by:

$$\lambda(t) = \mu(t) + \int_0^t g(t-u)\,dN(u), \tag{2}$$

where $\mu(t)$ is the background rate and $g(t)$ is the triggering density. The seismological case where $\mu(t)$ is constant is investigated by Hawkes and Adampoulos (1973). The following form for g is suggested by Lomnitz (1966) as

$$g(t) = \phi e^{-\theta t}, \tag{3}$$

which corresponds with Boltzman's theory of elastic aftereffect. Alternate forms for the trigger density are presented by Ogata (1988), e.g.

$$g(t) = \frac{\kappa}{(t+\phi)^{\theta}}, \tag{4}$$

which corresponds to the long-used modified formula originally due to Omori (1894) for aftershock frequency (see Utsu et al., 1995), and

$$g(t) = \sum_{k=1}^{K} \phi_k t^{k-1} e^{-\theta t}, \tag{5}$$

in accordance with Vere-Jones and Ozaki (1982) and Ogata and Akaike (1982). (The reader may be more familiar with the notation $\frac{K}{(t+c)^p}$ in Omori's formula. Throughout this paper, estimable parameters are represented with Greek characters rather than Roman, as is standard in most statistical literature.)

Note that when N is a marked point process (i.e. the magnitudes of events are included; see e.g. Brillinger, 1982), the marks may provide additional information and therefore the formulae for λ may be adjusted. For instance, Ogata (1988) lets the conditional rate λ assume the form (2), where for event i, the triggering density $g(t, m_i)$ is written as a function not only of time t but also of the magnitude m_i of the earthquake.

Now consider the second or strain-dependent type of earthquake sequence description. For many Markovian strain-release models for earthquake occurrences (e.g. Vere-Jones and Ogata, 1984; Ogata, 1994), the conditional rate may be written

$$\lambda(t) = f(t) - g(N[0,t)), \tag{6}$$

where $N[0,t]$ is the total number of events observed between time 0 and time t. For example, Ogata and Vere-Jones (1984) write λ as

$$\lambda(t) = \alpha + \beta t - \nu N[0,t). \tag{7}$$

For a renewal model with density f, the conditional rate is given by

$$\lambda(t) = s(t - \tilde{t}), \tag{8}$$

where $\tilde{t}$ is the time of the most recent event before time t, and $s(t)$ is the survivor function corresponding to f (see e.g. Bullen and Bolt, 1985). That is,

$$s(t) = \frac{f(t)}{1 - F(t)}, \tag{9}$$

where $F(t) = \int_0^t f(u)du$ is the cumulative distribution function corresponding to f. (Note that f is ordinarily taken to be a density on the half-line, i.e. $f(t) = 0$ for $t < 0$.)

Examples of SELC models

One way to define a SELC model is via a "U-shaped" renewal density f, as in Hong and Guo (1995). Such a model is shown by these researchers to provide reasonably good fit to a very small earthquake catalog. However, the model described by Hong and Guo is perhaps unrealistically simple, and more complex SELC models may be more applicable to actual earthquake data, particularly in light of recent evidence of differential release of strain by earthquakes occuring at different times and locations (e.g. Shen et al., 1994; Gomberg, 1996).

As an alternative, the assumption that the point process is a renewal process may be discarded, and the conditional rate may be defined in the more general form of (2). Note that as t increases, the triggering density g decays rapidly to 0 in models (3) - (5), like that of Omori. Although the empirical clustering behavior of earthquakes does appear often to decrease rapidly after an event, there is no physical reason why

g must converge to 0. Indeed, the theory of long-term strain and release by a fault rupture suggests that g should be negative for large t.

For example, g may be given by:

$$g(t) = \frac{\kappa}{(t+\phi)^\theta} - \nu \tag{10}$$

or

$$g(t) = \sum_{k=1}^{K} \phi_k t^{k-1} e^{-\theta t} - \nu, \tag{11}$$

where the constant ν represents the average long-term decrease in rate of earthquakes caused by the release of strain.

In addition, because strain is thought to gradually increase during the time interval between earthquakes, the background rate $\mu(t)$ may be given as in (7), i.e.

$$\mu(t) = \alpha + \beta t. \tag{12}$$

Several articles (e.g. Vere-Jones and Ogata, 1984; Wang et al., 1991) use $\exp(\alpha + \beta t)$ rather than $\alpha + \beta t$, in order to ensure that $\lambda(t)$ is nonegative for all t. Such a modification introduces no complications to the specification here.

