11. Two approaches to resolve ambiguities in aliased effects are:

(i) Use *a priori* knowledge to dismiss some of the aliased effects. If a lower order effect is aliased with a higher order effect, the latter may be dismissed.

(ii) Run a follow-up experiment.

12. Two methods for choosing a follow-up experiment are:

(i) Fold-over technique. Choose the second design by switching the signs of some or all columns in the first design. Its objectives are more narrow and the method is less flexible than the optimal design approach.

(ii) Optimal design approach. Apply an optimal design criterion like the $D$ or $D_0$ criterion to a model consisting of the effects (including all its aliases) identified as significant in the original experiment. It works for any model and shape of experimental region. By using fast optimal design algorithms, it can be used to solve large design problems.

**EXERCISES**

1. Because the $BCQ$ and $DEQ$ in the leaf spring experiment are aliased, the two-step procedure in Section 5.3 has another version by substituting $x_B x_C x_Q$ in (5.9) by $x_D x_E x_Q$. Repeat the analysis in Section 5.3 using the following steps:

   (a) Produce the $D \times E \times Q$ interaction plot. Determine the level of $E$ to reduce dispersion.

   (b) Perform step (i) of the two-step procedure.

   (c) Identify the adjustment factor.

   (d) Perform step (ii) of the two-step procedure.

   (e) Compare the results here with those in Section 5.3 in terms of the $\hat{\sigma}^2$ value, $\hat{y}$ value, and degree of extrapolation of the adjustment factor to move $\hat{y}$ on target.

   (f) If on physical grounds it is known that a heat time between 21 and 27 seconds has a significant effect on free height while the effect of hold-down time on free height is based only on the model for $\hat{y}$, which version of the two-step procedure would be preferred?

2. (a) Choose factor settings over $-1 \leq x_B, x_C, x_D, x_E, x_Q \leq 1$ to minimize $\text{MSE} = (\hat{y} - 8)^2 + \sigma^2$ based on models (5.8) and (5.9).

   (b) For the setting chosen in (a), compute its $\hat{y}$, $\hat{\sigma}^2$, and MSE values. Compare them with those obtained in Section 5.3 and in Exercise 1. If you want to further reduce these values, do you need to relax the range of the factor levels? Which factor would you consider first?
15. (a) What is the resolution of each of the fractional factorial designs indicated below? Which design do you prefer? Justify your answers.

(i) \(2^{6-2}\) with \(5 = 1234, 6 = 124\),

(ii) \(2^{6-2}\) with \(5 = 123, 6 = 124\).

(b) For the design in (ii), if we further know that any two-factor interaction involving factor 6 (i.e., 16, 26, 36, 46, 56) is negligible, which two-factor interactions are estimable under the usual assumptions that three-factor and higher interactions are negligible?

(c) Under the same assumptions as in (b), find a scheme to arrange the design in (ii) in two blocks each of size 8. Explain why your choice is the best.

16. Two choices of generators for a \(2^{6-2}\) design are being considered:

\[
A: \quad 5 = 1234, \quad 6 = 123, \\
B: \quad 5 = 123, \quad 6 = 234.
\]

(a) Which design, \(A\) or \(B\), would you recommend? Why?

(b) Show that it is impossible to have a \(2^{6-2}\) design. (Hint: Count the degrees of freedom.)

17. Find the most economical design that can estimate the main effects of five factors (each with two levels) and all their two-factor interactions. What is the resolution of this design? If it is known that the level combination (+, +, +, +, +) of the five factors can lead to disastrous results (e.g., explosion, burn-out), specify how the design should be chosen.

18. A \(2^{k-p}\) design is said to be saturated if the number of factors equals the run size minus 1. Show that it can be formally written as a \(2^{(2^r-1)-(2^r-1)}\) design. Denote by \(1, 2, \ldots, r\) the \(r\) independent columns that generate all the \(2^r\) runs. Show how the remaining columns in the \(2^{(2^r-1)-(2^r-1)}\) design can be generated from \(1, 2, \ldots, r\).

19. Suppose that a letter does not appear in any of the defining words of length 3 or 4 for a \(2^{k-p}\) design. Prove that the main effect represented by this letter is strongly clear and all the two-factor interactions involving this letter are clear. (Note: Two such examples can be found in Sections 5.2 and 5.4.1.)

20. An experimenter considers running a \(2^{7-3}\) design. He is contemplating two possibilities: design \(A\) with generators \(5 = 123, 6 = 124, 7 = 1234\) and design \(B\) with generators \(5 = 123, 6 = 124, 7 = 134\). Which design is better? Why?
21. (a) By comparing the two graphs in Figures 5.7 and 5.8, one would note that the former has one more line than the latter. Identify this line.

