

Quantification of Luck and Skill in Texas Hold'em

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1. Introduction. Las Vegas, Nevada, July 11, 2015. It is day 4 of the 2015 World Series of Poker Main Event. The tournament began with 6,420 players paying \$10,000 apiece to enter the tournament. Now only about 400 remain. Some are legendary poker professionals, while others are amateurs taking a shot at poker glory. The tension and drama increase each hand, as millions of dollars and the title of World Champion hang in the balance with each card dealt and each poker decision made. Winning the tournament will clearly require not only great skill but also a huge amount of luck as well.

The determination of whether poker is primarily a game of luck or skill has been the subject of intense legal debate. For instance, in 2007 an English court ruled that poker is primarily a game of luck in finding the owner of the Gutshot Club in central London guilty of violating the Gaming Act, which requires a license to host games of chance but not games of skill (Cacciottolo 2007). On the other hand, in 2009 the organizer of a poker club in Colorado was found not guilty of illegal gambling on games of chance in a trial where the jury agreed with a statistician's testimony contending that poker is a game of skill (Hannum and Cabot 2009). As laws differ around the world regarding the legality of gambling on games of skill or games of luck, a clear definition of whether poker is a game primarily of skill or luck might have broad legal implications.

This article offers a definition of luck and skill in poker, and highlights certain scenarios, involving real poker hands, in which the luck and skill components can readily be quantified. The focus here will be on the game Texas Hold'em, the most popular version of poker played today. We will begin with an example illustrating basic rules, concepts and terminology surrounding the game. For further details the reader is invited to read Sklansky (1989), Brunson and Addington (2002), Hellmuth (2003), Harrington and Robertie (2004), Chen and Ankenmann (2006), Gordon (2006), or Schoenberg (2017).

2. Background. Before giving the proposed definitions of luck and skill, we begin with a description of a sample hand illustrating the game and basic terminology of Texas Hold'em.

Continuing with the scene set forth in the introduction, at this stage of the tournament, the *blinds* are 5,000/10,000 and the *ante* is 1,000, which means before the cards were even dealt, one player called the *small blind* must bet 5,000 chips, the next player, called the *big blind*, must bet 10,000 chips, and in addition all 8 players must put 1,000 in chips into the pot. The dealer then deals two cards face down to each player. Each player acts in sequence, and can either *call*, which means matching the largest bet (currently 10,000), *raise*, which means betting more than the current bet, or *fold*.

In this hand, Ryan D'Angelo raises to 22,000 with A♦ K♠, five players fold, Daniel

Negreanu, the small blind, calls with A♠ 7♠ and Fernando Perez calls from the big blind with 3♥ 2♥. The pot now totals 74,000 chips and the first betting round is complete.

At this point, the dealer places 3 community cards, called the *flop*, face up on the board. The community cards can be used by any of the players, along with their own two cards, to form the best possible 5-card poker hand. In this hand, the flop is 3♠ 10♣ 9♠. Now there is another betting round. All three players *check*, which means betting 0 chips.

Next the dealer reveals another community card, called the *turn*, and in this hand it is 2♠. At this point Negreanu has a *flush*, meaning 5 cards of the same suit, because his cards are both spades (A♠ 7♠) and there are three more spades on the board (3♠, 9♠, and 2♠). Perez meanwhile has two pairs. After the turn is revealed there is another betting round, and Negreanu begins it by betting 35,000 chips. Perez raises to 105,000, D'Angelo folds, Negreanu reraises to 250,000, and Perez calls. The pot now totals 574,000 chips.

The dealer now places the fifth and final community card, called the *river*, on the board, and it is 5♣. There is then a final betting round. Negreanu goes *all-in* (bets all his 359,000 chips), and Perez folds. Negreanu collects the entire pot of 933,000 chips and the hand is over. Had Perez called, both players would *showdown* (reveal) their cards, and the player with the better 5-card hand, which in this case is Negreanu, would win the pot. In the event of a tie, the two players split the pot.