Putting together (2), (12), and either (10) or (11), the conditional rate of these two SELC processes may be written as:

$$\lambda(t) \quad = \quad \alpha + \beta t + \int_0^t \left(\frac{\kappa}{(t-s+\phi)^\theta} - \nu \right) dN(s) \tag{13}$$

or

$$\lambda(t) \quad = \quad \alpha + \beta t + \int_0^t \left(\sum_{k=1}^{K} \phi_k (t-s)^{k-1} e^{-\theta(t-s)} - \nu \right) dN(s). \tag{14}$$

When earthquake magnitudes are included in the model as marks, (13) and (14) may be modified. Both the short-term triggering effect and the long-term decrease in strain due to an earthquake depend on the magnitude of the event. Thus the triggering density g may be adjusted for example by introducing a factor $e^{\gamma(m-m_o)}$ as in Ogata (1988), where m is the magnitude of the event and m_o is the cutoff magnitude for the catalog. Similarly, the strain-drop constant ν in (10) and (11) may be replaced by a function of m; an exponential function such as $\nu e^{\gamma(m-m_o)}$ agrees with Vere-Jones and Ogata (1984) and Wang et al. (1991). The conditional rates in (13) and (14) are then replaced by

$$\lambda(t,m) \quad = \quad \alpha + \beta t + \int_0^t \int_{m_o}^{m_{max}} e^{\gamma(m-m_o)} \left(\frac{\kappa}{(t-s+\phi)^\theta} - \nu \right) dN(s,m) \tag{15}$$

and

$$\lambda(t,m) \quad = \quad \alpha + \beta t + \int_0^t \int_{m_o}^{m_{max}} e^{\gamma(m-m_o)} \left(\sum_{k=1}^{K} \phi_k (t-s)^{k-1} e^{-\theta(t-s)} - \nu \right) dN(s,m). \tag{16}$$

Spatial components, such as the locations of earthquake epicenters or hypocenters, can similarly be introduced quite readily. For instance one may replace the function $\mu(t)$ in (12) with a function $\mu(t, \mathbf{x})$, where the vector $\mathbf{x}$ represents spatial coordinates. The trigger density g and strain decrease term ν may also be replaced by decreasing functions of $\mathbf{x}$, as in Schoenberg (1997), for example.

Estimation and Inference

Given a catalog of observed earthquake origin times, standardized to begin at time 0 and ending at time T, the parameter vector θ of SELC models such as (13) or (14) may be estimated by maximizing the loglikelihood function

$$\int_0^T [log\lambda(t;\theta)]dN(t) - \int_0^T \lambda(t;\theta)dt. \tag{17}$$

When a matrix $\mathbf{x}$ of magnitudes and/or spatial or other information is also included, the function (17) is simply replaced by

$$\int_0^T \int_{\mathbf{x}} [log\lambda(t,\mathbf{x};\theta)]dN(t,\mathbf{x}) - \int_0^T \int_{\mathbf{x}} \lambda(t,\mathbf{x};\theta)d\mathbf{x}dt. \tag{18}$$

Approximate standard errors may be obtained via the diagonal of the inverse of the negative Hessian of the loglikelihood function (see e.g. Rathbun and Cressie, 1994).

Note that each computation of λ in equations such as (13)-(16) requires $O(n)$ calculations, where n is the number of data points in the sample. Thus it would appear that computing the likelihood function (17) would require $O(n^2)$ computations. However, by storing elements of the previous computation the procedure can be reduced to just $O(n)$ computations. Details, as well as sample C programs for maximizing the likelihood function for SELC models, can be found at http://pearson.stat.ucla.edu/selc/index.html.

Two earthquake catalogs are used to illustrate the estimation of some examples of the processes described above. The first dataset consists of a catalog of 2402 microearthquakes in Parkfield, California, between 1988 and 1995 (Nadeau et al., 1995). The earthquakes, occuring at a rate of approximately one per day, range in magnitude from 3 to approximately -1, and were detected by a high-resolution seismic network. The dataset is described in detail by Nadeau et al. (1995) and Schoenberg (1997). A histogram of the earthquake event times for the Parkfield catalog is shown in Figure 1. The second catalog consists of 580 earthquakes of magnitude 3.0 and higher (up to 5.5) occuring along a 35 kilometer portion of the San Andreas Fault stretching from Hollister to Bear Valley, California (latitude 36.5 to 37, longitude 120.5 to 121). The catalog was obtained from the Council of the National Seismic System at quake.geo.berkeley.edu and details about the data may be obtained there. A histogram of the earthquake origin times appears in Figure 2.

Using the datasets described above, a trigger process (2) with triggering density as in (3) and background rate as in (12), a strain-release model (7), and a SELC model (14) with $K = 1$ are fit by maximum likelihood.

