

Instructions Attached to this is a hard copy of computer code that you may use to do problem 3 in the homework. Enter the code, and modify it if you need to, to answer the questions.

Work on this problem with the Code given in separate hard copy The number of pump failures Y_i over time periods t_i in 10 power plants is given below:

Y_i	5	1	5	14	5	19	1	1	4	22
t_i	94.320	15.72	62.880	125.760	5.240	31.440	1.048	1.048	2.096	10.480

We consider a hierarchical event rate model with a Poisson likelihood $Y_i \sim Poi(\lambda_i t_i)$ and a prior model

$$\lambda_i \sim Ga(\alpha, \beta) \quad i = 1, \dots, 10$$

$$\beta \sim Ga(c, d)$$

where (α, c, d) are fixed hyperparameters.

- Write down the joint posterior distribution for the relevant parameters
- Show that the full conditional distributions needed to do a Gibbs sampling are:
 If λ_j ($j \neq i$) and β are given,

$$P(\lambda_i | \lambda_j, \beta, y) \propto \lambda_i^{\alpha+y_i-1} e^{-\lambda_i(\beta+t_i)}$$

$$\sim Ga(\alpha + y_i, \beta + t_i)$$

If all the λ_i 's are given, $i = 1, \dots, 10$

$$P(\beta | \lambda_1, \lambda_2, \dots, \lambda_{10}, y) \propto \beta^{(n\alpha+c-1)} e^{-\beta(\sum \lambda_i + d)}$$

$$\sim Ga(c + n\alpha, d + \sum \lambda_i)$$

- Use the code given to you in the lab, modified, if you need to do the remaining questions in this problem. First, run your gibbs 1000 times to simulate from the posterior distribution of the parameters using the following values for the hyperparameters: $c = 0.1, d = 0.1, \alpha = 1$ and the following initial values for the sampler:

$$\beta = 1, \lambda = (0.1, 0.1, \dots, 0.1)$$

Give the 5 number summary for your parameters (without the burn in) and comment on your findings. Include 95% posterior intervals. Check that the sampler has converged. Show trace plot of the 1000 iterations for each parameter.

- Run again the Gibbs sampler 5000 times, but now with initial values

$$\beta = 2, \lambda = (0.5, 0.5, \dots, 0.5)$$

Give the 5 number summary for your parameters (without the burn in) and comment on your findings. Check that the sampler has converged. Show trace plot of the 1000 iterations for all parameters. Compare with the results you got with the previous initial values.

- Plot the posterior distributions of the parameters in one graph containing boxplots for all the parameters. Comment on the results.
- Attach your code indicating question number. Make sure that you put comments in this code, saying what each section of the code is doing and commenting what the lines in the code are doing.