The Circus Example by D. Basu (1971)

The circus owner is planning to ship his 50 elephants and so he needs a rough estimate of the total weight of the elephants. As weighing an elephant is a cumbersome process, the owner wants to estimate the total weight by weighing just one elephant. Which elephant should he weigh? So the owner looks back on his records and discovers a list of the elephants' weights taken 3 years ago. He finds that 3 years ago, Sambo, the middle-sized elephant was the average (in weight) elephant in the herd. He checks with the elephant trainer who reassures him (the owner) that Sambo may still be considered to be the average elephant in the herd. Therefore, the owner plans to weigh Sambo and take $50y$ (where y is the present weight of Sambo) as an estimate of the total weight $Y = Y_1 + Y_2 + \cdots + Y_{50}$ of the 50 elephants. But the circus statistician is horrified when he learns of the owner's purposive sampling plan. "How can you get an unbiased estimate of Y this way?" protests the statistician. So, together they work out a compromise sampling plan. With the help of a table of random numbers they devise a plan that allots a selection probability of 99/100 to Sambo, and equal selection probabilities to each of the other 49 elephants. Naturally, Sambo is selected and the owner is happy. "How are you going to estimate Y?" asks the statistician. "Why? The estimate ought to be $50y$ of course," says the owner. "Oh! No! That cannot possibly be right," says the statistician, "I recently read an article in the *Annals of Matematical Statistics* where it is proved that the Horvitz-Thompson estimator is the unique hyperadmissible estimator in the class of all generalized polynomial unbiased estimators." "What is the Horvitz-Thompson estimate in this case?" asks the owner, duly impressed. "Since the selection probability of Sambo in our plan was 99/100," says the statistician, "the proper estimate of Y is $100y/99$ and not $50y$." "And, how would you have estimated Y," inquires the incredulous owner, "if our sampling plan made us select, say the big elephant Jumbo?" "According to what I understand of the Horvitz-Thompson estimation method," say the unhappy statistician, "the proper estimate of Y would then have been $4900y$, where y is Jumbo's weight." That is how the statistician lost his circus job (and perhaps became a teacher of statistics!).

Reference. Basu, D. (1971). An essay on the logical foundations of survey sampling, Part 1 (with discussion), In: Godambe and Sprott (Eds.), *Foundations of Statistical Inference*, 203–242, Holt, Reinhart and Winston, Toronto. pp. 212-213.