Quantification of Luck and Skill in Texas Hold’em

Introduction. The determination of whether Texas Hold’em is primarily a game of luck or skill has been the subject of intense legal debate. The terms luck and skill are extremely difficult to define, and surprisingly, rigorous definitions of these terms seldom appear in books and journal articles on game theory. A few articles have defined skill in terms of the variance in results among different players, with the idea that players should perform more similarly if a game is mostly based on luck, but their results may differ more substantially if a game is based on skill (Potter van Loon et al., 2015). Another definition of skill is the extent to which players can improve; poker does indeed involve a significant amount of potential for improvement (Dedonno and Detterman, 2008). This article offers a different take on the definitions of luck and skill in poker, and highlights certain scenarios, involving real Texas Hold'em hands, in which the luck and skill components can readily be quantified. The reader not well versed in Texas Hold'em rules, basic concepts, and strategy is invited to read Sklansky (1989), Brunson and Addington (2002), Hellmuth (2003), Harrington and Robertie (2004), Chen and Ankenmann (2006), Gordon (2006), and and Schoenberg (2017).

Before considering the definition of luck and skill used here, note that the alternative definitions referred to above in terms of variation or improvement among players are obviously extremely problematic for various reasons. The definitions seem somewhat arbitrary and only very loosely tied to one's conceptions of luck and skill, and it is easy to think of counterexamples and major flaws in the definitions especially when considering their application to other games. There are many contests of skill wherein the differences between players are small, or where one’s results vary wildly. For instance, in Olympic trials of the 100-meter sprints, the differences between finishers are typically quite small, often just hundredths of a second. This hardly implies that the results are based on luck. In other sporting events, for example pitching in baseball, an individual’s results may vary widely from one day to another, but that does not mean luck plays a major role. Some players might not be able to improve beyond a certain point in chess, but this does not render chess a game of luck.

Proposed definition. To quantify the amount of luck or skill in a particular game of poker, one possibility is to define luck as expected profit gained when cards are dealt by the dealer, and skill as expected profit gained by a player’s actions during betting rounds. A player might gain expected profit during a hand by several actions:

* The cards dealt by the dealer (whether the players’ hole cards or the flop, turn, or river) give the player in question a greater chance of winning a hand in a showdown, thus increasing her equity in the pot.

* The size of the pot is increased while the player in question's chance to win the hand in a showdown is better than those of her opponents.

* By betting, the player get others to fold and thus increases her probability of winning the pot.

Certainly, anyone would characterize the first case as luck, unless perhaps one believes in ESP or time travel. Thus, it may be possible to estimate skill in poker by looking at the second and third cases above. That is, we may view skill as the expected profit gained during the betting rounds, whereas luck is the expected profit gained simply by dealing the cards. Both are easily quantifiable, and one may dissect a particular poker game and analyze how much expected profit each player gained due to luck or skill.

Drawbacks and limitations. There are obvious objections to these proposed definitions. First, situations can occur where a terrible player may gain expected profit during betting rounds against even the greatest player in the world and attributing such gains to skill may be objectionable. For instance, in heads-up Texas Hold'em, if the two players are dealt AA and KK, one would expect the player with KK to put a great number of chips in while way behind. This situation seems more like bad luck for the player with KK than a deficit in skill. However, virtually any definition of skill can be objected to on such a basis. Most poker players, probably due to their large and fragile egos, tend to attribute nearly all losses to bad luck, and almost anything can be attributed to luck if the definition of luck is general enough. Even if a player makes an amazingly skillful poker play, such as folding a very strong hand because of an observed tell or betting pattern, one could argue that the player was lucky to observe the tell or even that he was lucky to have been born with the ability to discern the tell. On the other hand, situations like AA versus KK truly do seem like bad luck. It is difficult to think of any remedy to this problem. It may be that skill is too strong a word, and that when analyzing hands in terms of equity, one should perhaps instead refer to expected profit gained *during betting rounds* rather than expected profit gained *due to skill*. The word skill will nevertheless be used in what follows. Second, using the definitions proposed here, luck and skill will often be correlated in practice. This is explored further following Example 3 below. Third, one may question the idea of calculating expected profit, or equity, in a pot assuming no future betting. The assumption of no future betting may seem absurdly simplistic and unrealistic in some cases. Unlike implied equity, which accounts for betting on future betting rounds, ordinary equity is unambiguously defined and easy to compute, but admittedly does have its shortcomings, as shown in the following example.

