Lecture 6 Correlation

Stock example: stock prices are likely to be correlated. Need a measure of strength of correlation.

- Microarray example
- Defining correlation :
- Procedure of computing correlation
- (1)standardize x, (2)standardize y, (3)average product of standardized x and standardized y
- properties.: between -1 and 1
- Three special cases : perfect positive relationship= 1, perfect negative relationship= -1 and no correlation =0
- Back to the stock example.

Definition of correlation coefficient

Correlation remains the same under any scale changes

If X and Y both have mean 0 and variance 1, then correlation coefficient

$\rho = E(XY)$

For the general case, standardize each variable first.

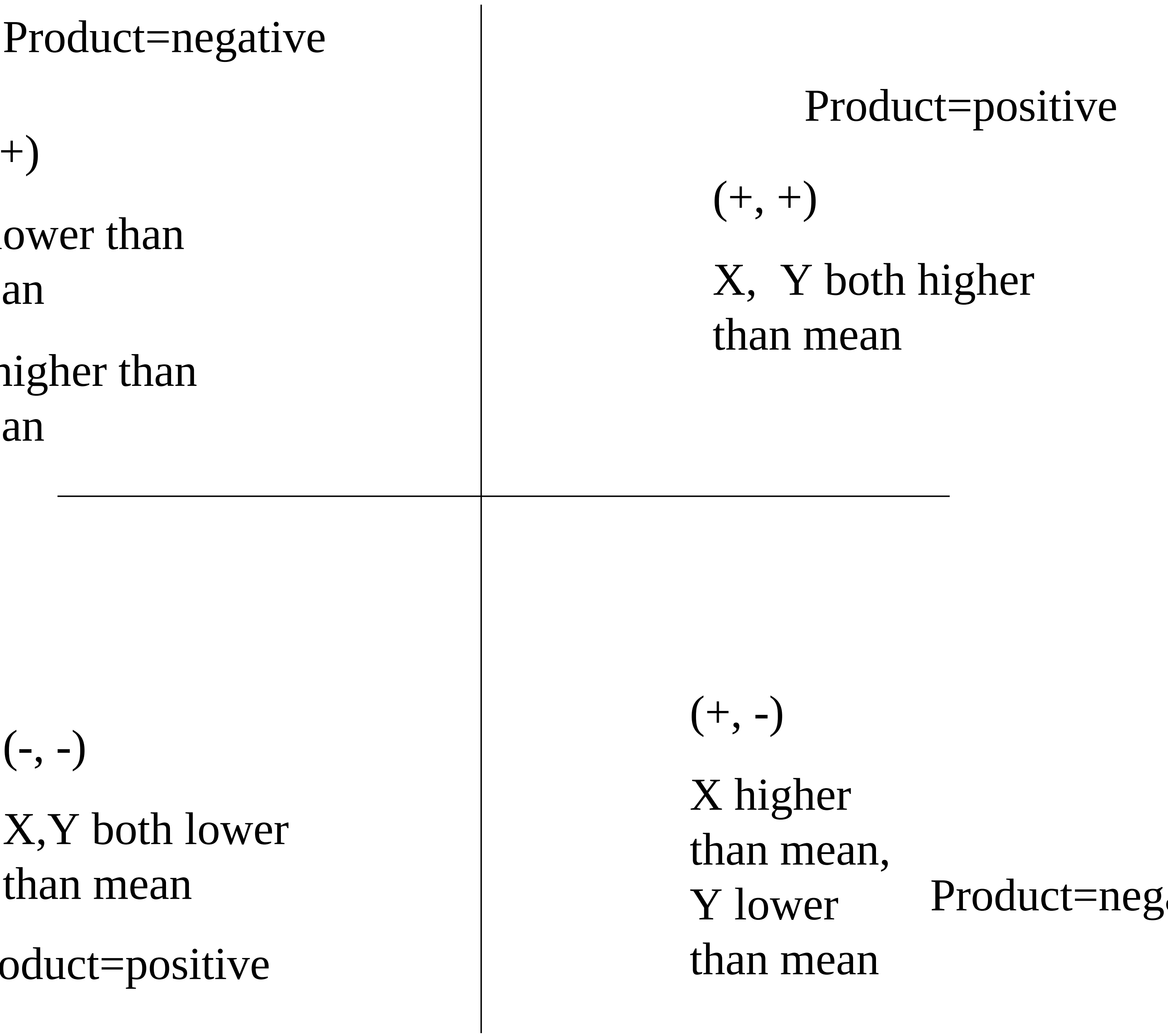
If you forgot to divide by SD, then you obtained a quantity ca
Covariance, which is still useful (see next page)

Cov (X, Y) = E (X-mean of X)(Y-mean of Y)

Without subtracting the mean, you got E(XY), a garbage !
A remedy : cov(X,Y) = E(XY) - E(X) E(Y)

Correlation coefficient= cov(X,Y)/SD(X)SD(Y), where cov(X,Y)= E [(X-mean) (Y-mean)]

- Use the independence example (from lecture 4) to construct positive correlation by cutting of the points on the edge
- Do a step by step calculation of corr. Coeff.
- Do a plotting showing 4 quadrants by drawing vertical and horizontal lines passing the means.


