Quantification of Luck and Skill in Texas Hold'em
Frederic Paik Schoenberg

Introduction. The determination of whether poker is primarily a game of luck or skill has been the subject of intense legal debate. For instance, in 2007 an English court ruled that poker is primarily a game of luck in finding the owner of the Gutshot Club in central London guilty of violating the Gaming Act, which requires a license to host games of chance but not games of skill (Cacciotollo 2007). On the other hand, in 2009 the organizer of a poker club in Colorado was found not guilty of illegal gambling on games of chance in a trial where the jury agreed with a statistician's testimony contending that poker is a game of skill (Hannum and Cabot 2009).

The terms luck and skill are difficult to define, and rigorous definitions of these terms seldom appear in books and journal articles on game theory. A few articles have defined skill in terms of the variance in results among different players, with the idea that players should perform more similarly if a game is mostly based on luck, but their results may differ more substantially if a game is based on skill (Potter van Loon et al., 2015). Another definition of skill is the extent to which players can improve; Dedonno and Detterman (2008), for instance, argued that poker is a game of skill by showing that participants who were given strategic instruction outperformed those who were given no instruction. This article offers a different take on the definitions of luck and skill in poker, and highlights certain scenarios, involving real poker hands, in which the luck and skill components can readily be quantified.

The focus here will be on the game Texas Hold'em, the most popular version of poker played today. We will begin with an example illustrating basic rules, concepts and terminology surrounding the game. For further details the reader is invited to read Sklansky (1989), Brunson and Addington (2002), Hellmuth (2003), Harrington and Robertie (2004), Chen and Ankenmann (2006), Gordon (2006), or Schoenberg (2017).

Example 1. It is day 4 of the World Series of Poker Main Event in 2015, and eight players are seated at the table in question. At this stage of the tournament, the *blinds* are 5,000/10,000 and the *ante* is 1,000, which means before the cards were even dealt, one player called the *small blind* must bet 5,000 chips, the next player, called the *big blind*, must bet 10,000 chips, and in addition all 8 players must put 1,000 in chips into the pot. The dealer then deals two cards face down to each player. Each player acts in sequence, and can either *call*, which means matching the largest bet (currently 10,000), *raise*, which means betting more than the current bet, or *fold*.

In this hand, Ryan D'Angelo raises to 22,000 with A♦ K♠, five players fold, Daniel Negreanu, the small blind, calls with A♠ 7♠ and Fernando Perez calls from the big blind with 3♥ 2♥. The pot now totals 74,000 chips and the first betting round is complete.

At this point, the dealer places 3 community cards, called the *flop,* face up on the board.

The community cards can be used by any of the players, along with their own two cards, to form the best possible 5-card poker hand. In this hand, the flop is 3♠ 10♣ 9♠. Now there is another betting round. All three players *check*, which means betting 0 chips.

Next the dealer reveals another community card, called the *turn,* and in this hand it is 2♠. At this point Negreanu has a *flush*, meaning 5 cards of the same suit, because his cards are both spades (A♠ 7♠) and there are three more spades on the board (3♠, 9♠, and 2♠). Perez meanwhile has two pairs. After the turn is revealed there is another betting round*,* and Negreanu begins it by betting 35,000 chips. Perez raises to 105,000, D'Angelo folds, Negreanu reraises to 250,000, and Perez calls. The pot now totals 574,000 chips.

The dealer now places the fifth and final community card, called the *river*, on the board, and it is 5♣. There is then a final betting round. Negreanu goes *all-in* (bets all his 359,000 chips), and Perez folds. Negreanu collects the entire pot of 933,000 chips and the hand is over. Had Perez called, both players would *showdown* (reveal) their cards, and the player with the better 5-card hand, which in this case is Negreanu, would win the pot. In the event of a tie, the two players split the pot.

In this hand, Negreanu profits 301,000 chips. At any interim point during the hand, one may consider what Negreanu's *expected* profit is at that point, and assess how much this expected profit changes with each action. In particular one might ask: how much expected profit does Negreanu gain during the turn betting round?

