EXERCISE 1
A new profit-sharing plan was introduced at an automobile parts manufacturing plant last year. Both management and union representatives were interested in determining how a worker’s years of experience influence his or her productivity gains. After the plan had been in effect for a while, the data shown below were collected:

<table>
<thead>
<tr>
<th>Years of experience (x)</th>
<th>Number of units daily (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.1</td>
<td>110</td>
</tr>
<tr>
<td>7.0</td>
<td>105</td>
</tr>
<tr>
<td>18.6</td>
<td>115</td>
</tr>
<tr>
<td>23.7</td>
<td>127</td>
</tr>
<tr>
<td>11.5</td>
<td>98</td>
</tr>
<tr>
<td>16.4</td>
<td>103</td>
</tr>
<tr>
<td>6.3</td>
<td>87</td>
</tr>
<tr>
<td>15.4</td>
<td>108</td>
</tr>
<tr>
<td>19.9</td>
<td>112</td>
</tr>
</tbody>
</table>

For your convenience:

\[
\sum_{i=1}^{9} y_i = 965, \sum_{i=1}^{9} x_i = 133.9, \sum_{i=1}^{9} y_i^2 = 104469, \sum_{i=1}^{9} x_i^2 = 2258.73, \sum_{i=1}^{9} x_i y_i = 14801.2.
\]

a. Construct a scatterplot of the number of units manufactured daily on the years of experience on the assembly line.

b. Find the least-squares regression line.

c. Predict the number of units manufactured daily by an employee who has 10 years of experience on the assembly line.

EXERCISE 2
Suppose that in the simple regression model the intercept \( \beta_0 \) is missing. The model regression equation is then given by \( y_i = \beta_1 x_i + \epsilon_i \). Find the least squares estimate of the slope \( \beta_1 \).

EXERCISE 3
For the regression model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \) show that the sum of the residuals is always equal to zero, i.e. \( \sum_{i=1}^{n} \epsilon_i = 0 \).

EXERCISE 4
Show that \( \hat{\beta}_1 \) (the estimate of the slope) in the regression model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \) is also equal to the following expression:

\[
\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}
\]

To do this expand the above expression and show that it is equal to the expression we discussed in class:

\[
\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2}
\]

Exercise 5
Suppose in the model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \), where \( i = 1, \cdots, n \), \( E(\epsilon_i) = 0, \text{var}(\epsilon_i) = \sigma^2 \) the measurements \( x_i \) were in inches and we would like to write the model in centimeters, say, \( z_i \). If one inch is equal to \( c \) centimeters (\( c \) is known), we can write the above model as follows \( y_i = \beta_0^* + \beta_1^* z_i + \epsilon_i \).

a. Suppose \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \) are the least squares estimates of \( \beta_0 \) and \( \beta_1 \) of the first model. Find the estimates of \( \beta_0^* \) and \( \beta_1^* \) in terms of \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \).

b. Show that the value of \( R^2 \) remains the same for both models.

c. Find the variance of \( \hat{\beta}_1^* \).

Exercise 6
Consider the regression model \( y_i = (\beta_0 + \beta_1 \bar{x}) + \beta_1 (x_i - \bar{x}) + \epsilon_i \)

This model is called the centered version of the regression model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \) that was discussed in class. If we let \( \gamma_0 = \beta_0 + \beta_1 \bar{x} \) we can rewrite the centered version as \( y_i = \gamma_0 + \beta_1 (x_i - \bar{x}) + \epsilon_i \). Find the least squares estimates of \( \gamma_0 \) and \( \beta_1 \).
EXERCISE 7
Consider the regression model \( y_i = \beta_1 x_i + \epsilon_i \) with \( \epsilon \sim N(0, \sigma^2) \).

a. Show that \( \hat{\beta}_1 \) is an unbiased estimator of \( \beta_1 \).

b. Find the variance of \( \hat{\beta}_1 \).

EXERCISE 8
Consider the regression model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \). Show that \( \text{Cov}(\bar{Y}, \hat{\beta}_1) = 0 \) where \( \bar{Y} \) is the sample mean of the \( y \) values, and \( \hat{\beta}_1 \) is the estimate of \( \beta_1 \).