(counter) Example 1. This hand from Season 7 of High Stakes Poker illustrates some of the limitations of making inferences based on equity, where one assumes no future betting (or folding) in calculating the expected winnings for each player. With blinds of $400 and $800 plus $100 antes from each of the eight players, after Bill Klein straddled for $1600, Phil Galfond raised to $3500 with Q♠ 10♥, Robert Croak called in the big blind with A♣ J♣, Klein called with 10♠ 6♠, and the other players folded. The flop came J♠ 9♥ 2♠, giving Croak top pair, Klein a flush draw, and Galfond an open-ended straight draw. Croak bet $5500, Klein raised to $17,500, and Galfond and Croak called. At this point, it is tempting to compute Klein's probability of winning the hand by computing the probability of exactly one more spade coming on the turn and river without making a full house for Croak, or the turn and river including two 6s, or a 10 and a 6. Counting combinations, and using the notation $C(n,k) = n! / [k! (n-k)!]$ to represent the number of distinct combinations of k different items out of n different possibilities, this would yield a probability of $[(8 \times 35 – 4 – 4) + C(3,2) + 2 \times 3] \div C(43,2) = 281/903 \sim 31.12\%$. Klein could also split the pot with a straight if the turn and river were KQ or Q8 without a spade, which has a probability of $[3 \times 3 + 3 \times 3] \div C(43,2) = 18/903 \sim 1.99\%$. These seem to be the combinations Klein needs, and one would not expect Klein to win the pot with a random turn and river combination not on this list, and especially not if the turn and river contain a king or a jack with no spades. However, look at what actually happened. The turn was the K♣, giving Galfond a straight, and Croak checked. Klein bet $28,000, Galfond raised to $67,000, Croak folded, and Klein called. The river was the J♥, Klein bluffed $150,000, and Galfond folded, giving Klein the $348,200 pot!

Returning to the proposed, and admittedly occasionally flawed, definitions of luck and skill as expected profit gained during the dealing of the cards and expected profit gained during the betting rounds, respectively, it is worth considering some examples of actual poker hands in order to see how luck and skill are quantified in these cases.

Example 2. On day 4 of the World Series of Poker (WSOP) Main Event in 2015, with blinds of 5,000 and 10,000 and antes of 1,000 from eight players, after Ryan D'Angelo raised to 22,000 with A♦ K♠, Daniel Negreanu called 17,000 more from the small blind with A♠ 7♠, and Fernando Perez called from the big blind with 3♥ 2♥. The pot was 74,000. The flop came 3♠ 10♣ 9♠ and everyone checked. The turn was 2♠, giving Negreanu the nuts and giving Perez two pairs. Negreanu bet 35,000, Perez raised to 105,000, D'Angelo folded, Negreanu reraised to 250,000, and Perez called. The pot was now 574,000. The river was 5♣, Negreanu bet all-in for 359,000, and Perez folded. How much expected profit did Negreanu gain (a) due to luck on the turn, (b) due to skill on the turn, (c) due to luck on the river, and (d) due to skill on the river?

Answer—(a) Counting combinations, before the turn was revealed,
P(Negreanu wins) = P(spade on turn or river) + P(77) + P(7x) + P(J*8*) + P(8*6*),
where x is a non-spade card that is not a 2, 3, or K, and * denotes non-spades,

$= [\{8/45+8/45 – C(8,2)\} + C(3,2) + 3\times34 + 3\times3 + 3\times3] / C(45,2)$

$\sim 45.15\%$.

(There was also a small [1/165] chance of a split pot with 10*9* but we will ignore that here.) When the turn was revealed, the probability that Negreanu would win, assuming nobody folded, was the probability that a 3 or 2 would not come on the river, which is $41/44 \sim 93.18\%$. Thus Negreanu's equity increased from $45.15\% \times 74{,}000$ to $93.18\% \times 74{,}000$ due to luck on the turn, an increase of 35,542.2 chips.

(b) During the betting on the turn, the pot increased from 74,000 chips to 574,000 chips. Thus Negreanu's expected share of the pot increased from $93.18\% \times 74{,}000 =$

68,953.2 to 93.18% × 574,000 = 534,853.2, for an increase of 465,900 chips. The cost to Negreanu on the turn was 250,000 chips, so his increase in expected profit on the turn due to skill was 465,900 – 250,000 = 215,900 chips.

(c) When the river card was revealed, Negreanu went from having a 93.18% chance of winning the hand in a showdown to 100%, so his equity increased from 534,853.2 chips to 574,000 chips, for an increase of 39,146.8 chips due to luck.

(d) The river betting did not increase Negreanu's profit, so Negreanu gained 0 due to skill on the river.