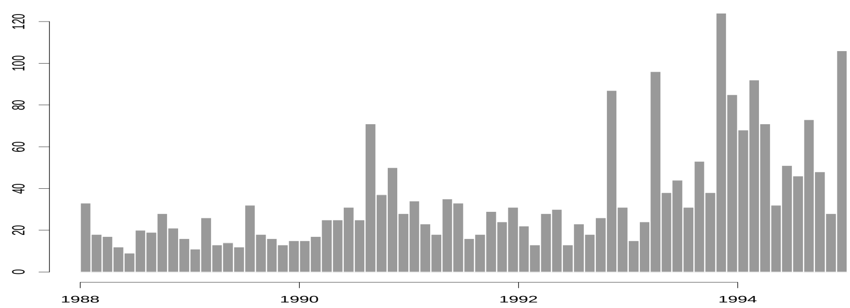

Figure 1: Histogram of Parkfield event times

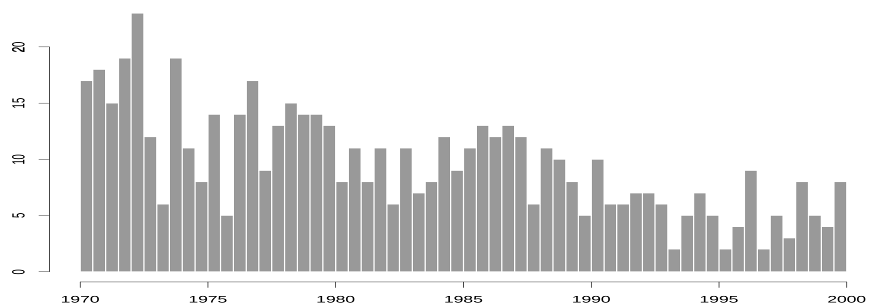

Figure 2: Histogram of Hollister-Bear Valley event times

The parameter estimates for the fitted models are shown in Tables 1 and 2 below. Approximate standard errors are given in parentheses. The column with the label "LR" contains the likelihood ratios of each of the models relative to the best-fitting (SELC) model. The relative likelihoods in Tables 1 and 2 can be used as a measure of goodness-of-fit: higher likelihood indicates better fit.

Note that the parameters of the SELC model are easily interpretable and may provide meaningful information about the earthquake catalog. The parameters α and β reflect the background rate of earthquake generation, with β representing the average increase in seismicity from one year to the next. The parameters ϕ and θ govern the triggering rate: ϕ represents the increase in conditional rate immediately following an event, and θ dictates the rate at which this triggering effect decays with time. Note also that the overall increase in seismicity caused by a single event is given by the ratio ϕ/θ; in the last row of Table 1 the ratio of the two estimates is approximately 1/3, meaning that roughly 1/3 of the events in the Parkfield catalog appear to have been triggered by other earthquakes. Not surprisingly, for the Hollister-Bear Valley catalog of moderate-sized earthquakes, the ratio decreases to approximately 1/5. The parameter ν reflects the long-term decrease in seismicity caused by a typical earthquake in the catalog due to its release of strain. As might be expected, this decrease is minimal for the Parkfield catalog which contains exclusively micro-earthquakes, and is over 50 times larger for the moderate earthquakes along the San Andreas segment.

Table 1. Parameter Estimates, using Parkfield Data

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\theta}$	$\hat{\nu}$	LR
Trigger	114 (6)	34 (2)	917 (39)	2805 (139)		.74
Strain-Release	112 (6)	64 (2)			.001 (.007)	.00
SELC	113 (6)	34 (2)	925 (38)	2787 (138)	.001 (.006)	1.00

Table 2. Parameter Estimates, using Hollister-Bear Valley Data

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\theta}$	$\hat{\nu}$	LR
Trigger	25.3 (.7)	-.59 (.04)	2270 (225)	10738 (1319)		.05
Strain-Release	28.7 (.8)	1.9 (.05)			.116 (.002)	.00
SELC	20.2 (.6)	.80 (.04)	837 (75)	3960 (387)	.058 (.002)	1.00

Two other important features which can be interpreted from the parameters of the SELC model are the scales at which the triggering and strain-release mechanisms appear to operate, for the catalogs in question. For the Parkfield dataset, immediately following one of the micro-earthquakes, the triggering effect typically represents $80-90\%$ of the conditional rate. However, after just one day (.0027 years), this ratio drops to less than .3%. The decrease is slower for the Hollister-Bear Valley earthquakes; according to the SELC model estimates of Table 2 the triggering effect decreases from 95% of the conditional rate immediately following an earthquake to approximately 2% one day later. Secondly, as for the strain-release from earthquakes, according to the SELC model estimates it would require over $2,000$ events to decrease the average conditional rate for the Parkfield micro-earthquakes by just 1%; at the rate observed this would take nearly a decade. For the Hollister-Bear Valley earthquakes however, only 50 earthquakes are needed to release enough strain to cause a decrease of 10% in the overall conditional rate.

Of course, such inferences based on parameter estimates must always be taken with some skepticism. If any portion of the model is mis-specified – e.g. if the background rate is nonlinear, the trigger density is not exponential, or the strain-release term varies with time – then all bets are off. Also, inferences based on the estimated strain decrease for the Parkfield micro-earthquakes are particularly suspect, since the estimated standard error is 6 times larger than the parameter estimate itself.

