13

Design and Analysis of Single-Factor Experiments: The Analysis of Variance

CHAPTER OUTLINE

13-1 Designing Engineering Experiments

Every experiment involves a sequence of activities:

1. Conjecture – the original hypothesis that motivates the experiment.
2. Experiment – the test performed to investigate the conjecture.
3. Analysis – the statistical analysis of the data from the experiment.
4. Conclusion – what has been learned about the original conjecture from the experiment. Often the experiment will lead to a revised conjecture, and a new experiment, and so forth.

13-2 The Completely Randomized Single-Factor Experiment

13-2.1 An Example

A manufacturer of paper used for making grocery bags is interested in improving the tensile strength of the product. Product engineering thinks that tensile strength is a function of the hardwood concentration in the pulp and that the range of hardwood concentrations of practical interest is between 5 and 20%. A team of engineers responsible for the study decides to investigate four levels of hardwood concentration: 5%, 10%, 15%, and 20%. They decide to make up six test specimens at each concentration level, using a pilot plant. All 24 specimens are tested on a laboratory tensile tester, in random order. The data from this experiment are shown in Table 13-1.

13-2 The Completely Randomized Single-Factor Experiment

13-2.1 An Example

Table 13-1 Tensile Strength of Paper (psi)

Hardwood Concentration (%)	Observations						Totals	Averages
	1	2	3	4	5	6		
5	7	8	15	11	9	10	60	10.00
10	12	17	13	18	19	15	94	15.67
15	14	18	19	17	16	18	102	17.00
20	19	25	22	23	18	20	127	21.17
							383	15.96

13-2 The Completely Randomized Single-Factor Experiment

13-2.1 An Example

- The levels of the factor are sometimes called **treatments**.
- Each treatment has six observations or **replicates**.
- The runs are run in **random** order.

13-2 The Completely Randomized Single-Factor Experiment

13-2.1 An Example

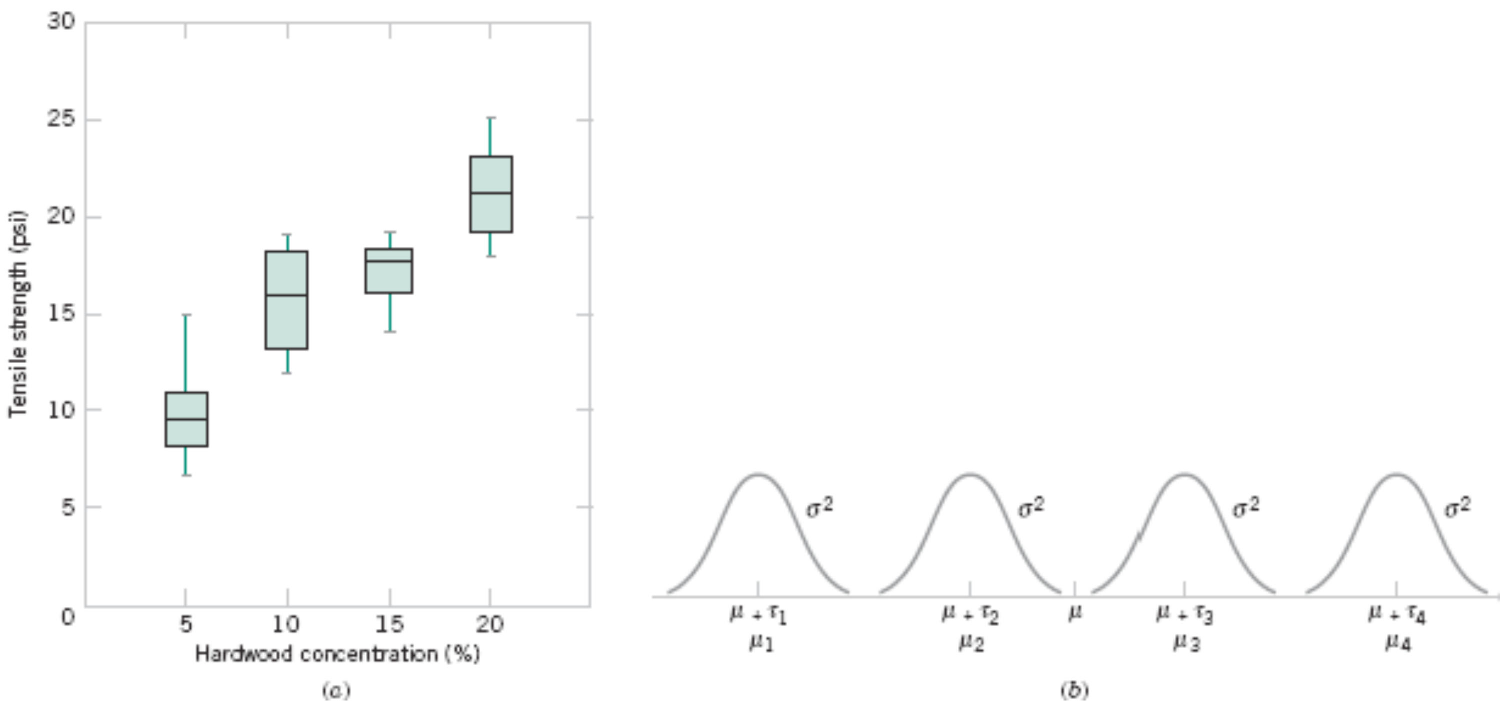


Figure 13-1 (a) Box plots of hardwood concentration data. (b) Display of the model in Equation 13-1 for the completely randomized single-factor experiment

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

Suppose there are *a* different levels of a single factor that we wish to compare. The levels are sometimes called **treatments**.

