Testing hypotheses about π

What is hypothesis testing about π? We evaluate a conjecture (which we will call a hypothesis) about the parameter π.

Procedure for testing Hypotheses about π : Steps are the same as for μ, but will use the sampling distribution of p to compute the P-value, assuming the null is true and the z statistic. Must have $n\pi > 10$ $n(1-\pi) > 10$ and random sample. We will see this with examples.

Example 1

With a perfectly balanced roulette wheel, in the long-run red numbers should turn up 18 times in 38. To test its wheel, one casino records the results of 3,800 plays finding 1000 red numbers. Is that too few reds or chance variation?

A roulette has 38 numbers, 18 reds, 18 black and 2 green. Let1 represents red and the 0 represents other (black and green).

Null hypothesis: $\pi = 18/38 = 0.47$

Alternative hypothesis: $\pi < 0.47$.

If the null hypothesis is true, then the percentage of reds we would expect in a sample of 1000 plays is 47% and the SE = $\frac{\sqrt{0.47 \times (1-0.47)}}{\sqrt{3800}}$ = 0.0080 or 0.811%.

We observe in our sample that 26.3% are red. This is 25.5 SE below the expected value

$$z = \frac{26.3\% - 47\%}{0.81\%} = -25.5$$

The p-value is approximately zero. This is the chance that the average is smaller than 26.3% just by chance if the null hypothesis is true.
Since the p-value < 0.01 , we reject the null hypothesis at the 0.01 level of significance. The data result is highly significant.

Answer: There is evidence to conclude that the roulette wheel is not fair (not working well), and it gives too few reds.

Example 2

With a perfectly balanced roulette wheel, in the long run red numbers should turn up 18 times in 38. To test its wheel, one casino records the results of 3,800 plays, finding 1,890. Is that too many reds?

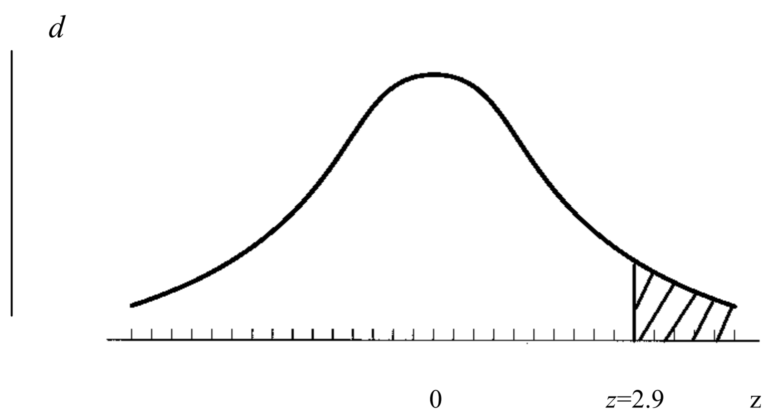
Null hypothesis: $\pi = 18/38 = 0.47$
Alternative hypothesis: $\pi > 0.47$.

If the null hypothesis is true, then the percentage of reds we would expect in a sample of 1000 plays is 47% and the SE = $\dfrac{\sqrt{0.47 \times (1-0.47)}}{\sqrt{3800}} = 0.0080$ or 0.811%.
We observe in our sample that 49.73% are red. This is 25.5 SE below the expected value

$$z = \frac{49.73\% - 47\%}{0.81\%} = 3.37$$

The p-value is approximately zero. This is the chance that the sample proportion p is larger than 49.73% just by chance if the null hypothesis is true or the chance that z is larger than 3.37.
Since the p-value < 0.01 we reject the null hypothesis at the 0.01 level of significance. The data result is highly significant.

Answer: There is evidence to conclude that the roulette wheel is not fair (not working well), and it gives too many reds.



Example 3
With a perfectly balanced roulette wheel, in the long run red numbers should turn up 18 times in 38. To test its wheel, one casino records the results of 3,800 plays, finding 1,890. Does that mean that the roulette may not be working properly?

(Notice how, here, we are not giving any hint as to the direction of the alternative hypothesis so we will have to do a two-sided hypothesis).

Null hypothesis: $\pi = 18/38 = 0.47$
Alternative hypothesis: $\pi \neq 0.47$.

Example 4

In the U.S. there are two sources of National statistics on crime rates: (I) the FBI's Uniform Crime Reporting Program, which publishes summaries on all crimes reported to police agencies in jurisdictions covering virtually 100% of the population; (ii) the National Crime Survey, based on interviews with a nationwide probability sample of households.
In 1992, 4.9% of the hoseholds in the sample told the interviewers they had experienced at least one burglary within the past twelve months. The same year, the FBI reported a burglary rate of 32 per 1000 households, or 3.2%. Can this difference be explained by chance error? If not, how would you explain it? You may assume that the survey is based on a simple random sample of 50,000 households out of 100 million households.