Graded: problem p.235 #3 (1 pt)
P 235 #10 (1pt)
p. 286 #9 (1pt)

Stat 10/Sanchez
Hwk 6 answer key

p.235, no 3.- Four cards will be dealt off the top of a well-shuffled deck. There are two options:

(i) To win \$1 if the first card is a club, and the second is a diamond, and the third is a heart, and the fourth is a spade.

(ii) To win \$1 if the four cards are of four different suits.

Which option is better? Or are they the same? Explain.

As explained in the textbook on page 226 section 2, a deck of cards has 4 suits: clubs, diamonds, hearts, spades. Thre are 13 cards in each suit: 2 through 10, jack, queen, king, ace. So there are 4x13=52 cards in the deck.

Option (i): Prob(the first card is a club, and the second is a diamond, and the third is a heart, and the fourth is a spade) = Prob(winning \$1)=
(13/52)(13/51)(13/50)(13/49) = 0.00439.

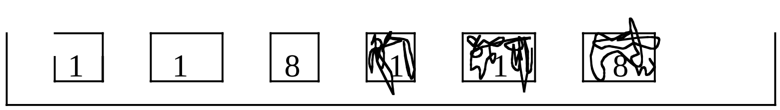
Option (ii): Prob(the four cards are of four different suits) = Prob(winning 1\$) =
(prob of being of any suit)(prob of being of another suit given that the first
(52/52)(39/51)(26/50)(13/49) = 0.10549

Since option (ii) has a higher probability of winning \$1, option (ii) is better.
Notice how this problem uses conditional probability.

p. 235, no. 4.- A poker hand is dealt. Find the chance that the first four cards are aces and the fifth is a king.

As explained in the textbook on page 226 section 2, a deck of cards has 4 suits: clubs, diamonds, hearts, spades. Thre are 13 cards in each suit: 2 through 10, jack, queen, king, ace. So there are 4x13=52 cards in the deck.
There are 4 aces and 4 kings in a deck of cards.
Prob(the first four cards are aces and the fifth is a king)=
(4/52)(3/51)(2/50)(1/49)(4/48)=
(1/3,000,000).

p. 235, no. 5.- One ticket will be drawn at random from the box below. Are color and number independent? Explain.



Yes, they are independent.
Let C be color and let N be number. Two events are independent if the
Prob(N|C) = Prob(N)
Is that true here?
Prob(1|white)=2/3 =Prob(1) = (4/6)=(2/3)
Prob(8/white)=1/3=Prob(8)=(2/6) = 1/3
This is the same as saying Prob(1/white)=prob(1/black)=2/3
And Prob(8/white)=Prob(8/black)=1/3

p. 235, no. 6.- A deck of cards is shuffled and the top two cards are placed face down on a table. True or false, and explain:

(a) There is 1 chance in 52 for the first card to be the ace of clubs
(b) There is 1 chance in 52 for the second card to be the ace of diamonds
(c) The chance of getting the ace of clubs and then the ace of diamonds is (1/52)x (1/52)

As explained in the textbook on page 226 section 2, a deck of cards has 4 suits: clubs, diamonds, hearts, spades. Thre are 13 cards in each suit: 2 through 10, jack, queen, king, ace. So there are 4x13=52 cards in the deck.

(a) True. They are not asking for the chance that the first card is an ace, but for the chance that it is a particular card: the ace of clubs (one among the 52 cards. So
Prob (first card is ace of clubs) =1/52
(b) True. If you have not seen the first card, the second one could be any of the other 52.
Prob(second card is ace of clubs) = 1/52.
(c) False. Once you get the ace of club, there are 51 cards left in the deck. So the chance is (1/52)(1/51)

p.235, no 7.- A coin is tossed six times. Two possible sequences of results are:
(i) H T T H T H (ii) H H H H H H
(The coin must land H or T in the order given; H=heads, T = tails) which of the following is correct? Explain.