Table 13-2 Typical Data for a Single-Factor Experiment

Treatment	Observations				Totals	Averages
1	y_{11}	y_{12}	$\cdots$	y_{1n}	$y_{1\cdot}$	$\bar{y}_{1\cdot}$
2	y_{21}	y_{22}	$\cdots$	y_{2n}	$y_{2\cdot}$	$\bar{y}_{2\cdot}$
$\vdots$	$\vdots$	$\vdots$	$\vdots\vdots\vdots$	$\vdots$	$\vdots$	$\vdots$
a	y_{a1}	y_{a2}	$\cdots$	y_{an}	$y_{a\cdot}$	$\bar{y}_{a\cdot}$
					$y_{\cdot\cdot}$	$\bar{y}_{\cdot\cdot}$

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

We may describe the observations in Table 13-2 by the linear statistical model:

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, n \end{cases} \tag{13-1}$$

The model could be written as

$$Y_{ij} = \mu_i + \epsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, n \end{cases}$$

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

Fixed-effects Model

The treatment effects are usually defined as deviations from the overall mean so that:

$$\sum_{i=1}^{a} \tau_i = 0$$

Also,

$$y_{i\cdot} = \sum_{j=1}^{n} y_{ij} \qquad \bar{y}_{i\cdot} = y_{i\cdot}/n \qquad i = 1, 2, \ldots, a$$

$$y_{\cdot\cdot} = \sum_{i=1}^{a} \sum_{j=1}^{n} y_{ij} \qquad \bar{y}_{\cdot\cdot} = y_{\cdot\cdot}/N$$

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

We wish to test the hypotheses:

$$H_0\colon \tau_1 = \tau_2 = \cdots = \tau_a = 0$$
$$H_1\colon \tau_i \neq 0 \quad \text{for at least one } i$$

The analysis of variance partitions the total variability into two parts.

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

The **sum of squares identity** is

$$\sum_{i=1}^{a} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2 = n \sum_{i=1}^{a} (\bar{y}_{i\cdot} - \bar{y}_{..})^2 + \sum_{i=1}^{a} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{i\cdot})^2 \qquad (13\text{-}5)$$

or symbolically

$$SS_T = SS_{\text{Treatments}} + SS_E \qquad (13\text{-}6)$$

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

The expected value of the treatment sum of squares is

$$E(SS_{\text{Treatments}}) = (a-1)\sigma^2 + n\sum_{i=1}^{a}\tau_i^2$$

and the expected value of the error sum of squares is

$$E(SS_E) = a(n-1)\sigma^2$$

The ratio $MS_{\text{Treatments}} = SS_{\text{Treatments}}/(a-1)$ is called the **mean square for treatments**.

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

The appropriate test statistic is

$$F_0 = \frac{SS_{\text{Treatments}}/(a-1)}{SS_E/[a(n-1)]} = \frac{MS_{\text{Treatments}}}{MS_E} \qquad (13\text{-}7)$$

We would reject H_0 if $f_0 > f_{\alpha,a-1,a(n-1)}$

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

The sums of squares computing formulas for the ANOVA with equal sample sizes in each treatment are

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{N} \tag{13-8}$$

and

$$SS_{\text{Treatments}} = \sum_{i=1}^{a} \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{N} \tag{13-9}$$

The error sum of squares is obtained by subtraction as

$$SS_E = SS_T - SS_{\text{Treatments}} \tag{13-10}$$

where $N=na$ is the total number of observations.

13-2 The Completely Randomized Single-Factor Experiment

13-2.2 The Analysis of Variance

Analysis of Variance Table

Table 13-3 The Analysis of Variance for a Single-Factor Experiment, Fixed-Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a - 1$	$MS_{\text{Treatments}}$	$\dfrac{MS_{\text{Treatments}}}{MS_E}$
Error	SS_E	$a(n - 1)$	MS_E	
Total	SS_T	$an - 1$		

13-2 The Completely Randomized Single-Factor Experiment

Example 13-1

Consider the paper tensile strength experiment described in Section 13-2.1. We can use the analysis of variance to test the hypothesis that different hardwood concentrations do not affect the mean tensile strength of the paper.

The hypotheses are

$$H_0\colon \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$$

$$H_1\colon \tau_i \neq 0 \text{ for at least one } i \quad .$$

13-2 The Completely Randomized Single-Factor Experiment

Example 13-1

We will use $\alpha = 0.01$. The sums of squares for the analysis of variance are computed from Equations 13-8, 13-9, and 13-10 as follows:

$$SS_T = \sum_{i=1}^{4} \sum_{j=1}^{6} y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$= (7)^2 + (8)^2 + \cdots + (20)^2 - \frac{(383)^2}{24} = 512.96$$

$$SS_{\text{Treatments}} = \sum_{i=1}^{4} \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{N}$$

$$= \frac{(60)^2 + (94)^2 + (102)^2 + (127)^2}{6} - \frac{(383)^2}{24} = 382.79$$

$$SS_E = SS_T - SS_{\text{Treatments}}$$

$$= 512.96 - 382.79 = 130.17$$

13-2 The Completely Randomized Single-Factor Experiment

Example 13-1

The ANOVA is summarized in Table 13-4. Since $f_{0.01,3,20} = 4.94$, we reject H_0 and conclude that hardwood concentration in the pulp significantly affects the mean strength of the paper. We can also find a P-value for this test statistic as follows:

$$P = P(F_{3,20} > 19.60) \simeq 3.59 \times 10^{-6}$$

Since $P \simeq 3.59 \times 10^{-6}$ is considerably smaller than $\alpha = 0.01$, we have strong evidence to conclude that H_0 is not true.