Product=negative
+)
ower than
an
higher than
an
Product=positive
(+, +)
X, Y both higher than mean
(-, -)
X,Y both lower than mean
oduct=positive
(+, -)
X higher than mean, Y lower than mean
Product=neg

sistency : if use n-1 in doing SD, then use n-1 for averaging prod

Conceptual : Step by step for Corr Coeff.

tdzd = standardized (remove mean, divided by SD)

X-EX	y	Y-EY	stdzd x	stdzd y	product
-5	4	-1.5	-5/SD(X)	-1.5/SD(Y)	7.5/SD(X)SD(Y
-3	3	-2.5	-3/SD(X)	-2.5/SD(Y)	7.5/SD(X)SD(Y
-1	6	0.5	-1/SD(X)	0.5/SD(Y)	-0.5/SD(X)SD(
1	5	-0.5	1 /SD(X)	-0.5/SD(Y)	-0.5/SD(X)SD(
3	8	2.5	3/SD(X)	2.5/SD(Y)	7.5/SD(X)SD(Y
5	7	1.5	5/SD(X)	1.5/SD(Y)	7.5/SD(X)SD(Y

=7 E Y=5.5 SD(X) =sqrt(35/3)=3.4 SD(Y)=1.7 Corr=(29/6)/3.4 tim 1.7=29/35=0.828

Use population version, so divided by n

sistency : if use n-1 in doing SD, then use n-1 for averaging prod

actice: Step by step for Covariance,varianc and correlation coefficients.

y	X-EX	Y-EY	product	$(X\text{-}EX)^2$	$(Y\text{-}EY)^2$
4	-5	-1.5	7.5	25	2.25
3	-3	-2.5	7.5	9	6.25
6	-1	0.5	-0.5	1	0.25
5	1	-0.5	-0.5	1	0.25
8	3	2.5	7.5	9	6.25
7	5	1.5	7.5	25	2.25

=7 E Y=5.5 SD(X) =3.4 SD(Y)=1.7 Cov =29/6 Corr=0.82

sqrt(35/3)=3.4

Use population version, so divided by n

=cov/sd(x)sd(y)

Positive correlations

- Corr = 0.9
- Corr =.8
- Corr = .5

On line illustration with

Xlispstat, using

(bi-normal r n)

Algebra for Variance, covariance

- $Var(X+Y) = Var\ X + Var\ Y + 2\ cov\ (X,Y)$
- $Var(X) = Cov\ (X, X)$
- $Var\ (X+a) = Var\ (X)$
- $Cov\ (X+a, Y+b) = Cov(X,Y)$
- $Cov\ (aX, bY) = ab\ Cov(X,Y)$
- $Var(aX) = a^2\ Var\ (X)$
- $Cov(X+Y, Z) = cov(X,Z) + cov\ (Y,Z)$
- $Cov\ (X+Y, V+W) = cov(X,V) + cov\ (X, W) + cov\ (Y, W) + cov(Y,W)$

TRICK : pretend all means are zero;
$(X+Y)(V+W) = XV + XW + YW + YW$

Stock prices are correlated

- Effect on variance of option 1 and option
- Recall the problem

Example

- Stock A and Stock B
- Current price : both the same, $10 per share
- Predicted performance a week later: similar
- Both following a normal distribution with
- Mean $10.0 and SD $1.0
- You have twenty dollars to invest
- Option 1 : buy 2 shares of A portfolio mean=?, SD=?
- Option 2 : buy one share of A and one share of B
- Which one is better? Why?

Assume that there is a correlation of .8 between the prices of stock A and stock B a week later

Better? In what sense?

- What is the probability that portfolio value will be higher than 22 ?
- What is the probability that portfolio value will be lower than 18?
- What is the probability that portfolio value will be between18 and 22?

(How about if correlation equals 1 ?)

For option 2, the key is to find variance

- Let X be the future price of stock A
- Let Y be the future price of stock B
- Let T = X + Y portfolio value
- E T = E X + E Y (same as done before)
- Var T = Var X + Var Y + 2 cov (X, Y)
- Cov (X, Y) = correlation times SD(X) SD(Y) = .8 times 1 times 1 = 0.8
- Var X = $(SD (X))^2 = 1^2 = 1$; Var Y = 1
- Var T = 1 + 1 + 2 times .8 = 3.6 (compared to 1+1=2 when assuming independence)

D (T) = squared root of 3.6=1.9 is still less than SD for option 1

Index

- Index is usually constructed as a weighted average of several variables
- Stock index
- Course grade = .2 midterm+ .45 Final + .15 HW + .2 LAB
- Find SD of course grade
- Independence; dependence