

Stat 13 Lecture 19 discrete random variables, binomial

- A random variable is discrete if it takes values that have gaps : most often, integers
- Probability function gives $P(X=x)$ for every value that X may take
- X = number of heads in 3 tosses
- A couple decides to have children till at least one of each sex or a maximum of 3; X =number of girls. (tree)

Expected value and standard deviation

- $E(X) = \text{sum of } P(X=x) \cdot x$ (weighted average; using probability as weight)
- $\text{Var}(X) = \text{sum of } P(X=x) \cdot (x - E(X))^2$

Binomial probability

- Coin tossing ; multiple choices ; formula of binomial ; combination number
- $P(X=x) = \binom{n}{x} p^x (1-p)^{(n-x)}$
- Sampling with replacement
- Sampling without replacement; infinite population
- Sampling without replacement, finite population; (opinion) survey sampling

Conditions for binomial to hold

- Model the number of successful trials out of n trials
- Must know n
- Must know (or be able to estimate) p (=prob of success in each trial)
- Must satisfy independence assumption in different trials
- p should be the same in each trial

Sampling without replacement

- Suppose in a population of N individuals, a random sample of n individuals are selected. Their opinions on a proposal are recorded. Suppose in the population the proportion of individuals saying yes is p . Then X , the number of individuals in the sample saying yes follows a hypergeometric distribution
- $P(X=x) = \frac{[Np \text{ choose } x][N(1-p) \text{ choose } (n-x)]}{[N \text{ choose } n]}$, which is approximately equal to binomial when N is large and the sampling fraction n/N is small.