Table 13-4 ANOVA for the Tensile Strength Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	f_0	P-value
Hardwood concentration	382.79	3	127.60	19.60	3.59 E-6
Error	130.17	20	6.51		
Total	512.96	23			

Table 13-5 Minitab Analysis of Variance Output for Example 13-1

One-Way ANOVA: Strength versus CONC

Analysis of Variance for Strength

Source	DF	SS	MS	F	P
Conc	3	382.79	127.60	19.61	0.000
Error	20	130.17	6.51		
Total	23	512.96			

Level	N	Mean	StDev
5	6	10.000	2.828
10	6	15.667	2.805
15	6	17.000	1.789
20	6	21.167	2.639

Pooled StDev = 2.551

```
Individual 95% CIs For Mean
Based on Pooled StDev
--+------+------+------+-
(--*--)
          (--*--)
              (--*--)
                  (--*--)
--+--------+--------+--------+-
 10.0     15.0     20.0     25.0
```

Fisher's pairwise comparisons

Family error rate = 0.192
Individual error rate = 0.0500

Critical value = 2.086

Intervals for (column level mean) − (row level mean)

	5	10	15
10	−8.739		
	−2.594		
15	−10.072	−4.406	
	−3.928	1.739	
20	−14.239	−8.572	−7.239
	−8.094	−2.428	−1.094

13-2 The Completely Randomized Single-Factor Experiment

A $100(1-\alpha)$ percent confidence interval on the mean of the ith treatment μ_i is

$$\bar{y}_{i\cdot} - t_{\alpha/2,a(n-1)}\sqrt{\frac{MS_E}{n}} \le \mu_i \le \bar{y}_{i\cdot} + t_{\alpha/2,a(n-1)}\sqrt{\frac{MS_E}{n}} \qquad (13\text{-}11)$$

The 95% CI on the mean of the 20% hardwood is

$$\left[\bar{y}_{4\cdot} \pm t_{0.025,20}\sqrt{MS_E / n}\right]$$

$$\left[21.167 \pm (2.086)\sqrt{6.51 / 6}\right]$$

$$19.00 \text{ psi} \le \mu_4 \le 23.34 \text{ psi}$$

13-2 The Completely Randomized Single-Factor Experiment

A 100(1 − α) percent confidence interval on the difference in two treatment means $\mu_i - \mu_j$ is

$$\bar{y}_{i\cdot} - \bar{y}_{j\cdot} - t_{\alpha/2,a(n-1)}\sqrt{\frac{2MS_E}{n}} \le \mu_i - \mu_j \le \bar{y}_{i\cdot} - \bar{y}_{j\cdot} + t_{\alpha/2,a(n-1)}\sqrt{\frac{2MS_E}{n}} \quad (13\text{-}12)$$

For the hardwood concentration example,

A 95% CI on the difference in means $\mu_3 - \mu_2$ is

$$\left[\bar{y}_{3.} - \bar{y}_{2.} \pm t_{0.025,20}\sqrt{2MS_E / n}\right]$$

$$\left[17.00 - 15.67 \pm (2.086)\sqrt{2(6.51)/6}\right]$$

$$-1.74 \le \mu_3 - \mu_2 \le 4.40$$

13-2 The Completely Randomized Single-Factor Experiment

An Unbalanced Experiment

The sums of squares computing formulas for the ANOVA with unequal sample sizes n_i in each treatment are

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{..}^2}{N} \tag{13-13}$$

$$SS_{\text{Treatments}} = \sum_{i=1}^{a} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N} \tag{13-14}$$

and

$$SS_E = SS_T - SS_{\text{Treatments}} \tag{13-15}$$

13-2 The Completely Randomized Single-Factor Experiment

13-2.3 Multiple Comparisons Following the ANOVA

The least significant difference (LSD) is

$$\text{LSD} = t_{\alpha/2, a(n-1)} \sqrt{\frac{2MS_E}{n}} \qquad (13\text{-}16)$$

If the sample sizes are different in each treatment:

$$\text{LSD} = t_{\alpha/2, N-a} \sqrt{MS_E\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

13-2 The Completely Randomized Single-Factor Experiment

Example 13-2

We will apply the Fisher LSD method to the hardwood concentration experiment. There are $a = 4$ means, $n = 6$, $MS_E = 6.51$, and $t_{0.025,20} = 2.086$. The treatment means are

$$\begin{aligned}
\bar{y}_{1\cdot} &= 10.00 \text{ psi} \\
\bar{y}_{2\cdot} &= 15.67 \text{ psi} \\
\bar{y}_{3\cdot} &= 17.00 \text{ psi} \\
\bar{y}_{4\cdot} &= 21.17 \text{ psi}
\end{aligned}$$

The value of LSD is LSD $= t_{0.025,20}\sqrt{2MS_E/n} = 2.086\sqrt{2(6.51)/6} = 3.07$. Therefore, any pair of treatment averages that differs by more than 3.07 implies that the corresponding pair of treatment means are different.

13-2 The Completely Randomized Single-Factor Experiment

Example 13-2

The comparisons among the observed treatment averages are as follows:

$$
\begin{aligned}
4 \text{ vs. } 1 &= 21.17 - 10.00 = 11.17 > 3.07 \\
4 \text{ vs. } 2 &= 21.17 - 15.67 = 5.50 > 3.07 \\
4 \text{ vs. } 3 &= 21.17 - 17.00 = 4.17 > 3.07 \\
3 \text{ vs. } 1 &= 17.00 - 10.00 = 7.00 > 3.07 \\
3 \text{ vs. } 2 &= 17.00 - 15.67 = 1.33 < 3.07 \\
2 \text{ vs. } 1 &= 15.67 - 10.00 = 5.67 > 3.07
\end{aligned}
$$

From this analysis, we see that there are significant differences between all pairs of means except 2 and 3. This implies that 10 and 15% hardwood concentration produce approximately the same tensile strength and that all other concentration levels tested produce different tensile strengths. It is often helpful to draw a graph of the treatment means, such as in Fig. 13-2, with the means that are *not* different underlined. This graph clearly reveals the results of the experiment and shows that 20% hardwood produces the maximum tensile strength.