The SELC model appears to offer significant improvement in fit over the simple strain-release model for both the Parkfield and Hollister-Bear Valley datasets. There is little benefit relative to the trigger model for the list of Parkfield event times, however, perhaps because the trigger model already offers satisfactory fit to this catalog. Indeed, using a penalized likelihood function such as the Akaike Information Criterion (see e.g. Ogata and Tanemura, 1984), the infinitesimal increase in loglikelihood would not outweigh the penalty from using one extra parameter (ν). Thus the Parkfield data do not suggest any significant strain-release effect at all. For the Hollister-Bear Valley dataset, however, where the strain-decrease parameter carries more weight, the improvement in fit using the SELC model is more substantial.

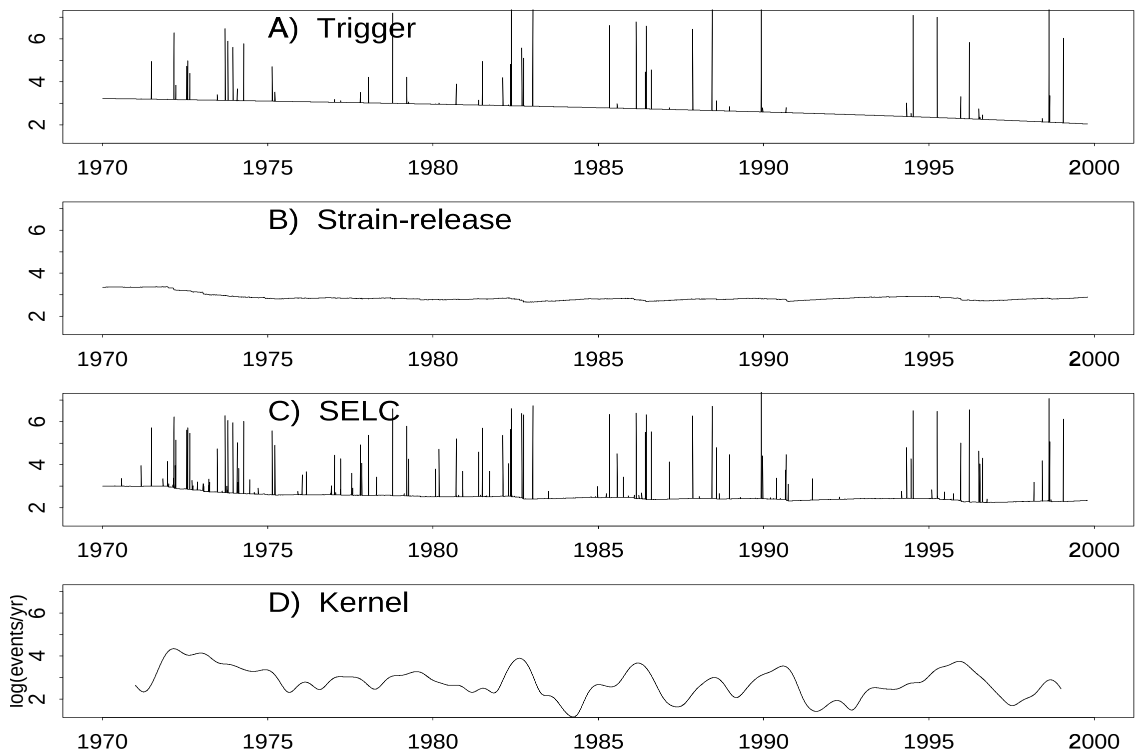

Figure 3: Conditional rate estimates for Hollister-Bear Valley data

Figure 3 displays the estimated conditional rates $\hat{\lambda}$ corresponding to trigger, strain-release, and SELC models fitted to the Hollister-Bear Valley dataset, sampled at 5000 equally spaced times between 1970 and 1999 and plotted on a logarithmic scale. For comparison, a nonparametric (kernel) rate estimate is given as well (see Vere-Jones, 1992). The conditional rates corresponding to the trigger and SELC models are highly volatile because of the exponential decay in the triggering density (3). By contrast, the conditional rate estimates based on the strain-release model estimates are smoother, because sharp fluctuations in seismicity corresponding to clustering behavior are not permitted by the model. The kernel estimate is by definition a smoothed version of the seismicity; hence the more gradual changes in the estimate of λ, compared to the trigger and SELC estimates (Vere-Jones, 1992).

