

Lecture 8 Confidence interval

- Parameter and estimate : population mean, sample mean
- Standard error of the mean (SE) Which one?
- 95% confidence interval
- Confidence level (coefficient) $1-\alpha$
- Using z score
- Two sample problem; matched sample problem
- Illustration with Computer simulation

Standard error refers to the standard deviation of an estimator

Measurement error= Reading from an instrument - true value

One biotech company specializing microarray gene expression profiling claims they can measure the expression level of a gene with an error of size .1 (that is, after testing their method numerous times, they found the standard deviation of their measurement errors is 0.1) The distribution of errors follow normal distribution with mean 0 (unbiased).

Cells from a tumor tissue of a patient are sent to this company for microarray assay. To assure consistency, the company repeat the assay 4 times. The result of one gene, P53 (the most well-studied tumor suppressor gene), is 1.1, 1.4, 1.5, 1.2.
Estimate the true level of P53. What is the SE ? Find a 95% confidence interval.

Answer :

- The sample mean is $(X_1 + X_2 + X_3 + X_4)/4 = \bar{X}$
- Standard deviation of each X random variable is .1
- So $SD(\bar{X})$ is $.1/\sqrt{4} = 0.05$, this is the SE
- Two SE is $0.05 \text{ times } 2 = .1$
- So 95% confidence interval runs from $1.3 - .1$
- To $1.3 + .1$; that is 1.2 to 1.4

- Another cell sample is prepared from a healthy person. The assay results are 1.5 1.6 1.4 1.7.

Question : Does the tumor cell have a lower P53 level? Find a 95% confidence interval for The difference.

Answer

The estimator is the difference $\bar{X} - \bar{Y}$,
Where the $\bar{Y}$ is average of 4 random variables
again, so it should have the same
standard deviation as SD of $\bar{X}$.

$$\text{Now } SD(\bar{X} - \bar{Y}) = \sqrt{\text{var}(\bar{X}) + \text{var}(\bar{Y})}$$
$$= \sqrt{.05^2 + .05^2} = 0.0707$$

So SE is 2 times $.0707 = 0.1414$

95% confidence interval goes from $(1.3-1.55) - .1414$
To $(1.3-1.55) + .1414$; that is from $-.2914$ to -0.0586

Using 95% confidence interval, there is a statistically
significant reduction in P53 expression

Rationale behind

- Box A , true mean=1.35 SD=.1
- Box B true mean= 1.50 SD=.1
- Generate sample from box A
- Generate sample from box B
- Using computer
- Find the difference of mean, check 2 SE interval to see if covering the true difference
- Repeat it many times to see how often the interval covers the true difference

Changing the confidence level

- For a 95% confidence interval, use 2SE rule
- This is because of normal distribution - central area $P\{-2 < Z < 2\}$ is about .95
- For 80% confidence interval, you look for
- $P\{-c < Z < c\} = .80$; equivalently $P\{Z < c\} = .80 + .10 = .90$ so c must be 1.28