13-2 The Completely Randomized Single-Factor Experiment

Example 13-2

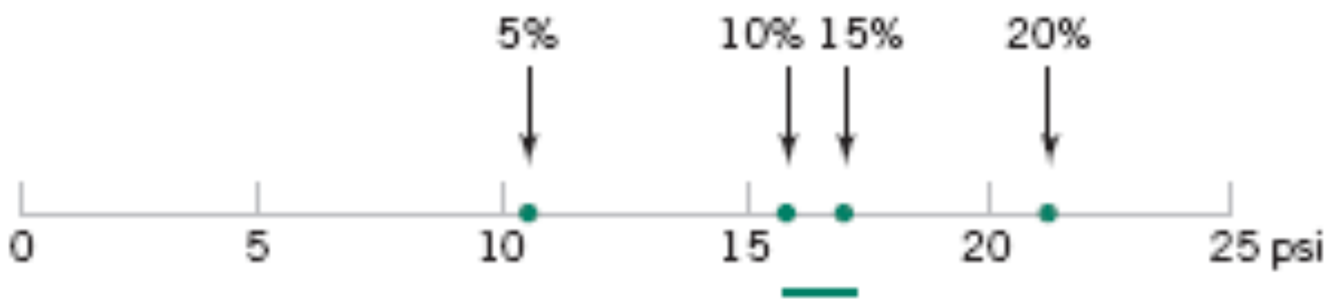


Figure 13-2 Results of Fisher's LSD method in Example 13-2.

Figure 13-2 Results of Fisher's LSD method in Example 13-2

13-2 The Completely Randomized Single-Factor Experiment

13-2.4 Residual Analysis and Model Checking

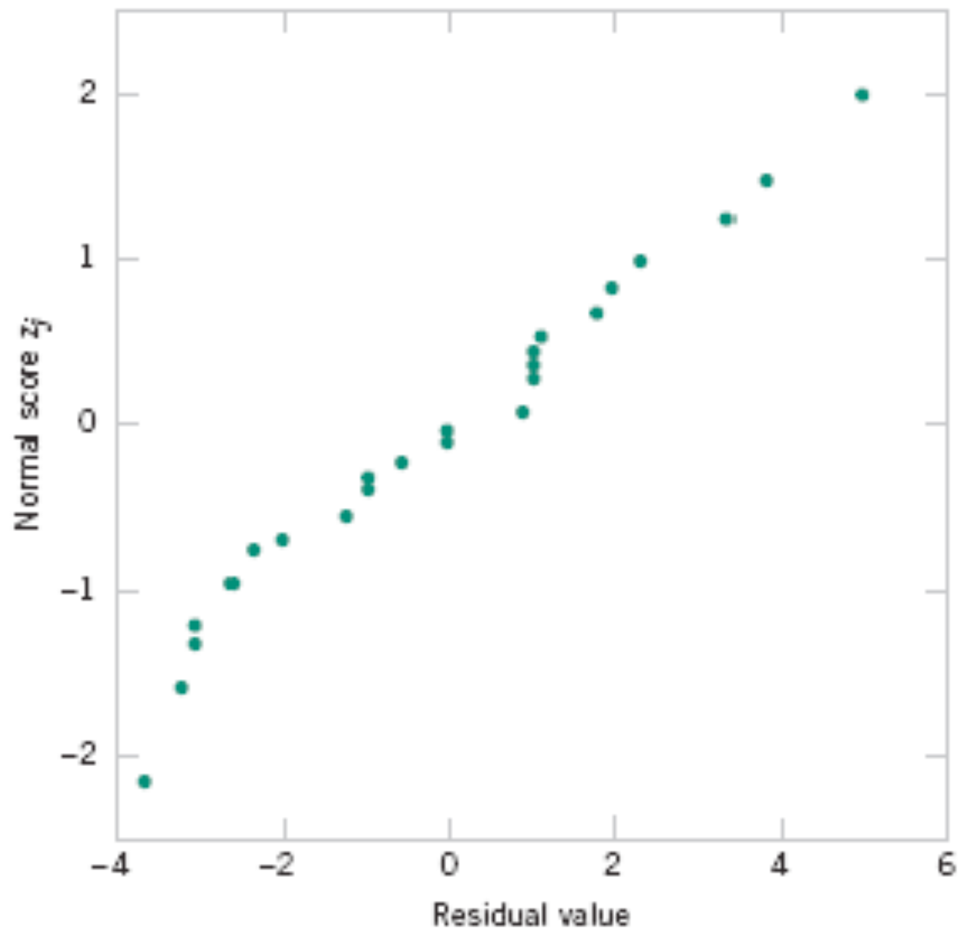
Table 13-6 Residuals for the Tensile Strength Experiment

Hardwood Concentration (%)	Residuals					
5	−3.00	−2.00	5.00	1.00	−1.00	0.00
10	−3.67	1.33	−2.67	2.33	3.33	−0.67
15	−3.00	1.00	2.00	0.00	−1.00	1.00
20	−2.17	3.83	0.83	1.83	−3.17	−1.17

13-2 The Completely Randomized Single-Factor Experiment

13-2.4 Residual Analysis and Model Checking

Figure 13-4 Normal probability plot of residuals from the hardwood concentration experiment.