Answer. To make the question well posed, let us define Negreanu's *equity* in the pot as the expected amount he would win assuming no further betting or folding, and assuming only knowledge of the cards previously mentioned or revealed in the hand. For example, after 2♠ is revealed as the turn, the pot consists of 74,000 chips, and assuming no folding, the probability of Negreanu winning this pot = the probability of the river being anything other than 2♣, 2♦, 3♣, or 3♦ = 1 - 4/42, because 10 of the 52 cards have either been revealed on the board or were in the hands of the three players, leaving 42 remaining cards equally likely to appear on the river. If the river were a 2 or 3 then Perez would have a *full house* (e.g. 3♠ 3♥ 3♣ 2♥ 2♠) which would defeat Negreanu's flush. Thus Negreanu's equity when the turn is revealed is (1 - 4/42) x 74,000 ~ 67,000 chips. With equity thus defined, Negreanu's expected profit from an action occurring during a poker hand is naturally defined as his increase in equity minus the cost Negreanu incurs from this action.

After the turn is revealed, the probability that Negreanu will win, assuming nobody folds, is the probability that a 3 or 2 will not come on the river, which is 38/42. During the betting on the turn, the pot increased from 74,000 chips to 574,000 chips. Thus, Negreanu's expected share of the pot increased from 38/42 × 74,000 ~ 67,000 to 38/42 × 574,000 ~ 520,000, for an increase of about 453,000 chips. The cost to Negreanu on the turn was 250,000 chips, so his increase in expected profit during the betting on the turn was 453,000 - 250,000 = 203,000 chips.

Before considering the definition of luck and skill used here, note that the alternative definitions referred to in the introduction, in terms of variation or improvement among players, are extremely problematic for various reasons. The definitions seem somewhat arbitrary and only very loosely tied to one's conceptions of luck and skill, and it is easy to think of counterexamples and major flaws in the definitions especially when considering their application to other games. There are many contests of skill wherein the differences between players are small, or where one's results vary wildly. For instance, in Olympic trials of the 100-meter sprints, the differences between finishers are typically quite small, often just hundredths of a second. This hardly implies that the results are based on luck. In other sporting events, for example pitching in baseball, an individual's results may vary widely from one day to another, but that does not mean luck plays a major role. Some players might not be able to improve beyond a certain point in chess, but this does not render chess a game of luck.

Proposed definition. To quantify the amount of luck or skill in a particular game of poker, one possibility is to define luck as expected profit gained when cards are dealt by the dealer, and skill as expected profit gained by a player's actions during betting rounds. A player might gain expected profit during a hand by several actions:

* The cards dealt by the dealer give the player in question a greater chance of winning a hand in a showdown, thus increasing her equity in the pot.

* The size of the pot is increased while the player in question's chance to win the hand in a showdown is better than those of her opponents.

* By betting, the player gets others to fold and thus increases her probability of winning the pot.

It seems natural to classify the first case as luck and the second and third cases as skill. That is, we may view skill as the expected profit gained during the betting rounds, and luck as the expected profit gained simply by dealing the cards. Both are easily quantifiable, and one may dissect a particular poker game and analyze how much expected profit each player gained due to luck or skill.

There are some problems with these proposed definitions. First, situations can occur where a terrible player may gain expected profit during betting rounds against even the greatest player in the world and attributing such gains to skill may be objectionable. For instance, if there are two players and one is dealt a pair of aces and the other a pair of kings, one would expect the player with the kings to put a great number of chips in while way behind, expecting that they had a decent shot of winning. This situation seems more like bad luck for the player than a deficit in skill. However, virtually any definition of skill can be objected to on such a basis. One might argue that skill is too strong a word, and that when analyzing hands, one should perhaps instead refer to expected profit gained *during betting rounds* rather than expected profit gained *due to skill*. The word skill will nevertheless be used in what follows.

