Using blocked fractional factorial designs to construct discrete choice experiments

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Outline

- Discrete choice experiments
- Block fractional factorial designs for discrete choice experiments
- Minimum aberration criteria
- Estimation capacity
- DCE application
- Discussion

Introduction

- The design of a DCE is critical because it determines which attributes' and their interactions are identifiable
- We present an approach for constructing a DCE using blocked fractional factorial designs (BFFDs)
- We consider the minimum aberration (MA) criteria for selecting BFFDs
  - Maximize the number of models with estimable two-factor interactions by minimizing the confounding or aliasing of two-factor interactions

Discrete Choice Experiments

- Method for understanding subjects preferences and their decision-making process
  - Present subjects with various choice sets of two or more options
  - Options consist of several attributes at one or more levels
  - Subjects are shown each choice set in turn and asked which option they prefer
  - The option chosen in each choice set is the most beneficial (utility)

Example DCE: Snack Nutritional Ingredients

- Prevalence of snack consumers in U.S. has progressively increased from 71% to 97% between 1977 and 2006.
- College students are challenged with the freedom to decide what they eat and how much they eat.
- Goal: Which nutritional ingredients influence college students decision making process

Which snack would you choose?

	Snack A	Snack B	Snack C	Snack D
Sugar	Low	Low	High	High
Salt	Low	Low	High	High
Fat	Low	High	Low	High
Calories	Low	High	Low	High
Fiber	Low	High	High	Low
All Natural	Low	High	High	Low

- ○ Snack A
- ○ Snack B
- ○ Snack C
- ○ Snack D
- ○ None

Previous DCE Studies

- The design of a DCE is a critical aspect
- A review of papers in health economics from 2009-2012 noted:
  - 54% of the designs focused on estimating main effects only
  - Only 13% considered main effects plus two-factor interactions
- Interactive effect between two attributes is key to gaining insight into subjects preferences
- Snack ingredients: may not be able to decide on a snack without considering both its sugar and calorie content

Fractional Factorial Design (FFD)

- A FFD with k two-level attributes: 2^{k-p}
  - Treatment defining contrast subgroup: p defining words and their products
- Resolution: length of the shortest word in the treatment defining contrast subgroup
- Let $A_{i,0}$ be the number of words of length i ($i = 1, \ldots, k$) in the treatment defining contrast subgroup
- Consider designs with resolution III or higher: $A_{1,0} = A_{2,0} = 0$
- Treatment wordlength pattern: $W_t = (A_{3,0}, \ldots, A_{k,0})$

Block Fractional Factorial Design (BFFD)

- Two-level BFFD: 2^{k-p} FFD in 2^q blocks with blocks of size 2^{k-p-q}
  - Two defining contrast subgroups: the treatment defining contrast subgroup and the block defining contrast subgroup
- Let $A_{i,1}$ be the number of treatment words of length i that are confounded with a block effect ($A_{1,1} = 0$)
- Block wordlength pattern: $W_b = (A_{2,1}, \ldots, A_{k,1})$
- A main effect or a two-factor interaction is *clear* in a BFFD if it is not aliased with any other main effects or two-factor interactions, or confounded with any block effects

Two-level BFFDs for DCEs with Symmetric Attributes

- We propose the use of BFFDs for constructing DCEs
- Advantage of BFFDs for DCEs is entire aliasing structure of a BFFD is known in advance
  - Hence, know which effects are estimable in the DCE
- Consider a 2^{k-p} FFD in 2^q blocks
  - The number of choice sets in a DCE is 2^q (number of blocks in the BFFD)
  - The number of options in each choice set is 2^{k-p-q} (size of the block)
- Consider designs of at least resolution IV - ensure clear estimation of main effects as well as possible two-factor interactions

Simulation

- MNL model common model for modeling responses and analyzing data from a DCE
- We construct a locally optimal design assuming nominal values for the parameters are available from pilot studies or experts' opinion

Example: Consider a DCE with 5 two-level attributes

- Assume true model with 5 main effects plus 3 two-factor interactions $\mu = 0.5x_A - 0.5x_B + 0.5x_C - 0.5x_D + 0.5x_E + 0.25x_A x_C - 0.25x_A x_D + 0.25x_B x_E$
- Consider three 2^{5-1} FFDs in 2^2 blocks and for each we fit two models:
  1. Main effects only
  2. All main effects and all clear two-factor interactions plus one two-factor interaction from each aliased set not confounded with block

Three BFFDs used in the simulation study

Design	Treatment defining words	Block defining words	W_t	W_b
S1	$I = ABCDE$	$b_1 = AB, b_2 = AC, b_3 = BC$	$(0,0,1)$	(3,3,0,0)
S2	$I = ABCE$	$b_1 = ACD, b_2 = BCD, b_3 = AB$	$(0,1,0)$	(2,4,0,0)
S3	$I = ABE$	$b_1 = AC, b_2 = ABCD, b_3 = BD$	$(1,0,0)$	(2,3,1,0)

