

Lecture 7 Accuracy of sample mean $\bar{X}$

$\text{Var}(\bar{X}) = \text{Var}(X)$ divided by sample size n

What is $\bar{X}$? Called sample mean.

Standard error of the mean $= \text{SD}(\bar{X})$

$= \text{SD}(X)$ divided by **squared root of n**

As sample size increases, the sample mean become more and more accurate in estimating the population mean

- Sample size needed to meet accuracy requirement

Sum of independent random variables from box A, mean μ , SD s

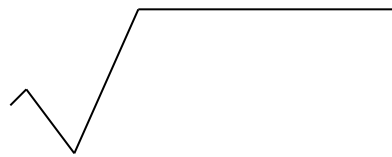
- Sample size $n = 2$
- Let X_1, X_2 be the length of two independent phone calls (the phone call example : mean=2.5 minutes, SD=1 minute)
- Let $T = X_1 + X_2$
- What is the variance of T ? ($=1+1$) SD of T ? ($=\sqrt{2}$)
- Sample size $n=4$
- $T = X_1 + X_2 + X_3 + X_4$
- $\text{Var}(T) = 1 + 1 + 1 + 1 = 4$ $\text{SD}(T) = \sqrt{4} = 2$

T = sum of n independent draws

$\bar{X}$ = average of n independent draws

$$\text{Var}(T) = n s^2 ;$$

$$\text{SD}(T) = \text{sqrt } n \text{ times } s ;$$



=Squared root
sqrt

$$\bar{X} = T/n$$

$$\text{SD}(\bar{X}) = \text{SD}(T) / n = s / \text{sqrt } n$$

Difference of two sample means

- Assume independence
- Example: Does body position for epidural injections affect labor pain? Page 303, textbook, National Women's Hospital in Auckland : number of spinal segments blocked by the anesthetic
- Lying group : $n_1 = 48$, $\text{mean} = 8.8$, $\text{SD} = 4.4$
- Sitting group : $n_2 = 35$, $\text{mean} = 7.1$, $\text{SD} = 4.5$
- Question : do we have enough evidence to tell which is better?
- SD of difference in mean = .9909, two-standard error interval : $[-0.3, 3.7]$. Ans. No.

Another example: from page 305 of book

- Individual 2-SE interval
- 2-SE interval for the difference
- Overlapping case
- Mean $\bar{X}=14$, SE =1.2
- Mean $\bar{Y}=10$, SE=1.1
- SE for difference of mean = $\text{sqrt}(1.2^2+1.1^2)=1.627$
- $\text{SD}(\bar{X}-\bar{Y}) = \text{sqrt} [\text{SD}(\bar{X})^2 + \text{SD}(\bar{Y})^2]$
< $\text{SD}(\bar{X}) + \text{SD}(\bar{Y})$