In order to address the question of whether Negreanu's profits should primarily be attributed to luck or skill, one needs first to define these concepts, and in order to define them we will rely on the key poker concepts of *equity* and *expected profit*. *Equity* is one's expected portion of the pot assuming no future betting, and assuming only knowledge of the cards previously mentioned or revealed in the hand. For instance, Negreanu's *equity* on the turn is the expected value of X , where X is the amount Negreanu would win assuming no further betting or folding, after the 2♠ is revealed as the turn. At this point the pot consists of 74,000 chips, and assuming no further betting, X will either be 74,000 or zero, depending upon whether Negreanu wins or loses the hand, so Negreanu's equity will simply be 74,000 times the probability of Negreanu winning the hand, assuming no player will fold. (In some cases X could also equal some fraction of the pot, if ties are possible.) A player's *expected profit* during a portion of a poker hand is her increase in equity minus the cost she incurs during this portion of the hand.

3. Prior attempts at defining luck and skill. The terms luck and skill are difficult to define, and rigorous definitions of these terms seldom appear in books and journal articles on game theory. A few articles have defined skill in terms of the variance in results among different players, with the idea that players should perform more similarly if a game is mostly based on luck, but their results may differ more substantially if a

game is based on skill (Potter van Loon et al., 2015). Another definition of skill is the extent to which players can improve; Dedonno and Detterman (2008), for instance, argued that poker is a game of skill by showing that participants who were given strategic instruction outperformed those who were given no instruction.

Before considering the definition of luck and skill used here, note that the alternative definitions referred to in the introduction, in terms of variation or improvement among players, are extremely problematic for various reasons. The definitions seem somewhat arbitrary and only very loosely tied to one's conceptions of luck and skill, and it is easy to think of counterexamples and major flaws in the definitions especially when considering their application to other games. There are many contests of skill wherein the differences between players are small, or where one's results vary wildly. For instance, in Olympic trials of the 100-meter sprints, the differences between finishers are typically quite small, often just hundredths of a second. This hardly implies that the results are based on luck. In other sporting events, for example pitching in baseball, an individual's results may vary widely from one day to another, but that does not mean luck plays a major role. Some players might not be able to improve beyond a certain point in chess, but this does not render chess a game of luck.

4. Proposed definitions. To quantify the amount of luck or skill in a particular game of poker, one possibility is to define luck as expected profit gained when cards are dealt by the dealer, and skill as expected profit gained by a player's actions during betting rounds. A player might gain expected profit during a hand by several actions:

- * The cards dealt by the dealer give the player a greater chance of winning a hand in a showdown, thus increasing her equity in the pot.
- * The size of the pot is increased while the player's chance to win the hand in a showdown is better than those of her opponents.
- * By betting, the player gets others to fold and thus increases her probability of winning the pot.

It seems natural to classify the first case as luck and the second and third cases as skill. That is, *we define skill as the expected profit gained during the betting rounds, and luck as the expected profit gained simply by dealing the cards.* Both are easily quantifiable, and one may dissect a particular poker game and analyze how much expected profit each player gained due to luck or skill.

Example 1. Continuing with the example hand described in Section 2, where Negreanu profits 301,000 chips in the hand, how much expected profit does Negreanu gain due to skill during the turn betting round?

Answer. After the 2♠ is revealed as the turn, the pot consists of 74,000 chips, and

assuming no folding, the probability of Negreanu winning this pot is the probability of the river being anything other than $2\clubsuit$, $2\spadesuit$, $3\clubsuit$, or $3\spadesuit = 1 - 4/42$, because 10 of the 52 cards have either been revealed on the board or were in the hands of the three players, leaving 42 remaining cards equally likely to appear on the river. If the river were a 2 or 3 then Perez would have a *full house* (e.g. $3\clubsuit 3\heartsuit 3\clubsuit 2\heartsuit 2\spadesuit$) which would defeat Negreanu's flush. Thus Negreanu's equity when the turn is revealed is $(1 - 4/42) \times 74,000 \sim 67,000$ chips.

