Lecture 14 chi-square test, P-value

- Measurement error (review from lecture 13)
- Null hypothesis; alternative hypothesis
- Evidence against null hypothesis
- Measuring the Strength of evidence by P-value
- Pre-setting significance level
- Conclusion
- Confidence interval

Some general thoughts about hypothesis testing

- A claim is any statement made about the truth; it could be a theory made by a scientist, or a statement from a prosecutor, a manufacture or a consumer
- Data cannot prove a claim however, because there
- May be other data that could contradict the theory
- Data can be used to reject the claim if there is a contradiction to what may be expected
- Put any claim in the *null* hypothesis H_0
- Come up with an alternative hypothesis and put it as H_1
- Study data and find a hypothesis testing statistics which is an informative summary of data that is most relevant in differentiating H_1 from H_0.

- Testing statistics is obtained by experience or statistical training; it depends on the formulation of the problem and how the data are related to the hypothesis.
- Find the strength of evidence by P-value :

from a future set of data, compute the probability that the summary testing statistics will be as large as or even greater than the one obtained from the current data. If P-value is very small , then either the null hypothesis is false or you are extremely unlucky. So statistician will argue that this is a strong evidence against null hypothesis.

If P-value is smaller than a pre-specified level (called significance level, 5% for example), then null hypothesis is rejected.

Back to the microarray example

- H_o : true SD σ=0.1 (denote 0.1 by σ_0)
- H_1 : true SD $\sigma > 0.1$ (because this is the main concern; you don't care if SD is small)
- Summary :
- Sample SD (s) = square root of (sum of squares/ (n-1)) = 0.18
- Where sum of squares = $(1.1-1.3)^2 + (1.2-1.3)^2 + (1.4-1.3)^2 + (1.5-1.3)^2 = 0.1$, n=4
- The ratio s/ σ =1.8 , is it too big ?
- The P-value consideration:
- Suppose a future data set (n=4) will be collected.
- Let s be the sample SD from this future dataset; it is random; so what is the probability that s/ will be
- As big as or bigger than 1.8 ? $P(s/\sigma_0 > 1.8)$

- P(s/ σ_0 >1.8)
- But to find the probability we need to use chi-square distribution :
- Recall that sum of squares/ true variance follow a chi-square distribution ;
- Therefore, equivalently, we compute
- P (future sum of squares/ σ_0^2 > sum of squares from the currently available data/ σ_0^2), (recall σ_0 is
- The value claimed under the null hypothesis) ;

Once again, if data were generated again, then Sum of squares/ true variance is random and follows a chi-squared distribution

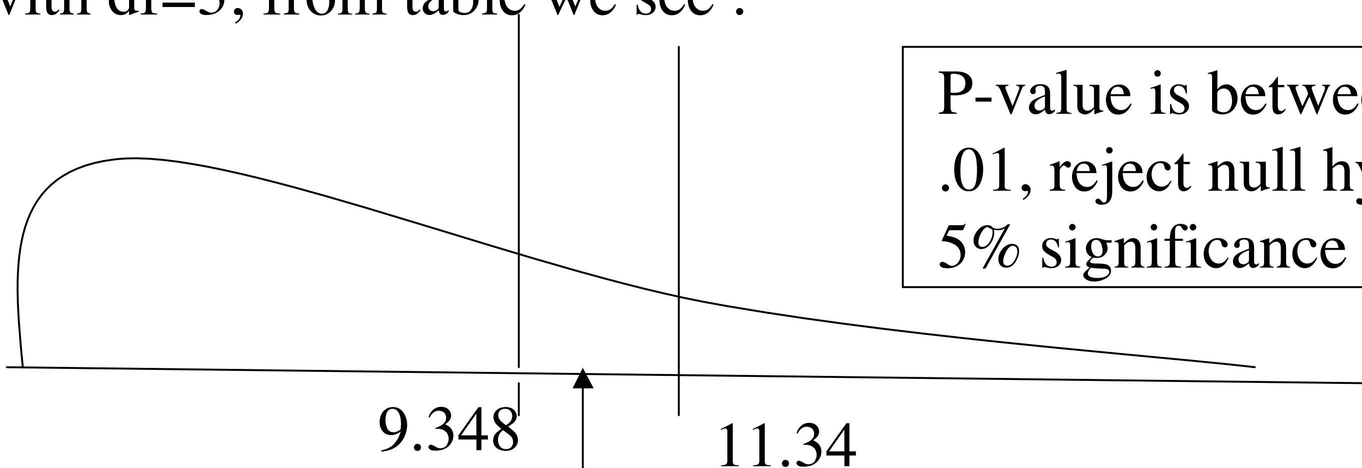
with n-1 degrees of freedom; where sum of squares= sum of squared distance between each data point and the sample mean

Note : Sum of squares= (n-1) sample variance = (n-1)(sample SD)^2

P-value = P(chi-square random variable> computed value from data)=P (chisquare random variable > 10.0)

For our case, n=4; so look at the chi-square distribution with df=3; from table we see :

P-value is between .025 and .01, reject null hypothesis at 5% significance level



The value computed from available data = .10/.01=10 (note sum of squares=.1, true variance =.1^2

Confidence interval

- A 95% confidence interval for true variance σ^2 is
- (Sum of squares/C_2, sum of squares/C_1)
- Where C_1 and C_2 are the cutting points from chi-square table with d.f=n-1 so that
- P(chisquare random variable > C_1)= .975
- P(chisquare random variable>C_2)=.025
- This interval is derived from
- P(C_1< sum of squares/ σ^2 <C_2)=.95

For our data, sum of squares= .1 ; from d.f=3 of table, C1=.216, C2=9.348; so the confidence interval of σ^2 is 0.1017 to .4629; how about confidence interval of σ ?