Example 3. In a captivating hand from the 2015 WSOP Main Event, Mike Cloud raised to 15,000 with A♣ A♠, Hellmuth called with A♥ K♠, Daniel Negreanu called from the big blind with 6♦ 4♥, and the flop came K♣ 8♥ K♥. Before the flop, the pot was 57,000 chips, and the probabilities shown on ESPN's broadcast of winning the hand in a showdown at this point were 74% for Cloud, 19% for Negreanu, and only 6% for Hellmuth. (The probabilities only add up to 99% because of an approximately 1% chance of a split pot.) After the flop, all three players checked, the turn was the J♥, Negreanu checked, Cloud bet 15,000, Hellmuth called, and Negreanu folded. The river was the 7♠, Cloud checked, Hellmuth bet 37,000, and Cloud called. How much expected profit did Hellmuth gain due to luck and how much due to skill (a) on the flop, (b) on the turn, and (c) on the river?

Answer—(a) Before the flop was revealed, Hellmuth's equity was 6% × 57,000 = 3,420 chips. After the flop was dealt, the only way Hellmuth could have lost in a showdown would have been if the turn or river contained the A♦ without the K♦, which, given the six cards belonging to the players and the three cards on the flop, had a probability of (1 × 41) / C(43,2) = 4.54%, so Hellmuth's equity suddenly increased to 95.46% × 57,000 = 54,412.2 chips. Thus on the flop Hellmuth gained 54,412.2 – 3420 = 50,992.2 chips in equity due to luck. There was no betting on the flop so Hellmuth gained 0 expected profit due to skill on the flop.

(b) When the turn was dealt, Hellmuth's probability of winning in a showdown increased to 41/42 ~ 97.62%, so his equity increased from 54,412.2 to 97.62% × 57,000 = 55,643.4, for an increase in expected profit of 1,231.2 due to luck on the turn.

During the betting on the turn, Hellmuth and Cloud each put 15,000 chips in the pot, so Hellmuth's expected return increased by 97.62% × 30,000 = 29,286 chips, but he put 15,000 chips into the pot on the turn, so his expected profit on the turn due to skill was 29,286 – 15,000 = 14,286 chips.

(c) After the betting on the turn was over, the pot was 87,000 chips. When the 7♠ was revealed on the river, Hellmuth's equity increased from 97.62% × 87,000 = 84,929.4 to 100% × 87,000, for an increase of 2070.6 chips due to luck. Hellmuth's expected profit gained due to skill on the river is simply 37,000 chips: the pot size increased by 74,000 while Hellmuth had a 100% chance of winning, but the cost to Hellmuth was 37,000, so his profit was 37,000.

Example 3 shows what one might consider a problem with defining skill and luck in terms of changes in expected profit or equity. Clearly Hellmuth got extremely lucky. The analysis here attributes 50,992.2 + 1231.2 + 2070.6 = 54,294 of his profits to luck. However, it also credits Hellmuth with 14,286 + 37,000 = 51,286 chips in profit due to skill. Luck and skill as defined here will tend to be correlated: players who are lucky enough to get better cards than their opponents will typically bet when they are ahead and thus gain in skill as well.

The extended example 4 below is intended to illustrate the division of luck and skill in a game of Texas Hold'em. It took place at the end of a tournament on Poker After Dark televised on NBC in October 2009. Dario Minieri and Howard Lederer were the final two players. Since this portion of the tournament involved only these two players, and since most of the hands were televised, this example allows one to parse out how much of Lederer's win was due to skill and how much to luck.

Technical note: Before we begin Example 4, we must clarify a few potential ambiguities. There is some ambiguity in the definition of expected profit before the flop, since the small and big blind put in different numbers of chips. The definition used here is the equity a player would have in the pot after calling minus cost, assuming the big blind and small blind call as well, or the (negative) profit a player would have by folding, whichever is greater. For example, in heads-up Texas Hold'em with blinds of 800 and 1600, the pre-flop expected profit for the big blind is 3200p – 1600, and max{3200p – 1600, –800} for the small blind, where p is the probability of the big blind winning the pot in a showdown. It makes sense to define increases in the size of the pot as relative to the big blind, i.e. increasing the pot size by calling preflop does not count as skill. The probability p of winning the hand in a showdown was obtained using the odds calculator at cardplayer.com, and the probability of a tie is divided equally between the two players in determining p.

Example 4. Table 1 summarizes all 27 hands shown on Poker After Dark in October 2009 for Dario Minieri and Howard Lederer in the heads-up segment of the tournament, with each hand's gains and losses in expected profit categorized as luck or skill. Each hand is analyzed from Minieri's perspective, i.e. a gain of -100 in skill for Minieri means Lederer gained 100 chips in expected profit during the betting rounds. The question we seek to address is how much of Lederer's win was due to skill and how much of it was due to luck?