In order to illustrate more local properties of the models, Figure 4 displays a small portion of the conditional rate processes λ shown in Figures 3A-C, plotted on a logarithmic scale. The plot in Figure 4 shows the estimated conditional rate for each of the three models during a one-day period containing the first earthquake in 1990. The choice of this earthquake (of magnitude 3.9) is arbitrary; the conditional rate estimates behave similarly after every earthquake in the catalog. The exponential decay in the conditional rate corresponding to the trigger and SELC models is clearly observable in Figure 4. This corresponds to short-term self-exciting behavior, because the conditional rate is much higher immediately following an event. Note that close inspection of the estimated conditional rate for the strain-release model reveals a sudden small decrease in λ immediately following the earthquake. This is self-correcting behavior; i.e. if many earthquakes occur in a short period of time, then the conditional rate will decrease, so that comparatively fewer events will occur shortly thereafter. Thus the cumulative number of events in any time period is moderated.

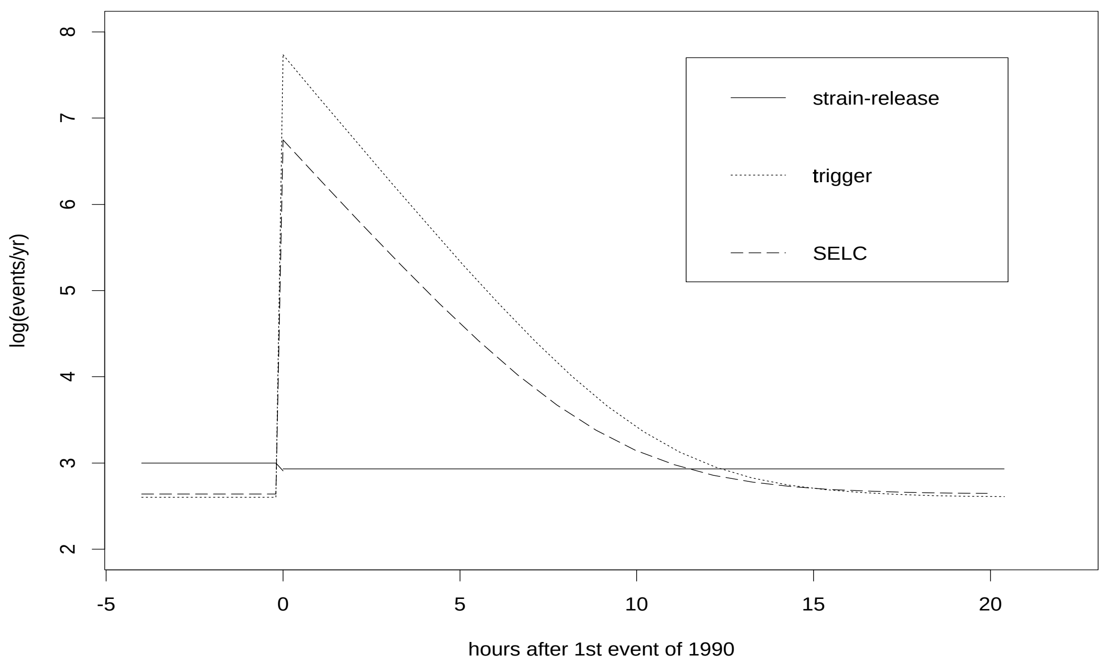

Figure 4: Local conditional rate estimates for Hollister-Bear Valley data

Simulation

Simulations are useful to demonstrate the different behavior of the various models described above. In Figure 5, histograms of simulated event times are presented for trigger, strain-release, and SELC models. The models used are those fit by maximum likelihood to the Hollister-Bear Valley earthquake dataset; i.e. the parameters are those which appear in Table 2. The three histograms look similar to one another and to the histogram of the Hollister-Bear Valley data in Figure 2. At this scale, the most obvious feature in each histogram is the overall trend; the rate of earthquakes is gradually decreasing in time, in agreement with the actual detection in Figure 2.

Consider next the second moment function $h(t)$ which represents the frequency of pairs of times $\{t_1, t_2\}$ in the catalog, where $t_2 - t_1 \approx t$ (see e.g. Ripley, 1979). Figures 6a-c depict the normalized empirical second moment functions $h(t)$ of the simulated events on a logarithmic scale and Figure 6d displays the same information for the Hollister-Bear Valley catalog. For the purpose of illustration, the second moment