(a) Sequence (i) is more likely
(b) Sequence (ii) is more likely
(c) Both sequences are equally likely

(c)
Assume: the coin is fair, so that the probability of a head is always 1/2 and does not change as you toss the coin.
Remember that if events are independent the probability of them happening together is the product of their probabilities. So the probability of the first sequence (i) is $\left(\frac{1}{2}\right)^6$

But this is also the probability of the second sequence. So (c) is correct.

p. 235, no. 8.- A die is rolled four times. What is the chance that

(a) all the rolls show 3 or more spots
(b) none of the rolls show 3 or more spots
(c) not all the rolls show 3 or more spots

For all answers assume the die is fair and the probabilities don't change as you roll it.

(a) Since what you get in one roll is independent of what you get in the other,

Prob(3 or more spots, 3 or more spots, 3 or more spots, 3 or more stops) = $(4/6)^4$

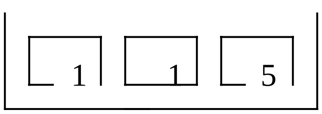
(b) Since what you get in one roll is independent of what you get in the other,

Prob(no 3 or more spots, no 3 or more spots, no 3 or more spots, no 3 or more spots) = Prob(2 or less spots, 2 or less spots, 2 or less spots, 2 or less spots) = $(2/6)^4$

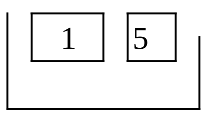
(c) Prob(anything that is not the event in part a) =Prob(complement of event in (a)) = $1-(4/6)^4$

p. 235. no 10.- One hundred draws will be made at random with replacement from one of the two boxes shown below. On each draw, you will be paid the number on the ticket, in dollars. Which box is better? About how much do you expect to make, using that box?

(i)



(ii)



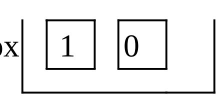
Prefer box (ii) because I expect to make more money in one hundred draws from it, than in 100 draws from box (i).

In box (i), prob(1) = 2/3
Prob(5) = 1/3
Expect to make : 100(2/3)1+100(1/3)5
= $233.1

In box (ii), prob(1) =1/2, prob(5)= 1/2
Expect to make: 100(1/2)*1 + (100)(1/2)(5)= $300.*

p.236, no. 11.- There are two options:

(i) You toss a coin 100 times; on each toss, if it lands heads you win $1., if it lands tails you lose $1.
(ii) You draw 100 times at random with replacement from the box



On each draw, you are paid (in dollars) the number on the Ticket.

Which option is better? Or are they the same? Explain briefly.

Assume the coin is fair. Then (ii) is preferred because you can expect more money from 100 draws from it than from 100 draws from box in (i).

(i) The box for this part is | *$1, -$1* |

So prob of winning a dollar (getting a head) is 1/2 and prob of losing a dollar (getting a tail) is 1/2.
If you draw 100 numbers from this box (equivalent to tossing a coin 100 times), you would expect to get: 100(1/2)1 -100(1/2)1 = 0

(ii) With the box in this part, you expect to make 100(1/2)1+100(1/2)0=50 dollars.

p. 285. no. 3.- A gambler loses ten times running at roulette. He decided to continue playing because he is due for a win, by the law of averages. A bystander advises him to quit, on the grounds that his luck is cold. Who is right? Or are both of them wrong?

None of them is right.
There is no "hot hand." The law of averages ***does not*** *say that after many plays you will win or not. It only says that after many plays the proportion of wins will approach the expected proportion of wins (the probability of winning) and the number of times you win will get farther and farther from the expected number of wins (i.e. chance error increases with n, but chance error as a percentage of the number of plays, decreases with n). Anything could happen in between, just by chance.*

p. 286. no. 6.- As genetic theory shows, there is very close to an even chance that both children in a two-child family will be of the same sex. Here are two possibilities.

(i) 15 couples have two children each. In 10 or more of these families, it will turn out that both children are of the same sex

(ii) 30 couples have two children each. In 20 or more of these families, it will turn out that both children are of the same sex.