13-2 The Completely Randomized Single-Factor Experiment

13-2.4 Residual Analysis and Model Checking

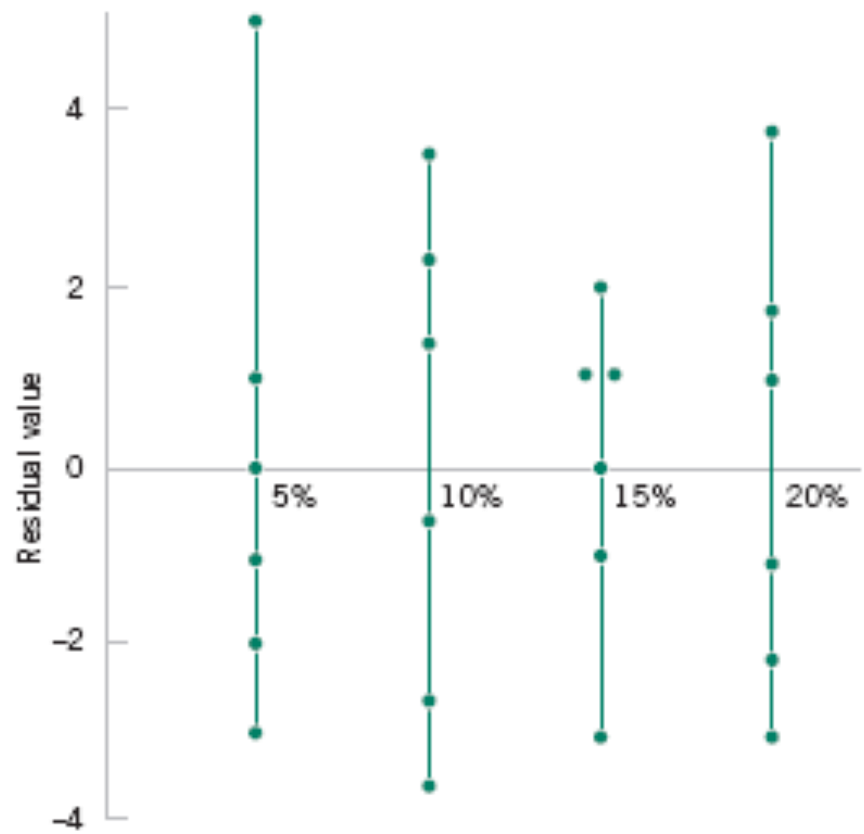


Figure 13-5 Plot of residuals versus factor levels (hardwood concentration).

13-2 The Completely Randomized Single-Factor Experiment

13-2.4 Residual Analysis and Model Checking

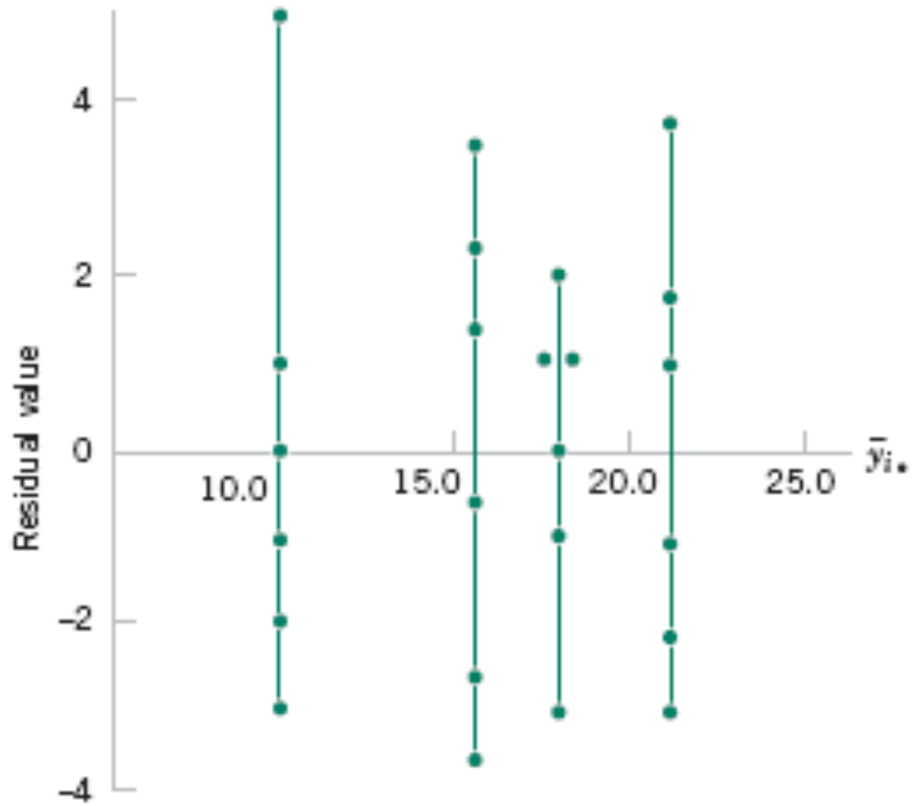


Figure 13-6 Plot of residuals versus $\bar{y}_i$

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

The **randomized block design** is an extension of the paired t-test to situations where the factor of interest has more than two levels.

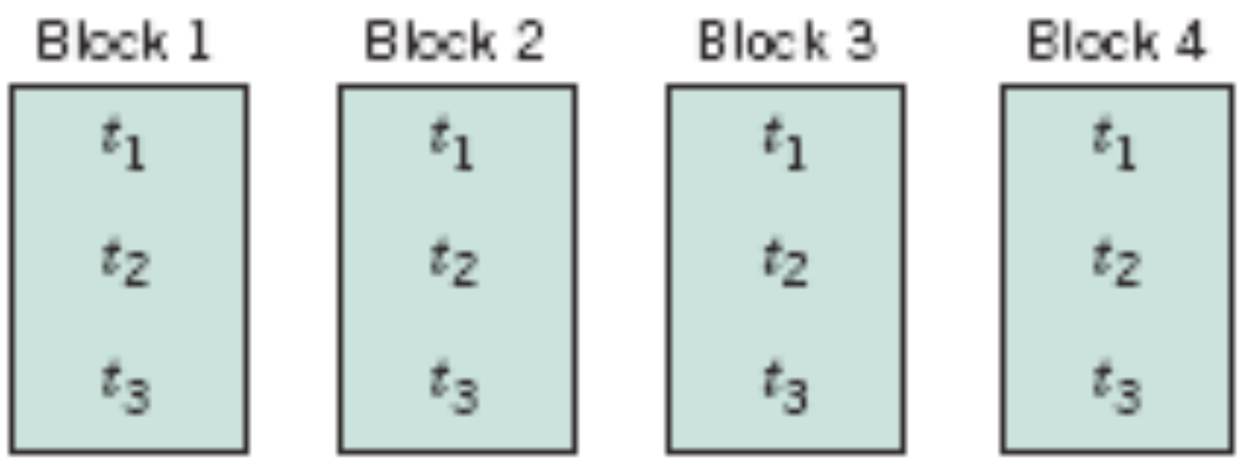


Figure 13-9 A randomized complete block design.