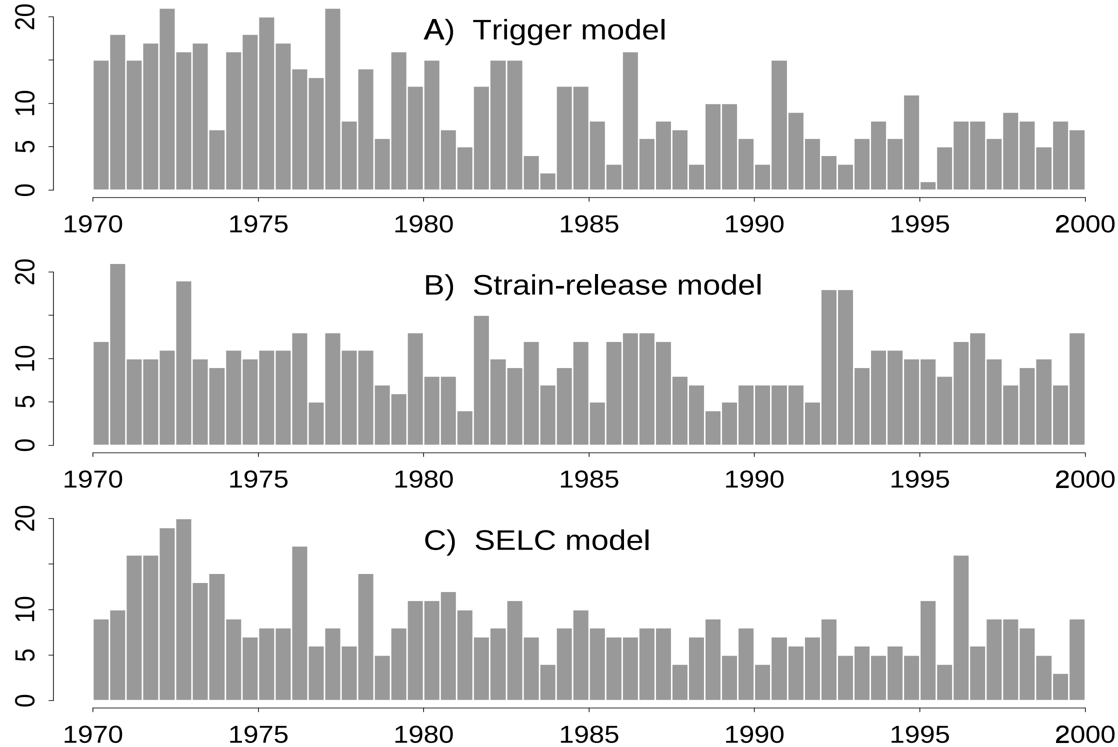

Figure 5: Histograms of Simulated Event Times

function of the stationary Poisson process is shown (dashed line), and the dotted lines mark the bounds of 95% confidence intervals for the second moment function of the stationary Poisson process. The significantly large values of $h(t)$ corresponding to small values of t for the simulated trigger process in Figure 6a are a direct result of the self-exciting nature of the process: an earthquake at time t causes earthquakes to occur shortly thereafter; hence there are many pairs of events with short inter-event times. A similar feature is evident in the SELC model as seen in Figure 6c.

By contrast, in Figure 6b the strain-release process does not have a significant number of pairs of events which are temporally very close together. Because the strain-release process in Figure 6b is self-correcting, an earthquake at time t decreases the subsequent conditional rate, thus decreasing the likelihood of an earthquake occurring shortly thereafter. Hence the normalized second moment is typically *less* than unity. Note that this property is possessed by the SELC model as well: although the second moment in Figure 6c is high for small values of t corresponding to self-exciting behavior immediately following an earthquake, $h(t)$ takes values less than unity thereafter.

It is readily apparent from Figure 6d that the SELC model appears to approximate the short-term second-order properties of the observed San Andreas seismicity catalog rather well. The large values of $h(t)$ in Figure 6d indicate very strong short-term temporal clustering of the Hollister-Bear Valley earthquakes. The small values of $h(t)$ for large t can be interpreted as the result of long-term self-correcting behavior due