Note: $W_t = (A_{3,0}, A_{4,0}, A_{5,0})$ and $W_b = (A_{2,1}, A_{3,1}, A_{4,1}, A_{5,1})$

Main Effects Only Models

Effect	Design S1	Design S2	Design S3
A	0.604 (0.032)	0.772 (0.033)	0.889 (0.041)
B	-0.455 (0.032)	-0.391 (0.033)	-0.567 (0.040)
C	0.509 (0.032)	0.609 (0.029)	0.471 (0.032)
D	-0.557 (0.027)	-0.502 (0.026)	-0.606 (0.028)
E	0.387 (0.026)	0.341 (0.029)	0.512 (0.037)

True Model: $\mu = 0.5x_A - 0.5x_B + 0.5x_C - 0.5x_D + 0.5x_E + 0.25x_Ax_C - 0.25x_Ax_D + 0.25x_Bx_E$

Main Effects Plus Two-Factor Interactions Models

Effect	Design S1	Design S2	Design S3
A	0.506 (0.048)	0.555 (0.044)	0.792 (0.051)
B	-0.524 (0.048)	-0.503 (0.045)	-0.533 (0.051)
C	0.549 (0.048)	0.464 (0.044)	0.497 (0.051)
D	-0.484 (0.048)	-0.435 (0.046)	-0.456 (0.041)
E	0.454 (0.048)	0.482 (0.045)	0.502 (0.051)
AB	–	–	–
AC	–	0.467 (0.045)	–
AD	-0.246 (0.048)	-0.28 (0.038)	-0.262 (0.039)
AE	0.028 (0.048)	-0.025 (0.041)	–
BC	–	–	0.029 (0.050)
BD	0.038 (0.048)	0.039 (0.045)	–
BE	0.239 (0.048)	–	–
CD	-0.005 (0.048)	-0.015 (0.038)	0.011 (0.029)
CE	0.012 (0.048)	–	-0.053 (0.051)
DE	-0.017 (0.048)	-0.041 (0.045)	-0.057 (0.041)

True Model: $\mu = 0.5x_A - 0.5x_B + 0.5x_C - 0.5x_D + 0.5x_E + 0.25x_Ax_C - 0.25x_Ax_D + 0.25x_Bx_E$

Simulation Results

- Illustrate consequences of confounding and aliasing
- Misspecified model can lead to biased and misleading estimates even if effects are clear
  - Main effects only model - estimates of main effects are biased by the significant two-factor interactions, even if main effects are clear
  - Importance of including all significant effects in model - particularly significant two-factor interactions

Advantages of BFFDs for DCEs

1. Effects confounded with block effects are not estimable, but do not bias estimate of other effects

2. Aliasing causes bias, but aliased effects are estimable if all aliases are negligible

3. Aliasing or missing a significant two-factor interaction can bias estimation of main effects even if all main effects and two-factor interactions are clear

Hence, it is essential at the design stage to know the aliasing and confounding structure of the design in order to construct a DCE.

BFFDs for DCEs

- Choice of BFFD depends on:
  - Number of attributes k
  - Desired size of the choice set (i.e., the number of options)
  - Effects to be identified as clear (number of clear two-factor interactions in a BFFD depends on design generators and block generators)
- Note: The number of options is a power of the attribute levels
- Previously, focused on choice of BFFDs to maximize number of clear main effects and two-factor interactions
  - However, this approach assumes it is known in advance which two-factor interactions are significant
  - Problematic as significant interactions are often unknown in practice

Minimum Aberration Criteria

- We propose the use of MA criteria for selecting BFFDs to construct DCEs assuming the Multinomial logit (MNL) model
- For any two 2^{k-p} designs d_1 and d_2, let r be the smallest integer such that $A_r(d_1) \neq A_r(d_2)$. Then d_1 is said to have less aberration than d_2 if $A_r(d_1) < A_r(d_2)$. If there is no design with less aberration than d_1, then d_1 has minimum aberration (Wu and Hamada, 2009)

Combined Wordlength Patterns

Various approaches for applying MA criteria to select a BFFD

- Two defining contrast subgroups:
  1. Treatment effects
  2. Block effects
- Choice of MA criteria to construct a DCE depends on the goals of the study
- Various combined wordlength patterns in the literature:
  - $W_{scf} = (A_{3,0}, A_{2,1}, A_{4,0}, A_{3,1}, A_{5,0}, A_{4,1}, \ldots)$
  - $W_{cc} = (3A_{3,0} + A_{2,1}, A_{4,0}, 10A_{5,0} + A_{3,1}, A_{6,0} \ldots)$
  - $W_1 = (A_{3,0}, A_{4,0}, A_{2,1}, A_{5,0}, A_{6,0}, A_{3,1}, \ldots)$
  - $W_2 = (A_{3,0}, A_{2,1}, A_{4,0}, A_{5,0}, A_{3,1}, A_{6,0}, \ldots)$

  Proposed by Sitter et al. (1997), Chen and Cheng (1999), and Cheng and Wu (2002).