After the turn is revealed, the probability that Negreanu will win, assuming nobody folds, is the probability that a 3 or 2 will not come on the river, which is $38/42$. During the betting on the turn, the pot increased from 74,000 chips to 574,000 chips. Thus, Negreanu's expected share of the pot increased from $38/42 \times 74,000 \sim 67,000$ to $38/42 \times 574,000 \sim 520,000$, for an increase of approximately 453,000 chips. The cost to Negreanu on the turn was 250,000 chips, so his increase in expected profit during the betting on the turn was $453,000 - 250,000 = 203,000$ chips.

Example 2. Table 1 summarizes a selection of the 27 hands from the end of a tournament on *Poker After Dark* televised on NBC in October 2009, involving Dario Minieri and Howard Lederer. This portion of the tournament involved only these two players and all of the hands were televised. (For a full description of all 27 hands, see Example 4.4.3 of Schoenberg 2017, which includes technical details on how expected profit is defined before the flop.) How much of Lederer's win was due to skill and how much of it was due to luck?

Answer. Overall, as seen from the 2nd to last row of Table 1, although Lederer's gains were primarily (80.9%) due to luck, Lederer also gained more expected profit due to skill than Minieri. On the first 19 hands, Minieri actually gained 20,836.41 in expected profit due to skill and appeared to be outplaying Lederer quite substantially. On hands 20 and 21, however, Minieri tried two huge unsuccessful bluffs, both on hands (especially hand 20) where he should probably have strongly suspected that Lederer would be likely to call. On those two hands combined, Minieri lost 40,117.40 in expected profit due to skill. Minieri's profitable plays on the first 19 hands could not overcome the huge loss of expected profit due to skill in hands 20 and 21.

Example 2 also shows that, though luck and skill tend to be correlated, the player who gains the most expected profit due to skill does not always win. In the first 19 hands of this example, for instance, Minieri gained 20,836.41 chips in expected profit attributed to skill, but because of bad luck, he lost a total of 2800 chips over these 19 hands. The bad luck Minieri suffered on hand 16 negated most of his gains due to skillful play.

The bottom row of Table 1 shows the proportion of variation (PV) in profits attributable to luck or skill, respectively. The 27 hands between Minieri and Lederer yield an estimate of 52.73% for the percentage of variation due to luck, and 47.27% for the percentage of variation due to skill. One might view these percentages as reflecting the

contributions of luck and skill to the game of poker, though further study should be done to see if these proportions are stable across different poker players and for different tournaments.

5. Drawbacks. There are some problems with these proposed definitions. First, situations can occur where a terrible player may gain expected profit during betting rounds against a better player, and attributing such gains to skill may be objectionable. For instance, if there are two players and one is dealt a pair of aces and the other a pair of kings, one would expect the player with the kings to put a great number of chips in while way behind, expecting that they had a decent shot of winning. This situation seems more like bad luck for the player than a deficit in skill. However, virtually any definition of skill can be objected to on such a basis. One might argue that skill is too strong a word, and that when analyzing hands, one should perhaps instead refer to expected profit gained *during betting rounds* rather than expected profit gained *due to skill*.

Another issue with the definitions proposed here is that luck and skill will often be correlated in practice. This is explored further in the following example.