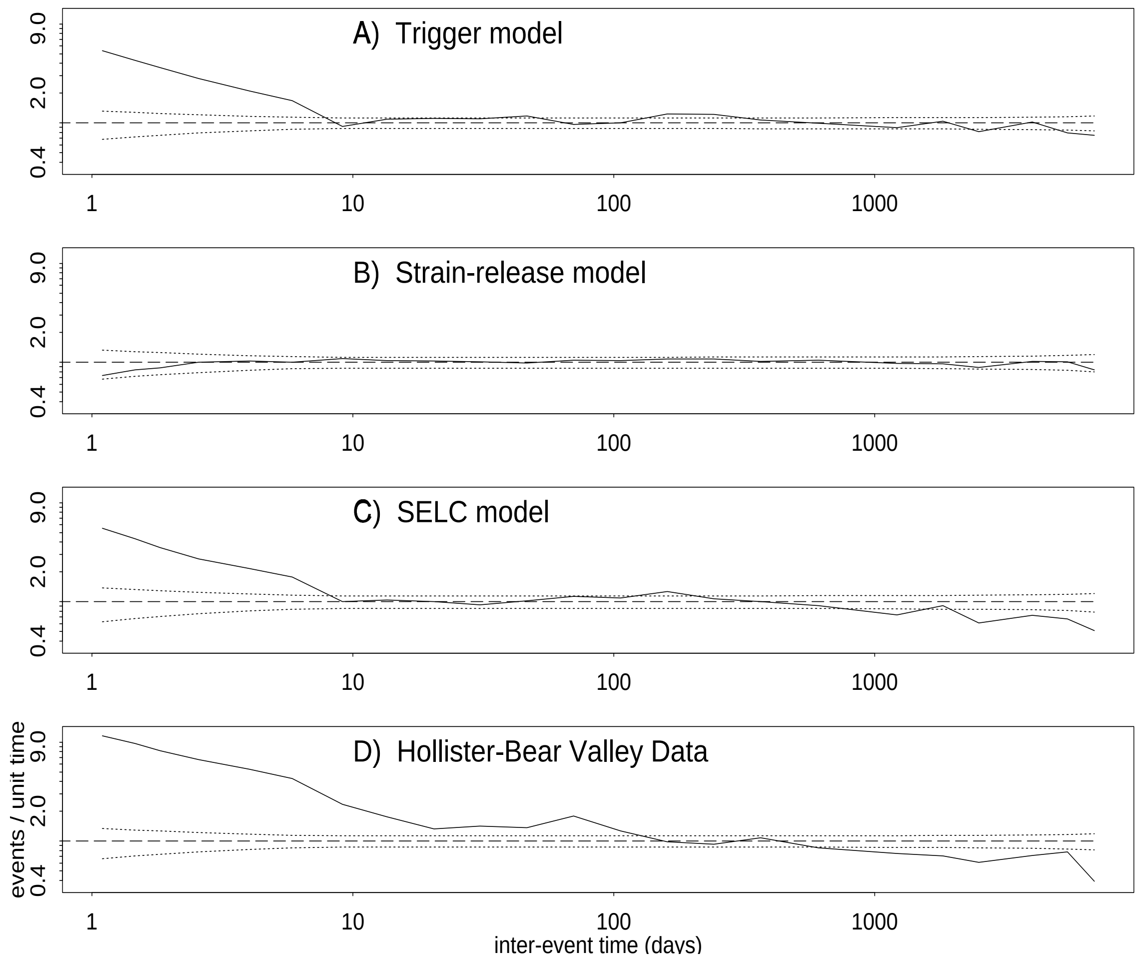

Figure 6: Second moments of simulated and Hollister-Bear Valley catalogs

to the release of strain.

Summary

The clustering nature of earthquake catalogs that contain aftershock sequences can be incorporated into the strain-release theory of elastic rebound which is often used to characterize catalogs of large earthquakes. Such an accomodation is achieved here by a general class of probability models for earthquake occurrence called Short-term Exciting, Long-term Correcting (SELC) processes. This class shares features of branching processes as well as Markovian strain-release processes.

The SELC models described have two desirable features. First, they represent a unified framework for analyzing earthquake catalogs. Such generality is important in order to eliminate both the practical and theoretical inconvenience in hazard studies of having to use two different classes of models to describe, for example, a 50-year catalog comprised only of large earthquakes and a 5-year catalog which includes small secondary events. Secondly, the parameters in the SELC models represent basic geophysical behavior and can be interpreted to offer meaningful descriptions of earthquake catalogs when fitted to data. In addition, tests with two reliable seismicity catalogs indicate that the SELC models introduced here appear to provide a satisfactory fit to actual sesmicity. Certain simulated SELC processes are shown to adequately approximate a 30-year catalog of moderate-sized earthquakes occuring along the San Andreas Fault near Hollister, California and a seven-year catalog of small earthquakes in Parkfield, California.

The results presented focus on the temporal character of earthquake clustering, in order to illustrate the main differences between SELC models and their predecessors. This restricted comparison, however, suggests that further statistical study of earthquake sequences is worthwhile in order to assess how broadly multi-dimensional SELC models fit different types of catalogs, including earthquakes from different regions.

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References

Bakun, W. and A. Lindh (1985). The Parkfield, California earthquake prediction experiment. *Science*, **229**, 619-624.

Brillinger, D. (1982). Seismic risk assessment: some statistical aspects. *Earthquake Prediction Research* **2**, *183-195.*

Bullen, K. and B. Bolt (1985). An Introduction to the Theory of Seismology, 4th ed. Cambridge University Press, Cambridge.

Daley, D. and D. Vere-Jones (1988). An Introduction to the Theory of Point Processes. Springer, Berlin.

