Instructions

(1) Homework must be typed and answered in the order given (problem 1(a)(b)(c)(d) first, problem 2(a)(b)... second, etc...)

(2) Undergrads and grads will answer all questions.

(3) Include in each part of the homework only the answer. R code and R output (without mistakes), must be included in the appendix to the question. For example, for question 1.a, write only the answer and your comments. The code and output for that part of the question will be in the appendix (the last part of question 1).

(4) No late homework under any circumstances.

(5) Write your name and ID this way: Last name, first name, UCLA ID, date, Homework number.

(6) Do not just give a number as an answer. For example, if asked for probability that posterior proportion is larger than 0.7, write $Prob(p > 0.7) = 0.3$, say and write comments or explanations if needed.

(7) The homework must be turned in in lecture (no mail box, no e-mail).

Problem 1. An experiment was performed to estimate the effect of beta-blockers on mortality of cancer patients. A group of patients were randomly assigned to treatment and control groups: out of 674 patients receiving the control, 39 died, and out of 680 receiving the treatment, 22 died. Assume that the outcomes are independent and binomially distributed, with probabilities of death of θ_1 and θ_2 under the control and treatment, respectively.

Let y_c be the number of patients that died in the control group and y_t the number of patients that died in the treatment group and let n_c be the number of patients in the control group and n_t the number of patients in the treatment group. Let also θ_1 be the population proportion of patients that died in the control group and θ_2 the population proportion of patients that died in the treatment group.

$n_c = 674 \qquad y_c = 39$ and $n_t = 680 \qquad y_t = 22$.

The likelihood function is

$$L \propto \theta_1^{39}(1-\theta_1)^{635}\theta_2^{22}(1-\theta_2)^{658}$$

Consider as prior a Dirichlet with $\alpha_1 = 1$ and $\alpha_2 = 1$. This means that $p(\theta_1, \theta_2) = 1$.

Under this non-informative prior, the posterior distribution is

$$p(\theta_1, \theta_2) \propto \theta_1^{39}(1-\theta_1)^{635}\theta_2^{22}(1-\theta_2)^{658}$$

(a) Obtain posterior simulations for the joint distribution of θ_1 and θ_2. Plot contour plots and a random sample from the joint distribution.

Hint: For this and the next part, use as template, and modify as needed, the template used in Outline 7 (our class notes). Comment and interpret a little the plots.

(b) The odds ratio is defined as $(\theta_2/(1-\theta_2))/(\theta_1/1-\theta_1))$. Summarize the posterior distribution for this estimand (mean, mode, median, standard deviation, 95% plot, etc..

Hint: to be able to obtain the posterior distribution of this function of θ_1 and θ_2, you will need first the marginal posterior distributions of θ_1 and θ_2 and then sample from them. Then, with the numbers obtained, you create the odds ratio and work with the distribution of this odds ratio.

(c) Attach a well documented R code, i.e., add comments that illustrate what the code is doing, and separate your functions with headers that make them easy to find.

Problem 2. This is a problem on material from Chapter 5 of Hoff's book. Thirty two students in a science class were randomly assigned to one of two study methods, A and B, so that $n_A = n_B = 16$ students were assigned to each method. After several weeks of study, students were examined on the course material with an exam designed to give an average score of 75 with a standard deviation of 10. The scores for the two groups are summarized by $\{\bar{y_A} = 75.2; s_A = 7.3\}$ and $\{\bar{y_B} = 77.5; s_B = 8.1\}$. Consider independent, conjugate normal prior distributions for each of θ_A and θ_B, with $\mu_0 = 75$ and $\sigma_0^2 = 100$ for both groups. For each $(\kappa_0, \upsilon_0) \in \{(1,1),(2,2),(4,4),(8,8),(16,16),(32,32)\}$, obtain $Pr(\theta_A < \theta_B \mid y_A, y_B)$ via Montecarlo sampling. Plot this probability as a function of $(\kappa_0 = \upsilon_0)$. Display how you may use this plot to convey the evidence that $\theta_A < \theta_b$ to people of a variety of prior opinions.

Hint: Do separately, for each group, the procedure described on page 77-78 of Hoffs book, to obtain random draws from the joint distribution and from there the marginal distribution of each theta. We have done draws like these before, in section 6.4 of our course notes, but applied to the other distributions. Read also section 6.4 to remind yourselves of what we did then.

Attach your R code.