Another issue with the definitions proposed here is that luck and skill will often be correlated in practice. This is explored further in the following example.

Example 2. In a hand from the 2015 WSOP Main Event, Mike Cloud raised to 15,000 with A♣ A♠, Hellmuth called with A♥ K♠, Daniel Negreanu called from the big blind with 6♦ 4♥, and the flop came K♣ 8♥ K♥. Before the flop, the pot was 57,000 chips, and the probabilities shown on ESPN's broadcast of winning the hand in a showdown at this point were 74% for Cloud, 19% for Negreanu, and only 6% for Hellmuth. (The probabilities only add up to 99% because of an approximately 1% chance of a split pot.) After the flop, all three players checked, the turn was the J♥, Negreanu checked, Cloud bet 15,000, Hellmuth called, and Negreanu folded. The river was the 7♠, Cloud checked, Hellmuth bet 37,000, and Cloud called. How much expected profit did Hellmuth gain due to luck and how much due to skill (a) on the flop, (b) on the turn, and (c) on the river?

Answer. (a) Before the flop was revealed, Hellmuth's equity was 6% × 57,000 = 3,420 chips. After the flop was dealt, the only way Hellmuth could have lost in a showdown would have been if the turn or river contained the A♦ without the K♦, which, given the six cards belonging to the players and the three cards on the flop, had a probability of (1 × 41) / C(43,2) = 4.54%, so Hellmuth's equity suddenly increased to 95.46% × 57,000 = 54,412.2 chips. Thus on the flop Hellmuth gained 54,412.2 – 3420 = 50,992.2 chips in equity due to luck. There was no betting on the flop so Hellmuth gained 0 expected profit due to skill on the flop.

(b) When the turn was dealt, Hellmuth's probability of winning in a showdown increased to 41/42 ~ 97.62%, so his equity increased from 54,412.2 to 97.62% × 57,000 = 55,643.4, for an increase in expected profit of 1,231.2 due to luck on the turn.

During the betting on the turn, Hellmuth and Cloud each put 15,000 chips in the pot, so Hellmuth's expected return increased by 97.62% × 30,000 = 29,286 chips, but he put 15,000 chips into the pot on the turn, so his expected profit on the turn due to skill was 29,286 – 15,000 = 14,286 chips.

(c) After the betting on the turn was over, the pot contained 87,000 chips. When the 7♠ was revealed on the river, Hellmuth's equity increased from 97.62% × 87,000 = 84,929.4 to 100% × 87,000, for an increase of 2070.6 chips due to luck. Hellmuth's expected profit gained due to skill on the river is simply 37,000 chips: the pot size increased by 74,000 while Hellmuth had a 100% chance of winning, but the cost to Hellmuth was 37,000, so his profit was 37,000.

Example 2 shows what one might consider a problem with defining skill and luck in terms of changes in expected profit. Clearly Hellmuth got extremely lucky. The analysis here attributes 50,992.2 + 1231.2 + 2070.6 = 54,294 of his profits to luck. However, it also credits Hellmuth with 14,286 + 37,000 = 51,286 chips in profit due to skill. Luck and skill as defined here will tend to be correlated: players who are lucky enough to get better cards than their opponents will typically bet when they are ahead and thus gain in skill as well.

In the above examples, we have calculated equity in a pot assuming no future betting. However, the assumption of no future betting may seem absurdly simplistic and unrealistic in some cases. Unlike implied equity, which accounts for betting on future betting rounds, ordinary equity is unambiguously defined and easy to compute, but admittedly does have its shortcomings, as shown in the following example.