Example 3. In a hand from the 2015 WSOP Main Event, Mike Cloud raised to 15,000 with $A\clubsuit A\spadesuit$, Phil Hellmuth Jr. called with $A\heartsuit K\spadesuit$, Daniel Negreanu called from the big blind with $6\diamondsuit 4\heartsuit$, and the flop came $K\clubsuit 8\heartsuit K\heartsuit$. Before the flop, the pot was 57,000 chips, and the probabilities shown on ESPN's broadcast of winning the hand in a showdown at this point were 74% for Cloud, 19% for Negreanu, and only 6% for Hellmuth. (The probabilities only add up to 99% because of an approximately 1% chance of a split pot.) After the flop, all three players checked, the turn was the $J\heartsuit$, Negreanu checked, Cloud bet 15,000, Hellmuth called, and Negreanu folded. The river was the $7\spadesuit$, Cloud checked, Hellmuth bet 37,000, and Cloud called. How much expected profit did Hellmuth gain due to luck and how much due to skill (a) on the flop, (b) on the turn, and (c) on the river?

Answer. (a) Before the flop was revealed, Hellmuth's equity was $6\% \times 57,000 = 3,420$ chips. After the flop was dealt, the only way Hellmuth could have lost in a showdown would have been if the turn or river contained the $A\diamondsuit$ without the $K\diamondsuit$, which, given the six cards belonging to the players and the three cards on the flop, had a probability of $(1 \times 41) / C(43,2) = 4.54\%$, using the notation $C(n,k) = n! / [k! (n-k)!]$, so Hellmuth's equity suddenly increased to $95.46\% \times 57,000 = 54,412.2$ chips. Thus on the flop Hellmuth gained $54,412.2 - 3,420 = 50,992.2$ chips in equity due to luck. There was no betting on the flop so Hellmuth gained 0 expected profit due to skill on the flop.

(b) When the turn was dealt, Hellmuth's probability of winning in a showdown increased to $41/42 \sim 97.62\%$, so his equity increased from 54,412.2 to $97.62\% \times 57,000 = 55,643.4$, for an increase in expected profit of 1,231.2 due to luck on the turn.

During the betting on the turn, Hellmuth's expected return increased by $97.62\% \times 30,000 = 29,286$ chips, but he put 15,000 chips into the pot on the turn, so his expected profit on the turn due to skill was $29,286 - 15,000 = 14,286$ chips.

(c) After the betting on the turn was over, the pot contained 87,000 chips. When the $7\spadesuit$ was revealed on the river, Hellmuth's equity increased from $97.62\% \times 87,000 = 84,929.4$ to $100\% \times 87,000$, for an increase of 2070.6 chips due to luck. Hellmuth's expected profit gained due to skill on the river is simply 37,000 chips, since the pot size increased by 74,000 while Hellmuth had a 100% chance of winning, but the cost to Hellmuth was 37,000.

Example 3 shows what one might consider a problem with defining skill and luck in terms of changes in expected profit. Clearly Hellmuth got extremely lucky. The analysis here attributes $50,992.2 + 1231.2 + 2070.6 = 54,294$ of his profits to luck. However, it also credits Hellmuth with $14,286 + 37,000 = 51,286$ chips in profit due to skill. Luck and skill as defined here will tend to be correlated: players who are lucky enough to get better cards than their opponents will typically bet when they are ahead and thus gain in skill as well.

In the above examples, we have calculated equity in a pot assuming no future betting. However, the assumption of no future betting may seem absurdly simplistic and unrealistic in some cases. Unlike implied equity, which accounts for betting on future betting rounds, ordinary equity is unambiguously defined and easy to compute, but admittedly does have its shortcomings, as shown in the following example.