Which possibility is more likely and why?
(*i) is more likely.*
The box model for this situation is the following:

| *A.-Couple with two Kids of the same sex* | *B.-Couple with two kids of different sex.* |

So, as the problem says, the chance of seeing a couple of each kind is 1/2. Now imagine

(i) *drawing 15 couples (observed proportion of As =10/15=2/3. Expected proportion of As = 1/2. Chance error as proportion of draws = (2/3)-(1/2)= 0.167 or 16.7%*

(ii) *drawing 30 couples (observed proportion of As = (20/30)=(2/3). Expected proportion of As = 1/2. Chance error as proportion of draws = 0.167 or 16.7%.*

The law of averages says that the chance error as a percentage of the number of draws gets smaller as n increases. So one would expect to see a smaller chance error (as %) in (ii). That is, (ii) is not very likely to be seen if (i) is the way it is. So choose (i) as the more likely possibility to be observed.

p. 286. no. 7.- A quiz has 25 multiple choice questions. Each question has 5 possible answers, one of which is correct. A correct answer is worth 4 points, but a point is taken off for each incorrect answer. A student answers all the questions by guessing at random. The score will be like the sum of ___________ draws from the box ___________. Fill in the first blank with a number and the second with a box of tickets. Explain your answers.

Answer is: 25 draws from the box | *4, -1, -1, -1, -1* |

With replacement.
Each question has 5 answers: one right and 4 wrong. So the chance of getting the right answerand therefore getting 4 points, is 1/5 and the chance of getting the wrong answer and therefore losing one point is 4/5. Doing the quiz of 25 questions at random is like drawing from the box given above 25 times with replacement.

p. 286.no 8.- A gambler will play roulette 50 times, betting a dollar on four joining numbers each time (like 23, 24, 25, 26 in figure 3, p.282). If one of these four numbers comes up, she gets the dollar back, together with winnings of \$8. If any other number comes up, she loses the dollar. So this bet pays 8 to 1, and there are 4 chances in 38 of winning. Her net gain in 50 plays is like the sum of __________ draws from the box ______; fill in the blanks. Explain.

A roulette has 38 numbers (p.281 in book):
Answer is: 50 draws with replacement from the box

| *4 tickets* [*\$8*] *34 tickets* [*-1*] |

p. 286. no 9.- A box contains red and blue marbles; there are more red marbles than blue ones. Marbles are drawn one at a time from the box, at random with replacement. You win a dollar if a red marble is drawn more often than a blue one. There are two choices:

(A) 100 draws are made from the box
(B) 200 draws are made from the box

Choose one of the four options below; explain your answer:

(i) A gives better chance of winning
(ii) B gives a better chance of winning
(iii) A and B give the same chance of winning
(iv) Can't tell without more information.

Answer is: I would choose ii.
The box we are talking about is:

More than 1/2 Red   less than 1/2 Blue

So the true prob(red) > 1/2.

You win a dollar if the proportion of red marbles is larger than that of blue marbles, that is, if you get a proportion > 0.5 of reds.
We know by the law of averages that chance error relative to the number of draws = (proportion of reds you get)- (proportion of reds expected) gets smaller and smaller as n increases. So use that to decide between the choices. So n=200 will get you closer to the true proportion of reds (> 1/2) than n=100, and therefore n=200 will get you closer to winning the $1.

p.286. no 10.- Two hundred draws will be made at random with replacement from the box

-3, -2, -1, 0, 1, 2, 3

(a) If the sum of the 200 numbers drawn is 30, what is their average?
(b) If the sum of the 200 numbers drawn is –20, what is their average?
(c) In general, how can you figure the average of the 200 draws, if you are told their sum?
(d) There are two alternatives:
   (i) Winning $1 if the sum of the 200 numbers drawn is between –5 and +5

(ii) Winning $1 if the average of the 200 numbers drawn is between –0.025 and +0.025.

Which is better, or are they the same?

The number of draws is n=200

(a) average=sum/n = (30/200) = 0.15

(b) average=sum/n = (-20/200) = -0.1

(c) Average = sum/200

(d) (i) and (ii) are the same. –0.025 = (-5/200) and 0.025=(5/200). So if –5 happens, -0.025 is happening, too, and if 5 happens,