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Statistics 19 **Instructor: Nicolas Christou**

Lower and upper bounds for the price of a European call and put

A. Lower bound for the price of a European call:

	Time $t=0$	Payoff at time $t=1$	
		$S_1 > E$	$S_1 \leq E$
Portfolio A:			
Buy 1 call	$-C$	$S_1 - E$	0
Cash (lend)	$-\frac{E}{1+r}$	$+E$	$+E$
Total		S_1	E
Portfolio B:			
Buy 1 share	$-S_0$	S_1	S_1

B. Lower bound for the price of a European put:

	Time $t = 0$	Payoff at time $t = 1$	
		$S_1 \geq E$	$S_1 < E$
Portfolio A:			
Buy 1 put	$-P$	0	$E - S_1$
Buy 1 share	$-S_0$	S_1	S_1
Total		S_1	E
Portfolio B:			
Cash (lend)	$-\frac{E}{1+r}$	$+E$	$+E$

C. Upper bound for the price of a European call:

No matter what happens, $C \leq S_0$

If not, there will be an opportunity for a riskless profit by buying the stock and selling the call option. How? Suppose $C > S_0$.

	Time $t = 0$	Payoff at time $t = 1$	
		$S_1 > E$	$S_1 \leq E$
Sell 1 call	C	$E - S_1$	0
Buy 1 stock	$-S_0$	S_1	S_1
Total	$C - S_0$	E	S_1

D. Upper bound for the price of a European put:

No matter what happens, $P \leq \frac{E}{1+r}$.
If not, there will be an opportunity for a riskless profit by selling the put and investing the proceeds at the risk free interest rate. How? Suppose $P > \frac{E}{1+r}$.

	Time $t = 0$	Payoff at time $t = 1$	
		$S_1 \geq E$	$S_1 < E$
Sell 1 put	$P > \frac{E}{1+r}$	0	$S_1 - E$

Put-Call Parity

This is an important relationship between the price of a put and the price of the call. A put and the underlying stock can be combined in such a way that they have the same payoff as a call at expiration. Consider the following two portfolios:
Portfolio A: Buy the call and lend an amount of cash equal to $\frac{E}{1+r}$.
Portfolio B: Buy the stock, buy the put.

This is shown on the table below:

	Time $t = 0$	Payoff at time $t = 1$	
		$S_1 > E$	$S_1 \leq E$
Portfolio A:			
Buy 1 call	$-C$	$S_1 - E$	0
Lend cash	$-\frac{E}{1+r}$	E	E
Total	$-C - \frac{E}{1+r}$	S_1	E

		$S_1 \geq E$	$S_1 < E$
Portfolio B:			
Buy 1 put	$-P$	0	$E - S_1$
Buy 1 stock	$-S_0$	S_1	S_1
Total	$-P - S_0$	S_1	E