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

For example, consider the situation of Example 10-9, where two different methods were used to predict the shear strength of steel plate girders. Say we use four girders as the experimental units.

Table 13-9 A Randomized Complete Block Design

Treatments (Method)	Block (Girder) 1	2	3	4
1	y_{11}	y_{12}	y_{13}	y_{14}
2	y_{21}	y_{22}	y_{23}	y_{24}
3	y_{31}	y_{32}	y_{33}	y_{34}

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

General procedure for a randomized complete block design:

Table 13-10 A Randomized Complete Block Design with a Treatments and b Blocks

Treatments	Blocks 1	2	…	b	Totals	Averages
1	y_{11}	y_{12}	…	y_{1b}	$y_{1\cdot}$	$\bar{y}_{1\cdot}$
2	y_{21}	y_{22}	…	y_{2b}	$y_{2\cdot}$	$\bar{y}_{2\cdot}$
⋮	⋮	⋮		⋮	⋮	⋮
a	y_{a1}	y_{a2}	…	y_{ab}	$y_{a\cdot}$	$\bar{y}_{a\cdot}$
Totals	$y_{\cdot 1}$	$y_{\cdot 2}$	…	$y_{\cdot b}$	$y_{\cdot\cdot}$	
Averages	$\bar{y}_{\cdot 1}$	$\bar{y}_{\cdot 2}$	…	$\bar{y}_{\cdot b}$		$\bar{y}_{\cdot\cdot}$

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

The appropriate linear statistical model:

$$Y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{cases}$$

We assume

- treatments and blocks are initially fixed effects
- blocks do not interact
- $\sum_{i=1}^{a} \tau_i = 0$ and $\sum_{j=1}^{b} \beta_j = 0$

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

We are interested in testing:

$$H_0: \tau_1 = \tau_2 = \cdots = \tau_a = 0$$
$$H_1: \tau_i \neq 0 \text{ at least one } i$$

The **sum of squares identity for the randomized complete block design** is

$$\sum_{i=1}^{a} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{..})^2 = b \sum_{i=1}^{a} (\bar{y}_{i\cdot} - \bar{y}_{..})^2 + a \sum_{j=1}^{b} (\bar{y}_{\cdot j} - \bar{y}_{..})^2 + \sum_{i=1}^{a} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{\cdot j} - \bar{y}_{i\cdot} + \bar{y}_{..})^2 \qquad (13\text{-}27)$$

or symbolically

$$SS_T = SS_{\text{Treatments}} + SS_{\text{Blocks}} + SS_E$$

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

The mean squares are:

$$MS_{\text{Treatments}} = \frac{SS_{\text{Treatments}}}{a - 1}$$

$$MS_{\text{Blocks}} = \frac{SS_{\text{Blocks}}}{b - 1}$$

$$MS_E = \frac{SS_E}{(a - 1)(b - 1)}$$

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

The expected values of these mean squares are:

$$E(MS_{\text{Treatments}}) = \sigma^2 + \frac{b \sum_{i=1}^{a} \tau_i^2}{a - 1}$$

$$E(MS_{\text{Blocks}}) = \sigma^2 + \frac{a \sum_{j=1}^{b} \beta_j^2}{b - 1}$$

$$E(MS_E) = \sigma^2$$

13-4 Randomized Complete Block Designs

13-4.1 Design and Statistical Analyses

Table 13-11 ANOVA for a Randomized Complete Block Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a - 1$	$\dfrac{SS_{\text{Treatments}}}{a - 1}$	$\dfrac{MS_{\text{Treatments}}}{MS_E}$
Blocks	SS_{Blocks}	$b - 1$	$\dfrac{SS_{\text{Blocks}}}{b - 1}$	
Error	SS_E (by subtraction)	$(a - 1)(b - 1)$	$\dfrac{SS_E}{(a - 1)(b - 1)}$	
Total	SS_T	$ab - 1$		

13-4 Randomized Complete Block Designs

Example 13-5

An experiment was performed to determine the effect of four different chemicals on the strength of a fabric. These chemicals are used as part of the permanent press finishing process. Five fabric samples were selected, and a randomized complete block design was run by testing each chemical type once in random order on each fabric sample. The data are shown in Table 13-12. We will test for differences in means using an ANOVA with $\alpha = 0.01$.

Table 13-12 Fabric Strength Data—Randomized Complete Block Design

	Fabric Sample					Treatment Totals	Treatment Averages
Chemical Type	1	2	3	4	5	$y_{i\cdot}$	$\bar{y}_{i\cdot}$
1	1.3	1.6	0.5	1.2	1.1	5.7	1.14
2	2.2	2.4	0.4	2.0	1.8	8.8	1.76
3	1.8	1.7	0.6	1.5	1.3	6.9	1.38
4	3.9	4.4	2.0	4.1	3.4	17.8	3.56
Block totals $y_{\cdot j}$	9.2	10.1	3.5	8.8	7.6	39.2($y_{\cdot\cdot}$)	
Block averages $\bar{y}_{\cdot j}$	2.30	2.53	0.88	2.20	1.90		1.96($\bar{y}_{\cdot\cdot}$)

13-4 Randomized Complete Block Designs

Example 13-5

Table 13-13 Analysis of Variance for the Randomized Complete Block Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	f_0	P-value
Chemical types (treatments)	18.04	3	6.01	75.13	4.79 E-8
Fabric samples (blocks)	6.69	4	1.67		
Error	0.96	12	0.08		
Total	25.69	19			

The ANOVA is summarized in Table 13-13. Since $f_0 = 75.13 > f_{0.01,3,12} = 5.95$ (the P-value is 4.79×10^{-8}), we conclude that there is a significant difference in the chemical types so far as their effect on strength is concerned.