Answer—Consider first a detailed breakdown of hand 4 in which the blinds were 800 and 1600, Minieri was dealt A♣ J♣, Lederer had A♥ 9♥, Minieri raised to 4300 and Lederer called. The flop was 6♣ 10♠ 10♣, Lederer checked, Minieri bet 6500, and

Lederer folded.

(a) Pre-flop dealing (luck): Minieri +642.08. Minieri was dealt a 70.065% probability of winning the pot in a showdown so his increase in expected profit is 70.065% × 3200 – 1600 = 642.08 in chips. Lederer was dealt a 29.935% probability to win the pot in a showdown, so his increase in expected profit is 29.935% × 3200 – 1600 = –642.08.

(b) Pre-flop betting (skill): Minieri +1083.51. The pot was increased to 8600. Since 8600 – 3200 = 5400, Minieri had 70.065% × 5400 = 3783.51 additional equity but paid an additional 2700, so his expected profit due to betting was 3783.51 – 2700 = 1083.51. Correspondingly, Lederer's expected profit due to betting was –1083.51 since 29.935% × 5400 – 2700 = –1083.51.

(c) Flop dealing (luck): Minieri +1362.67. After the flop was dealt, Minieri's probability of winning the 8600-chip pot in a showdown increased from 70.065% to 85.91%. Because of luck, he increased his equity by (85.91% – 70.065%) × 8600 = 1362.67 chips.

(d) Flop betting (skill): Minieri +1211.74. Because of betting on the flop, Minieri's equity went from 85.91% of the 8600 chip pot to 100% of the pot so he increased his equity by (100% – 85.91%) × 8600 = 1211.74 chips.

Hand	Minieri's cards	Lederer's cards	Betting actions	Minieri's luck gain	Minieri's skill gain
1	6♠ 6♦	A♣ 7♠	Blinds are 800/1600. L (43.535%) raises from 1600 to 4300, M raises to 47800, L folds.	206.88	4093.12
2	4♠ 2♦	K♠ 7♥	M (34.36%) raises to 4300, L raises all-in for 43500, M folds.	-500.48	-3799.52
3	A♦ 9♣	6♥ 3♦	L (34.965%) folds.	481.12	318.88
4	A♣ J♣	A♥ 9♥	M (70.065%) raises to 4300, L calls 2700. Flop 6♣ 10♠ 10♣. L (14.09%) checks, Minieri bets 6500, Lederer folds.	2004.75	2295.25
5	7♦ 6♠	5♠ 3♥	L (35.765%) folds.	455.52	344.48
6	K♥ 10♣	5♦ 2♦	M (61.41%) raises to 3200, L raises to 9700, M folds.	365.12	3565.12
7	10♦ 7♠	Q♣ 2♥	M (43.57%) raises to 3200, L calls 1600. Flop 8♠ 2♠ Q♥. L checks, M (7.27%) bets 3200, L calls. Turn 4♦. L (100%) checks, M	-3459.52	-12940.48

			bets 10000, Lederer calls. River A♥. L checks, M checks.		
8	9♣ 4♦	7♣ 2♦	L (35.72%) folds.	456.96	343.04
9	4♠ 2♣	8♥ 7♦	M (34.345%) raises to 3200, L calls 1600. Flop 3♦ 9♥ J♥. L (77.975%) checks, M bets 4800, L folds.	1289.44	4489.44
10	K♥ 7♣	K♠ 5♠	L (40.85%) calls 1600, M raises to 6400, L folds.	292.80	1307.20
11	A♥ 8♥	6♥ 3♠	M (66.85%) raises to 3200, L folds.	539.20	1060.80
12	7♦ 3♥	A♦ 4♦	L (65.345%) raises to 4300, M raises to 11500, L folds.	-491.04	4791.04
13	6♣ 3♣	K♠ 6♠	M (29.825%) raises to 4800, L calls. Flop 5♥ J♣ 5♣. L (52.575%) checks, M bets 6000, L folds.	1044.00	3756.00
14	8♦ 5♦	7♦ 5♠	L (30.56%) calls, M checks. Flop K♥ 10♠ 8♣. M (94.395%) checks, L bets 1800, M calls. Turn 7♠. M (95.45%) checks, L checks. River 6♥. Both check.	1801.78	1598.22
15	9♦ 5♠	A♥ 5♦	Blinds are now 1000/2000. M (26.755%) calls 1000, L raises to 7000, M raises to 14000, L calls. Flop 10♠ Q♦ 6♥. L (84.65%) checks, M bets 14,000, L folds.	-4123.20	18123.20
16	A♣ J♦	5♠ 5♥	L (53.915%) calls 1000, M raises all in for 26,800, L calls. The board is 3♠ 9♠ K♠ 10♦ 9♦.	-24858.16	-1941.84
17	K♣ 10♣	7♦ 5♦	M (62.22%) raises to 5000, L calls 3000. Flop J♠ J♦ 4♠. L (30.1%) checks, L checks. Turn 8♠. L (22.73%) bets 6000, M folds.	1993.80	-6993.80
18	10♠ 6♥	5♠ 5♣	L (53.88%) calls 1000, M checks. Flop 7♣ 8♣ Q♥. M (38.235%) checks, L bets 2000, M calls. Turn J♥. M (22.73%) bets 4000, L folds.	-1711.00	5711.00
19	K♣ 10♦	K♥ 5♠	L (26.825%) raises to 5000, M	3154.50	1845.50