Tables of MA BFFDs based on the W-criteria

- Sitter et al. (1997): provide MA BFFDs based on the W_{scf} criterion for all 8 and 16 run designs; for 32 run designs up to 15 attributes, and for 64 and 128 run designs up to 9 attributes

- Chen and Cheng (1999): provide MA BFFDs based on the W_{cc} criterion for 8, 16, and 32 runs up to 19 attributes

- Cheng and Wu (2002): provide MA BFFDs based on the W_1 and W_2 criteria for all 27 run designs, and for 81 run designs up to 10 attributes

- Xu and Lau (2006) and Xu (2006): provide MA BFFDs based on the W_{scf}, W_1, W_2, and W_{cc} criteria for all 32 run designs, for all 81 run designs, and for 64 runs up to 32 attributes

- Xu and Mee (2010): provide MA BFFDs based on the W_1 criterion for 128 runs and up to 64 attributes

Comparing W_1 and W_2 Criteria for selecting BFFDs to construct DCEs

- Several authors compared advantages and disadvantages of four sequences
- W_1 and W_2 are appropriate sequences because allow for large number of two-factor interactions to be estimated (Cheng and Wu, 2002)
- We focus on choice between W_1 and W_2 - depending on whether aliased effects or confounded effects are viewed as less desirable
  - Resolution III and IV FFDs: choice between W_1 and W_2 depends on:
    * whether $A_{4,0}$ or $A_{2,1}$ is less desirable, since both $A_{4,0}$ and $A_{2,1}$ pertain to either aliasing or confounding of two- factor interactions
  - Resolution V and VI FFDs: choice between W_1 and W_2 depends on:
    * whether $A_{6,0}$ or $A_{3,1}$ is less desirable, since both $A_{6,0}$ and $A_{3,1}$ pertain to either aliasing or confounding of three- factor interactions.

Comparing W_1 and W_2 Criteria for selecting BFFDs to construct DCEs.

$W_1 = (A_{3,0}, A_{4,0}, A_{2,1}, A_{5,0}, A_{6,0}, \ldots)$ vs $W_2 = (A_{3,0}, A_{2,1}, A_{4,0}, A_{5,0}, A_{3,1}, \ldots)$

- $A_{3,0}$ captures number of two-factor interactions aliased with main effects
- $A_{2,1}$ captures number of two-factor interactions confounded with block effects
- Hence, minimizing $A_{3,0}$ and $A_{2,1}$ maximizes number of estimable two-factor interactions besides estimation of main effects
- Minimizing aliasing and confounding of two-factor interactions, we maximize number of estimable two-factor interactions

Example: Comparing W_1 and W_2

- DCE with eight two-level attributes
  - 2^{8-3} FFD in 2^3 (8 choice sets) blocks of size 2^{8-3-3} (4 options)
  - Xu and Lau (2006) two possible MA BFFDs
- **Design D1:** 8-3.1/B3(W_1)
  - $W_t = (0,3,4,0,0,0)$ and $W_b = (8,16,11,\ldots)$
  - 8 main effects and 8 two-factor interactions are clear
- **Design D2:** 8-3.2/B3$(W_2 W_{scf})$
  - $W_t = (0,5,0,2,0,0)$ and $W_b = (7,18,10,\ldots)$
  - 8 main effects and 4 two-factor interactions are clear

Comparison of two 2^{8-3} designs in 2^3 blocks

Column	Design D1	Design D2
1	A	A
2	B	B
3	$AB = EG$	$AB = CG = DH$ = BLOCK
4	C	C
5	$AC = EH$	$AC = BG$
6	$BC = GH$	$BC = AG$
7	DF = BLOCK	G
8	D	D
9	AD = BLOCK	$AD = BH$
10	BD	$BD = AH$
11	CF	H
12	CD	$CD = GH$
13	BF	= BLOCK
14	AF = BLOCK	= BLOCK
15	F	$EF = DG = CH$
16	E	E
17	$AE = BG = CH$ = BLOCK	AE
18	$BE = AG$	BE
19	G	
20	$CE = AH$	$CE = FH$ = BLOCK
21	H	
22	= BLOCK	
23	$CG = BH$	$DF = EG$ = BLOCK
24	DE = BLOCK	$DE = FG$
25		= BLOCK
26	FH	= BLOCK
27	DG	$CF = EH$
28	FG	
29	DH	BF
30		AF
31	EF = BLOCK	F