13-4 Randomized Complete Block Designs

Minitab Output for Example 13-5

Table 13-14 Minitab Analysis of Variance for the Randomized Complete Block Design in Example 13-5

Analysis of Variance (Balanced Designs)

Factor	Type	Levels	Values				
Chemical	fixed	4	1	2	3	4	
Fabric S	fixed	5	1	2	3	4	5

Analysis of Variance for strength

Source	DF	SS	MS	F	P
Chemical	3	18.0440	6.0147	75.89	0.000
Fabric S	4	6.6930	1.6733	21.11	0.000
Error	12	0.9510	0.0792		
Total	19	25.6880			

F-test with denominator: Error

Denominator MS = 0.079250 with 12 degrees of freedom

Numerator	DF	MS	F	P
Chemical	3	6.015	75.89	0.000
Fabric S	4	1.673	21.11	0.000

13-4 Randomized Complete Block Designs

13-4.2 Multiple Comparisons

Fisher's Least Significant Difference for Example 13-5

$$\bar{y}_{1.} = 1.14 \quad \bar{y}_{2.} = 1.76 \quad \bar{y}_{3.} = 1.38 \quad \bar{y}_{4.} = 3.56$$

$$\text{LSD} = t_{0.025,12}\sqrt{\frac{2MS_E}{b}} = 2.179\sqrt{\frac{2(0.08)}{5}} = 0.39$$

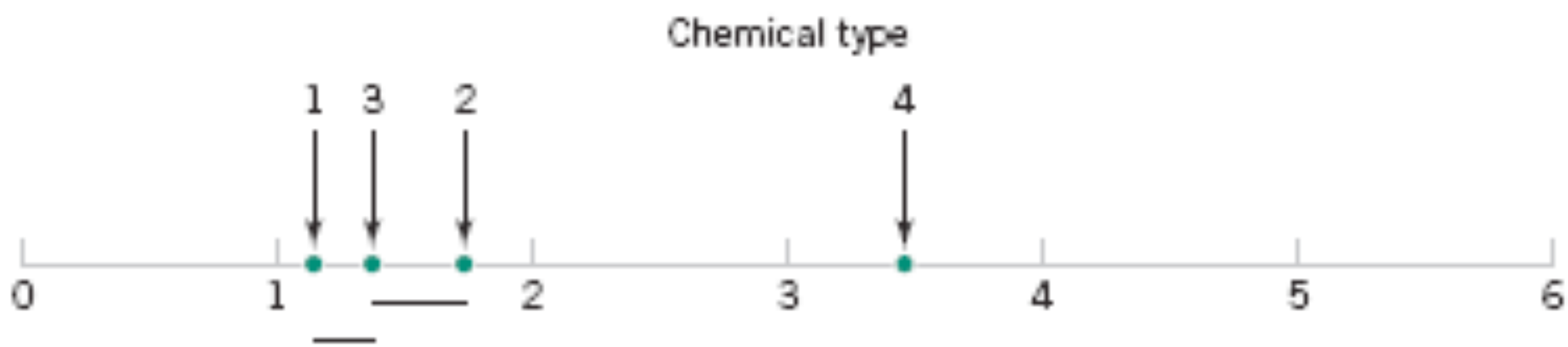
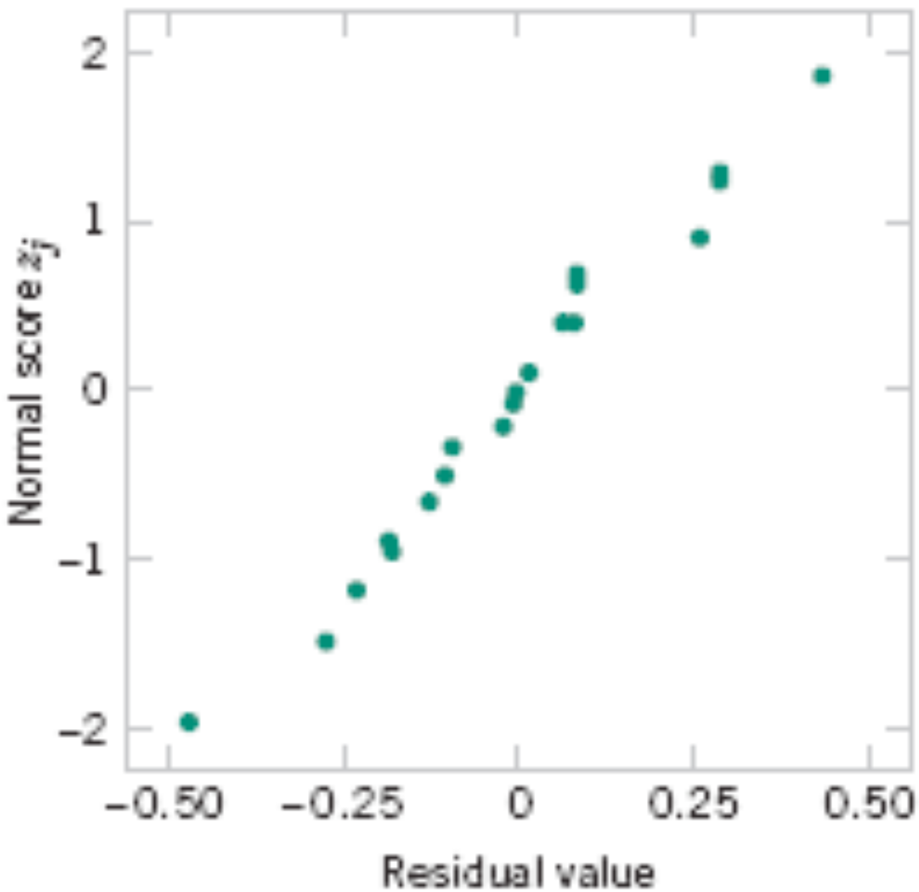