			calls 3000. Flop J♦ 8♥ 10♥. M (92.575%) checks, L checks. Turn 5♦. M (95.45%) bets 6000, L folds.		
20	7♣ 2♠	Q♠ 9♠	M (30.205%) raises to 6000, L calls 4000. Flop A♦ A♠ Q♦. L (98.835%) checks, M bets 6000, L calls. Turn J♣. L (100%) checks, M bets 14,000, L raises all in for 35,800, M folds.	-556.20	-21443.80
21	10♥ 3♦	Q♥ J♠	M (30.00%) calls 1000, L checks. Flop 8♠ 4♥ J♣ . L (95.66%) checks, M bets 2000, L raises to 7500, M raises to 18,500, L raises all-in, M folds.	-1826.40	-18673.60
22	5♣ 3♥	A♠ 2♦	L (57.655%) calls 1000, M checks. Flop K♠ 10♣ 3♠ . M (80.10%) checks, L checks. Turn Q♠. L (34.09%) bets 2000, M folds.	636.40	-2636.40
23	7♥ 7♣	8♦ 3♦	Blinds are now 1500/3000. M (68.175%) goes all in for 21,700, L folds.	1090.50	1909.50
24	Q♥ 5♥	8♦ 5♦	M (68.37%) goes all in for 26,200, L folds.	1102.20	1897.80
25	5♦ 2♦	9♣ 3♣	L (59.37%) folds.	-562.20	2060.20
26	10♣ 2♠	7♣ 7♥	M (29.04%) folds.	-1257.60	-242.40
27	A♣ 5♠	Q♣ 9♣	L (44.63%) goes all in for 29,200, M calls. Board is 7♣ 6♣ 10♠ Q♠ 6♦.	-32013.88	2813.88
Total				-61023.59	-13478.41

Table 1: Quantification of luck and skill for the 27 hands played between Dario Minieri and Howard Lederer on Poker After Dark, NBC, Oct 2009.

Overall, as seen from the last row of Table 1, although Lederer's gains were primarily (about 81.9%) due to luck, Lederer also gained more expected profit due to skill than Minieri. On the first 19 hands, Minieri actually gained 20,836.41 in expected profit due to skill and appeared to be outplaying Lederer quite substantially. On hands 20 and 21, however, Minieri tried two huge unsuccessful bluffs, both on hands (especially hand 20) where he should probably have strongly suspected that Lederer would be likely to call. On those two hands combined, Minieri lost 40,117.40 in expected profit due to skill. Although Minieri played very well on every other hand, all those good plays could not overcome the huge loss of expected profit due to skill in hands 20 and 21.

Example 4 shows that, though luck and skill tend to be correlated as previously mentioned, the player who gains the most expected profit due to skill does not always win. In the first 19 hands of this example, for instance, Minieri gained 20,836.41 chips in expected profit attributed to skill, but because of bad luck, he lost a total of 2800 chips over these 19 hands. The bad luck Minieri suffered on hand 16 negated most of his gains due to skillful play.

Summary. The definitions proposed here for luck and skill in poker can be used to quantify precisely how much of one's winnings in a given poker hand, session, or tournament are attributable to luck and how much are attributable to skill. In principle, if one were to analyze a sufficient number of poker sessions, one could perhaps estimate the contribution of each of these components to poker in general, to get a sense of whether poker is indeed primarily a game of chance or a game of skill. That is, one could use this method to assess, for a typical game of poker, what proportion of the wins and losses are due to luck and what proportion are due to skill.

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