Example: Comparing W_1 and W_2

$W_1 = (A_{3,0}, A_{4,0}, A_{2,1}, A_{5,0}, A_{6,0}, \ldots)$ vs $W_2 = (A_{3,0}, A_{2,1}, A_{4,0}, A_{5,0}, A_{3,1}, \ldots)$

- **Design D1:**
  - $W_1 = (0, 3, 8, 4, \ldots)$ and $W_2 = (0, 8, 3, 4, \ldots)$
  - W_1 criterion favors D1 smaller $A_{4,0}$ (3 vs. 5)
- **Design D2:**
  - $W_1 = (0, 5, 7, 0, \ldots)$ and $W_2 = (0, 7, 5, 0, \ldots)$
  - W_2 favors D2 because it only confounds seven two-factor interactions with blocks, $A_{2,1} = 7$ (vs. 8)

Hence, Design D1 under the MA W_1 criterion may be preferred.

Estimation Capacity

- MA criteria justified by the concept of estimation capacity
- MA criterion is a good substitute for some model-robustness criteria for unblocked FFDs (Cheng, Steinberg and Sun, 1999)
  - We extend this justification for blocked FFDs

Estimation Capacity

- Let $E_i(D)$ be the number of models containing all main effects and i two-factor interactions which can be estimated by design D $\left(i = 1, \ldots, \binom{k}{2}\right)$
  - Goal: $E_i(D)$ as large as possible
- Maximum estimation capacity if it maximizes $E_i(D)$ for all i (Chen and Cheng, 1999; Cheng and Mukerjee, 2001)
- 2^{k-p} design in 2^q blocks
  - k main effects and $2^q - 1$ block effects
  - Estimate at most $f = 2^{k-p} - k - 2^q$ two-factor interactions so that $E_i(D) = 0$ for $i > f$ and we only consider $(E_1, \ldots, E_f)$

Estimation Capacity: Designs S1 and S2

Design	E_1	E_2	E_3	E_4	E_5	E_6	E_7
S1	7	21	35	35	21	7	1
S2	8	26	44	41	20	4	0

- Design S1: Estimate all main effects and up to 7 two-factor interactions (as $E_7 = 1$)
- Design S2: Estimate all main effects and at most 6 two-factor interactions (as $E_7 = 0$).
- S1 can estimate more models than S2 if more than 4 two-factor interactions are important
- Whereas, S2 can estimate more models containing all main effects and up to 4 two-factor interactions than S1

Estimation Capacity

- This shows the W_1 criterion would be a better choice if the number of possible two-factor interactions is large, while the W_2 criterion would be a better choice if that number is thought to be smaller.

Example DCE: Snack Nutritional Ingredients

- An application was constructed using the aforementioned design to investigate the nutritional ingredients that most influence college students snack selection.
  - An electronic survey was administered to 792 undergraduate students at CSU Fullerton
  - Each student was presented 8 choice sets with 4 options
  - Six out of the 12 two-factor interactions between attributes had significant impacts on snack choices
    - Three interactions with sugar and three interactions with salt
  - Our results provide insight into factors that influence college students snack choices and suggest that healthiness and low sugar are the most important factors.

Summary

- Used ideas from BFFDs to construct various DCEs so that it is known in advance which attribute effects and their interactions can be identified
- Further extension to MA criteria for selecting BFFDs for constructing DCEs
  - Maximize number of estimable models involving two-factor interactions by minimizing confounding or aliasing of two-factor interactions
  - MA criteria choice depends on goals of the study
  - Demonstrate MA designs have large estimation capacity
- Generally, our proposed designs are easy to construct and for many practical scenarios are already available from the literature

Discussion

- Methodology can be extended for constructing DCEs with
  - Three-level symmetric attributes
  - Asymmetric attributes for identification of main effects
- Potential for future work considering various models other than MNL model
- Our designs are optimal for estimating parameters in the MNL model under the assumption that all options are equally attractive (Bush, 2014)
  - Locally optimal (or D-optimal) designs
  - A potential problem with this approach is that if the nominal values are misspecified, the locally optimal design may be potentially inefficient
- Alternative design approaches: Bayesian approach, maximin approach, or sequential approach

Discussion

- Question: Are our locally optimal designs robust to model misspecifications?
  - Kessels et al. (2011) conducted simulations to compare 9 Bayesian optimal designs to a locally optimal design
    * Concluded Bayesian optimal designs appear to be more robust than locally optimal designs
    * However, construction of Bayesian optimal designs is computationally intensive and becomes a very challenging task when the numbers of attributes and choice sets are large
    * Whereas, BFFDs are readily available and our method provides an attractive option for practitioners to implement DCEs to ensure identification

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Thank you for your time.