Stats 100C: HW2 due Friday, 4/10/09 (in class)

Exercise 1 A new profit-sharing plan was introduced at an automobile parts manufacturing plant last year. Both management and union represenatives were interested in determining how a worker's years of experience influence his or her productivity gains. After the plan had been in effect for a while, the data shown below were collected:

Years of experience (x)	Number of units daily (y)
15.1	110
7.0	105
18.6	115
23.7	127
11.5	98
16.4	103
6.3	87
15.4	108
19.9	112

For your convenience:

$$n = 9, \sum_{i=1}^{9} y_i = 965, \sum_{i=1}^{9} x_i = 133.9, \sum_{i=1}^{9} y_i^2 = 104469, \sum_{i=1}^{9} x_i^2 = 2258.73, \sum_{i=1}^{9} x_i y_i = 14801.2.$$

a. Construct a scatterplot of the number of units manufactured daily on the years of experience on the assebly line.

b. Find the least-squares regression line (round up to 2 decimals). Show your work (no computer output).

c. Predict the number of units manufactured daily by an employee who has 10 years of experience on the assembly line.

d. Find the fitted values and residuals (round up to 2 decimals).

e. Find the LS estimate of σ^2.

f. Compute the standard error of $\hat{\beta}_1$.

g. Construct a 95% confidence interval for β_1.

h. Test $H_0 : \beta_1 = 1$ against $H_1 : \beta_1 \neq 1$. Use $\alpha = 0.05$.

Exercise 2 Suppose in the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where $i = 1, \cdots, n$, $E(\epsilon_i) = 0$, $\text{var}(\epsilon_i) = \sigma^2$ the measurements x_i were in inches and we would like to write the model in centimeters, say, z_i. If one inch is equal to c centimeters (c is known), we can write the above model as follows $y_i = \beta_0^* + \beta_1^* z_i + \epsilon_i$.

a. Suppose $\hat{\beta}_0$ and $\hat{\beta}_1$ are the least squares estimates of β_0 and β_1 of the first model. Find the estimates of β_0^* and β_1^* in terms of $\hat{\beta}_0$ and $\hat{\beta}_1$.

b. Find the variance of $\hat{\beta}_1^*$.

Exercise 3 Consider the regression model

$$y_i = (\beta_0 + \beta_1 \bar{x}) + \beta_1 (x_i - \bar{x}) + \epsilon_i$$

This model is called the *centered* version of the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ that was discussed in class. If we let $\gamma_0 = \beta_0 + \beta_1 \bar{x}$ we can rewrite the *centered* version as $y_i = \gamma_0 + \beta_1 (x_i - \bar{x}) + \epsilon_i$. Find the least squares estimates of γ_0 and β_1.

Exercise 4 Consider the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. Show that $Cov(\bar{y}, \hat{\beta}_1) = 0$ where $\bar{y}$ is the sample mean of the y values, and $\hat{\beta}_1$ is the estimate of β_1.