(counter) Example 4. This hand from Season 7 of High Stakes Poker illustrates some of the limitations of making inferences based on equity, where one assumes no future betting or folding in calculating the expected winnings for each player. With eight players at the table, after Phil Galfond raised to \$3500 with $Q\spadesuit 10\heartsuit$, the next five players folded, Robert Croak called with $A\clubsuit J\clubsuit$, and Bill Klein called with $10\spadesuit 6\spadesuit$. The flop came $J\spadesuit 9\heartsuit 2\spadesuit$, Croak bet \$5500, Klein raised to \$17,500, and Galfond and Croak called. At this point, it is tempting to compute Klein's probability of winning the hand by computing the probability of exactly one more spade coming on the turn and river without making a full house for Croak, or the turn and river including two 6s, or a 10 and a 6, yielding a probability of $281/903 \sim 31.12\%$. Klein could also split the pot with a straight if the turn and river were KQ or Q8 without a spade, which has a probability of $18/903 \sim 1.99\%$. These seem to be the combinations Klein needs, and one would not expect Klein to win the pot with a random turn and river combination not on this list, and especially not if the turn and river contain a king or a jack with no spades. However, look at what actually happened. The turn was $K\clubsuit$, giving Galfond a straight, and Croak checked. Klein bet \$28,000, Galfond raised to \$67,000, Croak folded, and Klein called. The river was $J\heartsuit$, Klein bluffed \$150,000, and Galfond folded, giving Klein the \$348,200 pot! This example illustrates how in simple equity calculations the assumption of no future betting or folding can lead to significant overestimates or underestimates of the probability of

winning the hand.

6. Summary. The definitions proposed here for luck and skill in poker can be used to quantify how much of one's winnings in a given poker hand, session, or tournament are attributable to luck and how much are attributable to skill. Further, combining results from many sessions, one may estimate the contribution of luck and skill to poker generally, to get a sense of whether poker is indeed primarily a game of chance or a game of skill. That is, one could use this method to assess, for a typical game of poker, what proportion of the variation in profits are due to luck and what proportion are due to skill. The proposed definitions have limitations and caveats, however, as they rely on the assumption of no future betting or folding and will tend to yield positively correlated estimates of luck and skill.

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Table 1.

Hand	Minieri's cards	Lederer's cards	Betting actions	Minieri's luck gain	Minieri's skill gain
1	6♠ 6♦	A♠ 7♠	L raises from 1600 to 4300, M raises to 47800, L folds.	206.88	4093.12
12	7♦ 3♥	A♦ 4♦	L raises to 4300, M raises to 11500, L folds.	-491.04	4791.04
16	A♠ J♦	5♠ 5♥	L calls 1000, M raises all in for 26,800, L calls. The board is 3♠ 9♠ K♠ 10♦ 9♦.	-24858.16	-1941.84
18	10♠ 6♥	5♠ 5♣	L calls 1000, M checks. Flop 7♣ 8♣ Q♥. M checks, L bets 2000, M calls. Turn J♥. M bets 4000, L folds.	-1711.00	5711.00
20	7♣ 2♠	Q♠ 9♠	M raises to 6000, L calls 4000. Flop A♦ A♠ Q♦. L checks, M bets 6000, L calls. Turn J♠. L checks, M bets 14,000, L raises all in for 35,800, M folds.	-556.20	-21443.80
21	10♥ 3♦	Q♥ J♠	M calls 1000, L checks. Flop 8♠ 4♥ J♠. L checks, M bets 2000, L raises to 7500, M raises to 18,500, L raises all-in, M folds.	-1826.40	-18673.60
27	A♠ 5♠	Q♣ 9♣	L goes all in for 29,200, M calls. Board is 7♣ 6♣ 10♠ Q♠ 6♦.	-32013.88	2813.88

Total				-61023.59	-13478.41
PV				52.73%	47.27%

Table 1: Quantification of luck and skill for some of the 27 hands played between Dario Minieri (M) and Howard Lederer (L) on Poker After Dark, NBC, Oct 2009. Each hand is assessed from Minieri's perspective, i.e. a skill gain of -1948.84 for Minieri means Lederer gained 1948.84 chips in expected profit during the betting rounds. The total and PV (Proportion of Variation) in the bottom two rows are for all 27 hands. $PV_{\text{luck}} = \text{sum of squared luck gains} / (\text{sum of squared luck gains} + \text{sum of squared skill gains})$. Blinds were initially 800/1600 and switched to 1000/2000 in hand 15.