Gonzalez-Ruiz, J. and K. McNally (1988). Stress accumulation and release since 1882 in Ometepec, Guerrero, Mexico: implications for failure mechanisms and risk assessments of a seismic gap. Journal of Geophysical Research, **93**, *6297-6317.*

Gomberg, J. (1996). Stress/strain changes and triggered seismicity following the M subscript w 7.3 Landers, California, earthquake. Journal of Geophysical Research, **101**, *751-764.*

Hawkes, A. and L. Adamopoulos (1973). Cluster models for earthquakes – regional comparisons. Bull. Int. Statist. Inst. **45** *(3), 454-461.*

Hong, L. and S. Guo (1995). Nonstationary Poisson model for earthquake occurrences. Bull. Seis. Soc. Amer. **85**, *814-824.*

Kagan, Y. (1997). Are earthquakes predictable? Geophys. J. Int. **131**, *505-525.*

Kagan, Y. and D. Jackson (1991). Long-term earthquake clustering. Geophys. J. Int. **104**, *117-133.*

Kagan, Y. and D. Jackson (1999). Worldwide doublets of large shallow earthquakes. Bull. Seismol. Soc. Amer., 1-11 (in submission).

Kagan, Y. and Knopoff, L. (1984). A stochastic model for earthquake occurrence. Proc. 8th Int. Conf. Earthq Eng. **1**, *295-302.*

Lomnitz, C. (1966). Magnitude stability in earthquake sequences. Bull. Seis. Soc. Amer. **56**, *247-249.*

McEvilly, T. and A. Lindh (1986). The Parkfield, California earthquake prediction experiment. Science **229**, *619-624.*

Nadeau, R., W. Foxall, and T. McEvilly (1995). Clustering and periodic recurrence of microearthquakes on the San Andreas Fault at Parkfield, California. Science **267**, *503-507.*

Nishenko, S. and K. Jacob (1990). Seismic potential of the Queen Charlotte-Alaska-Aleutian seismic zone. Journal of Geophysical Research, **95**, *2511-2532.*

Ogata, Y. (1988). Statistical models for earthquake occurrences and residual analysis for point processes. J. Amer. Statist. Assoc. **83**, *9-27.*

Ogata, Y. (1994). Inference for earthquake models: a self-correcting model. Stochastic Processes and their Applications **17**, *337-347.*

Ogata, Y. and D. Vere-Jones (1984). Inference for earthquake models: A self-correcting model. Stoch. Proc. Appl. **17**, *337-347.*

Ogata, Y. and H. Akaike (1982). On linear intensity models for mixed doubly stochastic Poisson and self-exciting point processes. J. R. Statist. Soc. Ser. B **44**, *102-107.*

Ogata, Y. and K. Katsura (1988). Likelihood analysis of spatial inhomogeneity for marked point patterns. Ann. Inst. Stat. Math. 40, 29-39.

Ogata, Y. and M. Tanemura (1984). Likelihood analysis for spatial point patterns. Proceedings of the International Biometric Conference **12**, *276-286.*

Omori, F. (1984). On the after-shocks of earthquakes. J. College Sci., Imp. Univ. Tokyo **7**, *111-200.*

Rathbun, S. and N. Cressie (1994). Asymptotic properties of estimators for the parameters of spatial inhomogeneous Poisson point processes. Ad. Appl. Prob. **26**, *122-154.*

Reid, H. (1911). The elastic-rebound theory of earthquakes. Bull. Dep. Geol. Univ. Calif. **6**, *412-444.*

Ripley, B. (1979). Tests of 'randomness' for spatial point patterns. J. Royal Stat. Soc. B **39**, *172-192.*

Schoenberg, F. (1997). Assessment of Multi-dimensional Point Process Models. Ph.D. thesis, Statistics Department, University of California, Berkeley.

Shen, S., D. Jackson, Y. Feng, M. Cline, M. Kim, P. Feng, and Y. Bock (1994). Postseismic Deformation Following the Landers Earthquake, California, 28 June 1992. Bull. Seis. Soc. Amer. **84**, *780-791.*

Utsu, T., Y. Ogata, amd R. Matsu'ura (1995). The centenary of the Omori formula for a decay law of aftershock activity. J. Phys. Earth **43**, *1-33.*

Vere-Jones, D. (1992). Statistical methods for the description and display of earthquake catalogs. In Statistics in Environmental and Earth Sciences, A. Walden and P. Guttorp, eds. Halstead, NY, 220-246.

Vere-Jones, D. and T. Ozaki (1982). Some examples of statistical inference applied to earthquake data. Ann. Inst. Statist. Math. **34**, *189-207.*

Vere-Jones, D. and Y. Ogata (1984). On the moments of a self-correcting process. J. Appl. Probab. **21**, *335-342.*

Wang, A., D. Vere-Jones, and X. Zheng (1991). Simulation and estimation procedures for strain release models. In Stoch. Proc. Appl. Proc. Symp. Indian Inst. Tech., ed. M. Beckmann, M. Gopalan, R. Subramanian, p. 11-27. Lecture Notes in Economics and Mathematical Systems **370**, *Springer-Verlag, Berlin.*

Zheng, X. and D. Vere-Jones (1994). Further applications of the stochastic stress release model to historical earthquake data. Tectonophysics **229**, *101-121.*