

Lecture 13 Chi-square and sample variance

- Finish the discussion of chi-square distribution from lecture 12
- Expected value of sum of squares equals $n-1$.
- Why dividing by $n-1$ in computing sample variance
- It gives an unbiased estimate of true variance of measurement error
- Testing hypothesis about true SD of measurement error
- Confidence interval about the true SD of measurement error.

Brownian motion-continued

- How big the radius should be in order to find the particle within the circle with probability .95 at two minutes after releasing it from the origin?
 - Let (X_1, Y_1) be the position at one minute
 - Then $X_1, Y_1 \sim \text{Normal}(0,1)$
 - Let $(X_2, Y_2) \sim \text{Normal}(0,1)$
 - Then the position at 2 minute can be represented as (X_1+X_2, Y_1+Y_2)
 - Let $X=X_1+ X_2$ which has variance $1+1=2$
 - Let $Y= Y_1+ Y_2$, which has variance $1+1=2$
- So squared distance from origin is $D^2=X^2+ Y^2$

- How is D^2 related to chi-square?
- It is not a chi-square distribution with $df=2$
- But $D^2/2$ is.
- Therefore we can use chi-square table to find the cutting point C as done before ($C=5.991$)
- Then set up the equation $P(D^2/2 < C)=0.95$
- This means $P(D^2 < 2C)=.95$
- So the radius must be square root of $(2C)$

Measurement error = reading from an instrument - true value

One biotech company specializing microarray gene expression profiling claims they can measure the expression level of a gene with an error of size .1 (that is, after testing their method numerous times, they found the standard deviation of their measurement errors is 0.1) The distribution of errors follow normal distribution with mean 0 (unbiased).

Cells from a tumor tissue of a patient are sent to this company for Microarray assay. To assure consistency, the company repeat the assay 4 times. The result of one gene, P53 (the most well-studied tumor suppressor gene), is 1.1, 1.4, 1.5, 1.2.

Is there enough evidence to reject the company's claim about the accuracy of measurement? Note that sample SD is $\sqrt{0.1/3}$, Bigger than 0.1.

This problem can be solved by using chi-squared distribution. We ask How likely it is to observe a sample SD this big and if the probability is Small, then we have good evidence that the claim may be false . (next lecture)