(counter) Example 3. This hand from Season 7 of High Stakes Poker illustrates some of the limitations of making inferences based on equity, where one assumes no future betting (or folding) in calculating the expected winnings for each player. With eight players at the table, after Phil Galfond raised to $3500 with Q♠ 10♥, the next five players folded, Robert Croak called with A♣ J♣, and Bill Klein called with 10♠ 6♠. The flop came J♠ 9♥ 2♠, Croak bet $5500, Klein raised to $17,500, and Galfond and Croak called. At this point, it is tempting to compute Klein's probability of winning the hand by computing the probability of exactly one more spade coming on the turn and river without making a full house for Croak, or the turn and river including two 6s, or a 10 and a 6. Counting combinations, and using the notation C(n,k) = n! / [k! (n-k)!] to represent the number of distinct combinations of k different items out of n different possibilities, this would yield a probability of [(8 × 35 – 4 – 4) + C(3,2) + 2 × 3] ÷ C(43,2) = 281/903 ~ 31.12%. Klein could also split the pot with a straight if the turn and river were KQ or Q8 without a spade, which has a probability of [3 × 3 + 3 × 3] ÷ C(43,2) = 18/903 ~ 1.99%. These seem to be the combinations Klein needs, and one would not expect Klein to win the pot with a random turn and river combination not on this list, and especially not if the turn and river contain a king or a jack with no spades. However, look at what actually happened. The turn was K♣, giving Galfond a straight, and Croak checked. Klein bet $28,000, Galfond raised to $67,000, Croak folded, and Klein called. The river was J♥, Klein bluffed $150,000, and Galfond folded, giving Klein the $348,200 pot!

Returning to the proposed definitions of luck and skill as expected profit gained during the dealing of the cards and expected profit gained during the betting rounds, respectively, we may consider some collections of poker hands in order to see how luck and skill vary in these cases.

Example 4. Table 1 summarizes a selection of the 27 hands from the end of a tournament on Poker After Dark televised on NBC in October 2009, involving Dario Minieri and Howard Lederer. This portion of the tournament involved only these two players and all of the hands were televised. (For a full description of all 27 hands, see Example 4.4.3 of Schoenberg 2017, which includes technical details on how expected profit is defined before the flop.) How much of Lederer's win was due to skill and how much of it was due to luck?

Answer. Overall, as seen from the 2nd to last row of Table 1, although Lederer's gains were primarily (80.9%) due to luck, Lederer also gained more expected profit due to skill than Minieri. On the first 19 hands, Minieri actually gained 20,836.41 in expected profit due to skill and appeared to be outplaying Lederer quite substantially. On hands 20 and 21, however, Minieri tried two huge unsuccessful bluffs, both on hands (especially hand 20) where he should probably have strongly suspected that Lederer would be likely to call. On those two hands combined, Minieri lost 40,117.40 in expected profit due to skill. Minieri's profitable plays on the first 19 hands could not overcome the huge loss of expected profit due to skill in hands 20 and 21.

Example 4 also shows that, though luck and skill tend to be correlated, the player who gains the most expected profit due to skill does not always win. In the first 19 hands of this example, for instance, Minieri gained 20,836.41 chips in expected profit attributed to skill, but because of bad luck, he lost a total of 2800 chips over these 19 hands. The bad luck Minieri suffered on hand 16 negated most of his gains due to skillful play.

The bottom row of Table 1 shows the proportion of variation (PV) in profits attributable to luck or skill, respectively. The 27 hands between Minieri and Lederer yield an estimate of 52.73% for the percentage of variation due to luck, and 47.27% for the percentage of variation due to skill. One might view these percentages as reflecting the contributions of luck and skill to the game of poker, though further study should be done to see if these proportions are stable across different poker players and for different tournaments.

References.

Brunson, D. and Addington, C. (2002). *Doyle Brunson's Super System: A Course in Power Poker*, 3rd ed. Cardoza, New York.

Cacciottolo, M. (2007). Does luck or skill make poker ace? *BBC News*, Jan 16 http://news.bbc.co.uk/2/hi/uk_news/magazine/6264387.stm .

Chen, W., and Ankenmann, J. (2006). *The Mathematics of Poker*. ConJelCo, Pittsburgh.

Dedonno, M.A. and Detterman, D.K. (2008). Poker is a skill. *Gaming Law Rev.* **12**, 31-36.