(b) By using the assignment of factors A, B, C, D, E to columns 2, 5, 3, 4, 6 as in Section 5.5, identify which additional interaction among the factors can be estimated clearly.

22. Consider the $2^{16-10}$ design with generators $7 = 123, 8 = 124, 9 = 134, 
   t_0 = 234, t_1 = 125, t_2 = 135, t_3 = 145, t_4 = 126, t_5 = 136, t_6 = 123456$.
   (Note that, as in Appendix 5A, $t_0, t_1, \ldots, t_6$ denote factors 10, 11, \ldots, 16.)

(a) Show that the following 18 two-factor interactions (2fi’s) are clear: $t_1t_6, t_2t_6, t_3t_6, t_4t_6, t_5t_6, t_3t_4, t_3t_5, t_3t_6, t_4t_5, t_5t_6$.

(b) Represent the 18 2fi’s in (a) in a graph.

(c) Draw a graph to represent the 29 clear 2fi’s for the second $2^{16-10}$
   design in Table 5A.5.

(d) Show that the graph in (b) is isomorphic to a subgraph of the graph in (c). (Note: Two graphs are isomorphic if they are identical after relabeling the nodes of one graph.)

(e) Based on the finding in (d), argue that the design considered in this exercise is inferior to the one in part (c) in terms of the ability to clearly estimate two-factor interactions.

23. Consider the $2^{15-9}$ design with generators $7 = 123, 8 = 124, 9 = 134, 
   t_0 = 234, t_1 = 125, t_2 = 135, t_3 = 126, t_4 = 146, t_5 = 123456$.

(a) Show that it has the following 19 clear 2fi’s: $t_1t_5, t_2t_5, t_3t_5, t_4t_5, t_5t_5, 
   t_6t_5, t_7t_5, t_8t_5, t_9t_5, t_{10}t_5, t_{11}t_5, t_{12}t_5, t_{13}t_5, t_{14}t_5$.

(b) Represent the 19 2fi’s in (a) in a graph.

(c) Draw a graph to represent the 27 clear 2fi’s for the second $2^{15-9}$ design
   in Table 5A.5.

(d) Show that the graph in (b) is not a subgraph of the graph in (c) even though the latter has eight more lines.

(e) Argue that, except in rare situations, the design considered in this exercise is inferior to the one in part (c) in terms of the ability to clearly estimate two-factor interactions. Identify these rare situations.

24. An experimenter who wishes to use a $2^{8-2}$ design can only do 16 runs in a day and would like to include “day” as a blocking variable. What design would you recommend? Why? Give the treatment generators and the block generators for your design and the collection of clear effects.

25. It was shown in Section 5.2 that the $2^{11-2}$ design with $I = 125 = 1346 = 23456$ has nine clear effects: $3, 4, 6, 23, 24, 26, 35, 45, 56$. If this design
is arranged in four blocks with \( B_1 = 13 \) and \( B_2 = 14 \), show that the same nine effects are still clear. \( \text{Hint: } 13 \text{ and } 14 \text{ do not appear in the five-letter word } 23456. \)

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26. An experimenter used the following design for studying five variables in eight runs in four blocks of size 2.

(a) By reordering the runs, write down the design matrix in four blocks of size 2. This design matrix should contain five columns and eight rows. Indicate which two run numbers occur in each block.

(b) Explain why and under what conditions the main effect 3 is (or is not) confounded with a block effect.

(c) Explain why and under what conditions the main effect 4 is (or is not) confounded with a block effect.

*27. (a) Prove that it is impossible to arrange the \( 2^{5-1} \) design with \( I = 12345 \) in eight blocks without confounding the main effects.

(b) Show that the \( 2^{5-1} \) design with \( I = 1235 \) can be arranged in eight blocks with \( B_1 = 14, B_2 = 24, \) and \( B_3 = 34 \). It is somewhat counterintuitive that a resolution IV design can be arranged in the largest possible number (eight in this case) of blocks while a resolution V design cannot. Explain in intuitive terms why maximum resolution alone cannot guarantee maximal blocking. [Note: A theoretical characterization and explanation of this phenomenon can be found in Mukerjee and Wu (1999).]

28. In a resistance spot welding experiment, five factors were chosen to study their effects on the tensile strength, which is the maximum load a weld can sustain in a tensile test. The five factors are: button diameter (A), welding