Midterm Exam 2 Review — Chapters 5–8

1. A basketball player has made about 70% of his free throws over several years. Suppose that his free throws are independent trials with probability 0.7 of a success on each trial.

(a) What is the probability that he makes all in 6 attempts?

(b) What is the probability that he makes none in 6 attempts?

(c) What is the probability that he makes 2 or fewer in 6 attempts?

(d) What is the expected (mean) balls that he will make in 6 attempts? What is the standard deviation?

2. An opinion poll asks a random sample of 500 adults whether they favor giving parents of school-age children vouchers that can be exchanged for education at any public or private school of their choice. Suppose that in fact 45% of the population favor this idea. What is the probability that at least half of the sample are in favor?

3. "Random number generators" produce numbers that are distributed uniformly between 0 and 1.

(a) Sketch the distribution for the numbers generated. Include labels and some values on each axis.

(b) What proportion of the outcomes are less than 0.25?

(c) What proportion of outcomes lie between 0.1 and 0.9?

4. The distribution of scores for adults on the Wechsler Adult Intelligence Scale (WAIS) is approximately normal with mean 100 and standard deviation 15.

(a) What is the probability that a randomly chosen individual has a WAIS score of 105 or higher?

(b) What are the mean and standard deviation of the average WAIS score $\overline{X}$ for a random sample of 60 people?

(c) What is the probability that the average WAIS score of a random sample of 60 people is 105 or higher?

(d) Would your answers to any of (a), (b), or (c) be affected if the distribution of WAIS scores in the adult population were distinctly nonnormal? Explain.

5. Repeat weighing a compound of 2 grams (g) with two scales. The first scale produces readings X that have mean 2.000 g and standard deviation 0.002 g; the second scale's readings Y have mean 2.001 g and standard deviation 0.001 g.

(a) What are the mean and standard deviation of the difference $Y - X$ between the readings? (The readings X and Y are independent.)

(b) Let $Z = (X + Y)/2$ be the average of the two readings. What are the mean and standard deviation of Z?

(c) Is the average Z more or less variable than the reading Y from the second scale?

6. In recent years over 70% of the first-year college students responding to a national survey have identified "being well-off financially" as an important personal goal. A state university finds that 132 of a random sample of 200 of its first-year students say that this goal is important.

(a) Give a 95% CI for the proportion of all first-year students at the university who would identify being well-off as an important personal goal.

(b) How large a sample would be required to obtain a margin of error of ± 0.01 in a 95% CI for the proportion of all first-year students at the university who would identify being well-off as an important personal goal?

7. Degree of Reading Power (DRP) scores for a random sample of 44 third-grade students in a suburban school district have a mean of 35.1 and standard deviation 11.2. Assume DRP scores are approximately normal. Give a 95% CI for the population mean score in this district.

Some important concepts for review

Joint, marginal and conditional probability distributions

Independence, covariance and correlation between random variables

What does a random sample mean?

What does the 95% confidence level mean?

Midterm Exam 2

- Time: Friday, November 19 , 10-10:50am.
- Material: Chapters 5–8 (including lectures, homework 4-5, labs)
- It will be a **closed book exam**.
- Bring your calculator, ruler, pen, pencil, eraser, etc.

NOTE: The following formulas will be provided. Normal and t Tables will be provided if needed.

Formulas.

Sample Mean and Standard Deviation

mean $\bar{x} = \frac{1}{n}\sum x_i$, standard deviation $s = \sqrt{\frac{1}{n-1}\sum(x_i - \bar{x})^2}$

Random Variables

mean $\mu_X = E(X) = \sum x f(x)$ and variance $\mathrm{var}\,(X) = E[(X-\mu_X)^2] = E(X^2) - [E(X)]^2$

covariance $\mathrm{cov}\,(X,Y) = E(X - E(X))(Y - E(Y)) = E(X-\mu_X)(Y-\mu_Y)$

correlation $\rho = \mathrm{cor}\,(X,Y) = \frac{\mathrm{cov}\,(X,Y)}{\sqrt{\mathrm{var}\,(X)\mathrm{var}\,(Y)}}$

Rule for means, variances and expectation

$E(a+bX) = a + bE(X)$ and $E(aX+bY) = aE(X) + bE(Y)$

$\mathrm{var}\,(a+bX) = b^2\mathrm{var}\,(X)$ and $\mathrm{var}\,(aX+bY) = a^2\mathrm{var}\,(X) + b^2\mathrm{var}\,(Y) + 2a\,b\,\mathrm{cov}\,(X,Y)$.

If X and Y are independent, then $\mathrm{var}\,(aX+bY) = a^2\mathrm{var}\,(X) + b^2\mathrm{var}\,(Y)$.

$E[g(X)] = \sum g(x)f(x)$ and $E[g_1(X) + g_2(X)] = E[g_1(X)] + E[g_2(X)]$

$E[g(X,Y)] = \sum_x \sum_y g(x,y)f(x,y)$ and $E[g_1(X,Y) + g_2(X,Y)] = E[g_1(X,Y)] + E[g_2(X,Y)]$

Binomial distribution $B(n,p)$

$P(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$ for $k = 0, 1, \ldots, n$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

mean $\mu_X = np$ and standard deviation $\sigma_X = \sqrt{np(1-p)}$.

For large n, X is approximately $N(np, \sqrt{np(1-p)})$.

For large n, $\hat{p} = \frac{X}{n}$ is approximately $N(p, \sqrt{\frac{p(1-p)}{n}})$.

Sample Mean $\overline{X}$ of a random sample of size n from a population with mean μ and standard deviation σ.

mean $\mu_{\overline{X}} = \mu$ and standard deviation $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$

If the population has a normal distribution, $\overline{X}$ is $N(\mu, \sigma/\sqrt{n})$.

For large n, $\overline{X}$ is approximately $N(\mu, \sigma/\sqrt{n})$.

If the population has a normal distribution, $T = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ has t_{n-1} distribution.

CI for Population Mean and Proportion

CI for population mean μ: $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$ or $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$.

CI for a population proportion p : $\widehat{p} \pm z^* \left(\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}\right)$.

Sample size given error margin: $n = \left(\frac{z^*}{m}\right)^2 \hat{p}(1-\hat{p})$.

Conservative sample size given error margin: $n = \left(\frac{z^*}{2m}\right)^2$.