Gordon, P. (2006). *Phil Gordon's Little Blue Book: More Lessons and Hand

Analysis in No Limit Texas Hold'em. Simon & Schuster, New York.
Hannum, R.C. and Cabot, A.N. (2009). Toward legalization of poker: the skill vs. chance debate. *UNLV Gaming Research and Review Journal* **13**(1), 1-20.
Harrington, D. and Robertie, B. 2004. *Harrington on Hold'em: Expert Strategy for No Limit Tournaments.* Vol. 1: *Strategic Play.* Two Plus Two Publishing, Henderson, NV.
Hellmuth, P. (2003). *Play Poker Like the Pros.* HarperCollins, New York.
Potter van Loon, R.J.D., van den Assem, M.J., and van Dolder, D. (2015). Beyond Chance? The Persistence of Performance in Online Poker. *PLoS One* 10(3), e0115479.
Schoenberg, F. (2017). *An Introduction to Probability with Texas Hold'em Examples, 2nd ed.* CRC Press, Boca Raton.
Sklansky, D. (1989). *Theory of Poker*, 3rd ed. Two Plus Two Publishing, Henderson, NV.

Table 1.

Hand	Minieri's cards	Lederer's cards	Betting actions	Minieri's luck gain	Minieri's skill gain
1	6♠ 6♦	A♣ 7♠	Blinds are 800/1600. L (43.535%) raises from 1600 to 4300, M raises to 47800, L folds.	206.88	4093.12
12	7♦ 3♥	A♦ 4♦	L (65.345%) raises to 4300, M raises to 11500, L folds.	-491.04	4791.04
15	9♦ 5♠	A♥ 5♦	Blinds are now 1000/2000. M (26.755%) calls 1000, L raises to 7000, M raises to 14000, L calls. Flop 10♠ Q♦ 6♥. L (84.65%) checks, M bets 14,000, L folds.	-4123.20	18123.20
16	A♣ J♦	5♠ 5♥	L (53.915%) calls 1000, M raises all in for 26,800, L calls. The board is 3♠ 9♠ K♠ 10♦ 9♦.	-24858.16	-1941.84
18	10♠ 6♥	5♠ 5♣	L (53.88%) calls 1000, M checks. Flop 7♣ 8♣ Q♥. M (38.235%) checks, L bets 2000, M calls. Turn J♥. M (22.73%) bets 4000, L folds.	-1711.00	5711.00
20	7♣ 2♠	Q♠ 9♠	M (30.205%) raises to 6000, L calls 4000. Flop A♦ A♠ Q♦. L (98.835%) checks, M bets 6000, L calls. Turn J♣. L (100%) checks, M bets 14,000, L raises	-556.20	-21443.80

			all in for 35,800, M folds.		
21	10♥ 3♦	Q♥ J♠	M (30.00%) calls 1000, L checks. Flop 8♠ 4♥ J♣ . L (95.66%) checks, M bets 2000, L raises to 7500, M raises to 18,500, L raises all-in, M folds.	-1826.40	-18673.60
27	A♣ 5♠	Q♣ 9♣	L (44.63%) goes all in for 29,200, M calls. Board is 7♣ 6♣ 10♠ Q♠ 6♦.	-32013.88	2813.88
Total				-61023.59	-13478.41
PV				52.73%	47.27%

Table 1: Quantification of luck and skill for some of the 27 hands played between Dario Minieri (M) and Howard Lederer (L) on Poker After Dark, NBC, Oct 2009. Percentages indicate probability of winning the hand in a showdown and were calculated using the online calculator at cardplayer.com. Probabilities corresponding to ties are split evenly between both players. Each hand is assessed from Minieri's perspective, i.e. a skill gain of -1948.84 for Minieri means Lederer gained 1948.84 chips in expected profit during the betting rounds. The total and PV (Proportion of Variation) in the bottom two rows are for all 27 hands. PV_{luck} = sum of squared luck gains / (sum of squared luck gains + sum of squared skill gains).