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Constructing the optimal portfolios - Single index model
Calculation steps

a. **Step 1:** By regressing the returns of each stock on the returns of the market obtain for each stock: $\hat{\beta}, \hat{\alpha}, \hat{\sigma^2_\epsilon}$ and construct the table below:

Stock i	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\bar{R}_i$	$\hat{\sigma^2_{\epsilon i}}$	$\frac{R_i - R_f}{\hat{\beta}_i}$
IBM					
GOOGLE					
$\vdots$					

b. **Step 2:** Sort the table above based on the excess return to beta ratio:

$$\frac{\bar{R}_i - R_f}{\beta_i}$$

c. **Step 3:** Create 5 columns to the right of the sorted table as follows:

Stock i	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\bar{R}_i$	$\hat{\sigma^2_{\epsilon i}}$	$\frac{R_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\hat{\sigma^2_{\epsilon i}}}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\hat{\sigma^2_{\epsilon j}}}$	$\frac{\hat{\beta}_i^2}{\hat{\sigma^2_{\epsilon i}}}$	$\sum_{j=1}^{i} \frac{\hat{\beta}_j^2}{\hat{\sigma^2_{\epsilon j}}}$	C_i
						k_1	k_1	l_1	l_1	C_1
						k_2	$k_1 + k_2$	l_2	$l_1 + l_2$	C_2
						k_3	$k_1 + k_2 + k_3$	l_3	$l_1 + l_2 + l_3$	C_3
						$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
						k_n	$k_1 + k_2 + \cdots + k_n$	l_n	$l_1 + l_2 + \cdots + l_n$	C_n

Note: Compute all the $C_i, i = 1, \cdots, n$ (last column) as follows:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma^2_{\epsilon j}}}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma^2_{\epsilon j}}} = \frac{\sigma_m^2 \times \text{COL2}}{1 + \sigma_m^2 \times \text{COL4}}$$

Once the $C_i's$ are calculated we find the C^* as follows:

If short sales are allowed, C^* is the last element in the last column.
If short sales are not allowed, C^* is the element in the last column for which $\frac{\bar{R}_i - R_f}{\beta_i} > C^*$.

In both cases the $z_i's$ are computed as follows

$$z_i = \frac{\beta_i}{\sigma^2_{\epsilon i}} \left(\frac{\bar{R}_i - R_f}{\beta_i} - C^* \right)$$

and the $x_i's$

$$x_i = \frac{z_i}{\sum_{i=1}^{n} z_i}$$