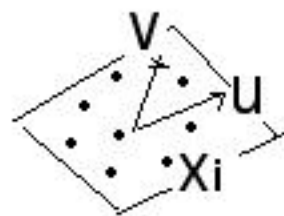


RANK 2  $x_i$



$$x_i = c_i v + d_i u$$

where  $v$  and  $u$  are two vectors ( $p$ -dim)

$c_i$  and  $d_i$  are scalars

for  $i = 1, \dots, n$

so the data matrix (means removed)  
becomes

$$X = c v' + d u' ; \text{ where } \\ c = (c_1, \dots, c_n)' \text{ and } d = (d_1, \dots, d_n)'$$

If  $v$  and  $u$  are orthogonal and have unit length,  $\|v\| = \|u\| = 1$ , then

$$X'X = c^2 v v' + d^2 u u'$$

This says that  $v$  and  $u$  are two eigenvectors (easy to check that

$X'X v$  is proportional to  $v$ )

Note :  $X'X = n$  covariance matrix