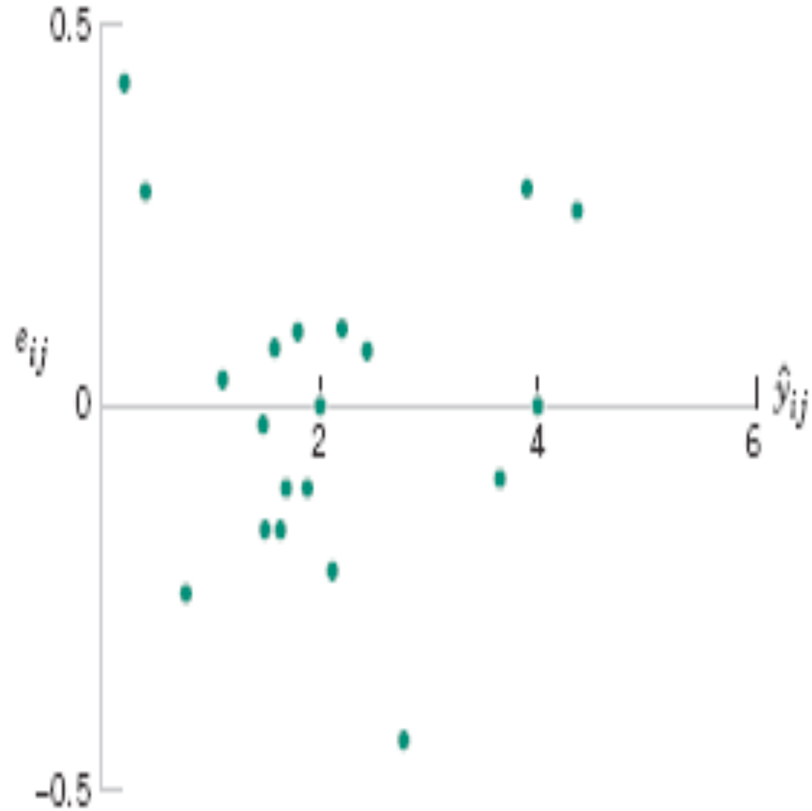
Figure 13-10 Results of Fisher's LSD method.

13-4 Randomized Complete Block Designs

13-4.3 Residual Analysis and Model Checking

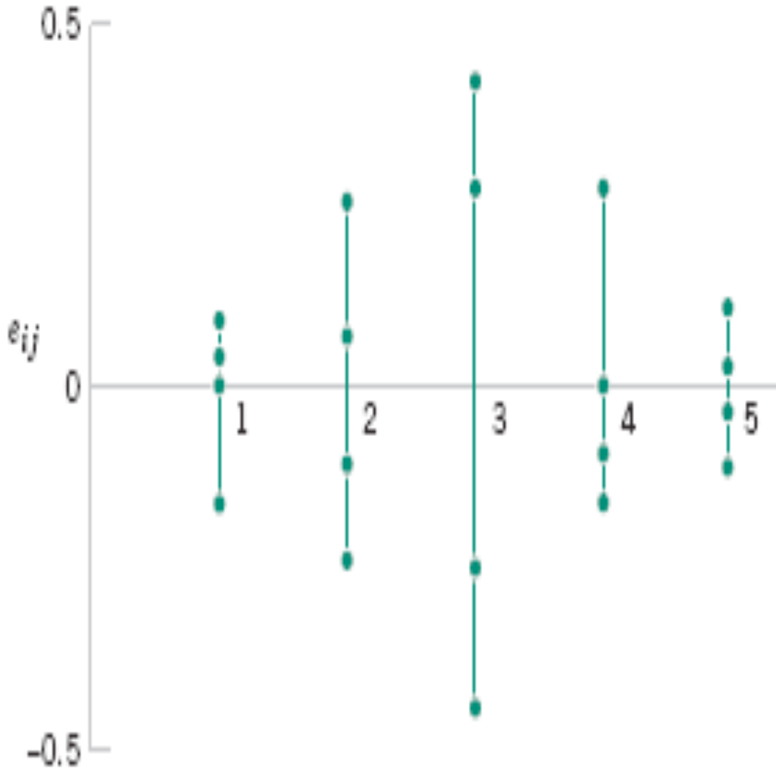


(a) Normal prob. plot of residuals

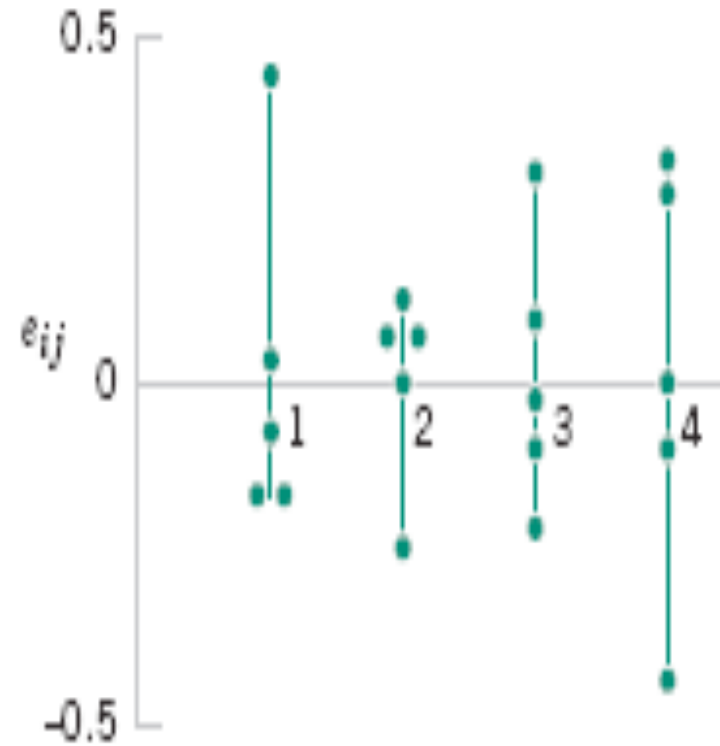


(b) Residuals versus $\hat{y}_{ij}$

13-4 Randomized Complete Block Designs



(a) Residuals by block.



(b) Residuals by treatment